

# Chapter 24

## Undrained Behaviour of Soils

If no drainage is possible from a soil, because the soil has been sealed off, or because the load is applied so quickly and the permeability is so small that there is no time for outflow of water, there will be no consolidation of the soil. This is the *undrained behavior* of a soil. This chapter contains an introduction to the description of this undrained behavior.

### 24.1 Undrained Tests

In an undrained triaxial test on a saturated clay each increase of the cell pressure will lead to an increase of the pore water pressure. As discussed in the previous chapter this can be described by Skempton's formula

$$\Delta p = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]. \quad (24.1)$$

The coefficient  $B$  can be expected to be about

$$B = \frac{1}{1 + n\beta K}, \quad (24.2)$$

where  $\beta$  is the compressibility of the pore fluid (including possible air bubbles) and  $K$  is the compression modulus of the grain skeleton. The value of the coefficient  $B$  will be close to 1, as the water is practically incompressible.

Increasing the cell pressure can be expected to result in an increment of the pore pressure by the same amount as the increment of the cell pressure, or slightly less, and thus there will be very little change in the effective stresses. If there is a possibility for drainage, and there is sufficient time for the soil to drain, the pore pressures will be gradually reduced, with a simultaneous increase of the effective stresses. This is

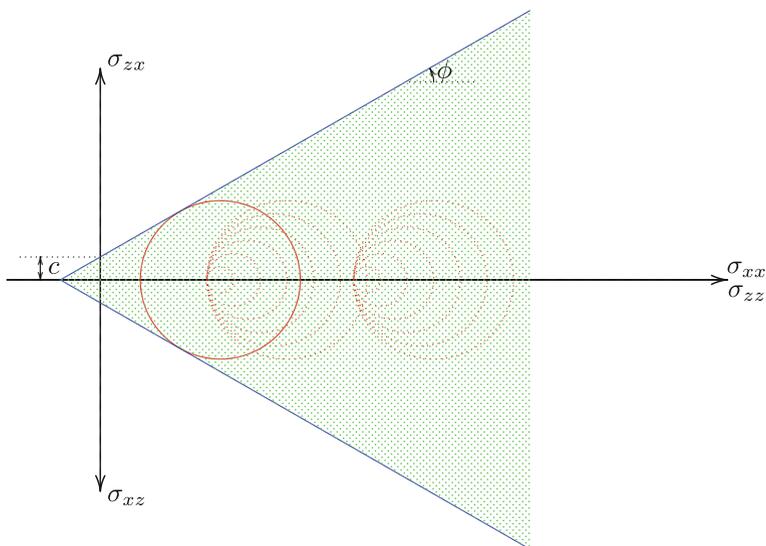
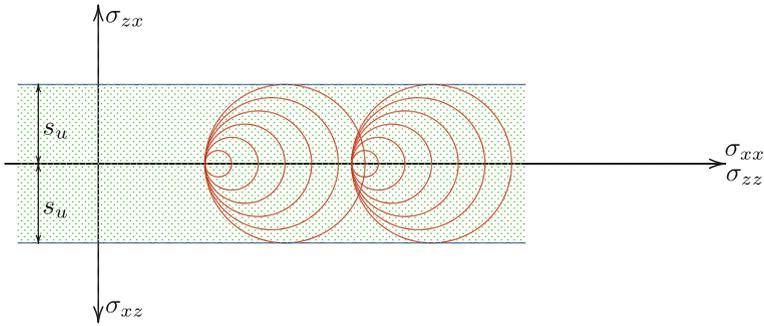


Fig. 24.1 Mohr circles for two undrained tests

the consolidation process. If there is no possibility for drainage, because the sample has been completely sealed off, or because the test is done so quickly that there is no time for consolidation, the test is called *unconsolidated*. In the second stage of a triaxial test, in which only the vertical stress is increased, distinction can also be made in drained or undrained tests. If in this stage no drainage can take place, the test is called *unconsolidated undrained* (a UU-test). If a second UU-test is done at a higher cell pressure, the only difference with the first test will be that the pore pressures are higher. The effective stresses in both tests will be practically the same. If the test results are plotted in a Mohr diagram, there would be just one critical circle for the effective stresses, but in terms of total stresses there will be two clearly distinct circles, of practically the same magnitude, see Fig. 24.1. In this figure the critical Mohr circles for the total stresses in the two tests have been dotted. The critical circles for the effective stresses can be obtained by subtracting the pore pressure, and these are represented by full lines. The two circles practically coincide, if the sample is saturated with water. These test results appear to be insufficient to determine the shear strength parameters  $c$  and  $\phi$ , because only one critical circle for the effective stresses is available. In order to determine the values of  $c$  and  $\phi$  the sample should be allowed to consolidate after the first loading stage in at least one of the tests, so that the isotropic effective stress at the beginning of the second stage, the vertical loading, is different in the two tests. This would mean that this test should be a *Consolidated Undrained* test, or a CU-test.

Admitting that undrained tests can not be used to determine the correct values of the shear strength parameters  $c$  and  $\phi$ , they may still be very useful, because in engineering practice there are many situations in which no (or very little) drainage



**Fig. 24.2** Mohr circles for total stresses

will occur, for instance in case of loading of a soil of very low permeability (clay) for a short time. Examples are a temporary loading for some building operation, or a temporary excavation for the construction of a pipe line. In order to predict the behavior of the clay in these circumstances it makes sense to just consider the total stresses, and to make use of the results of an undrained test, analyzing the test results in terms of total stresses also. That there may be considerable pore pressures in the test as well as in the field, is perhaps interesting, but irrelevant if the period of loading is so short that no consolidation can occur.

The analysis of the tests in terms of total stresses is illustrated in Fig. 24.2. As explained above, all critical stress circles will be of the same magnitude, and when the results are interpreted in terms of total stresses only it seems that the friction angle  $\phi$  is practically zero. The strength of the soil can be characterized by a cohesion only, which is then usually denoted as  $s_u$ , the *undrained shear strength* of the soil. The analysis, in which the friction of the material and the pore pressures are neglected, is called an *undrained analysis*. Because the analysis of the safety of a structure on a purely cohesive material (with  $\phi = 0$ ) is much simpler than the analysis for a material with internal friction, an undrained analysis is often used in engineering practice.

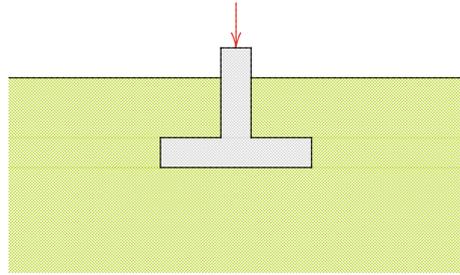
In general, the undrained shear strength  $s_u$  is simply determined as one half of the maximum shear stress, expressed in total stresses,

$$s_u = \frac{1}{2}(\sigma_1 - \sigma_3). \tag{24.3}$$

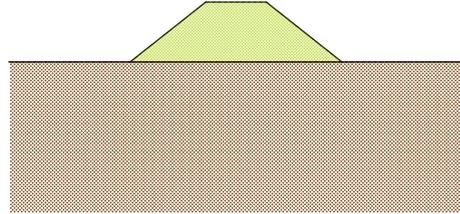
The applicability of undrained tests, and the use of undrained strength parameters is also justified if it can be expected that the most critical situation will be the undrained state immediately after loading. In many cases of loading of a soil by a constant load, it can be expected that the largest pore pressures will be developed immediately after loading, and that these pore pressures will gradually dissipate during consolidation of the soil, with the effective stresses increasing.

For instance, in the case of a permanent load applied to a shallow foundation slab, see Fig. 24.3, it can be expected that pore pressures will be developed below the foundation, and that these pore pressures will dissipate in course of time due to

**Fig. 24.3** Shallow foundation



**Fig. 24.4** Embankment

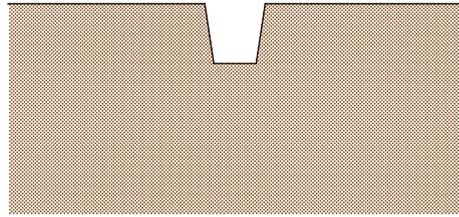


consolidation. If the load remains constant, it can be expected that the pore pressures are highest, and thus the effective stresses are smallest, just after the application of the load.

Later, after consolidation, the effective stresses will be higher, so that the Mohr circle will be shifted to the right. This means that the most critical situation occurs immediately after application of the load, in the undrained state. If the structure is safe immediately after application of the load it will certainly be safe at later times, when the pore pressures have been dissipated, the effective stresses have increased, and thus the strength of the soil has been further developed.

In the case of the construction of an embankment, for a dyke or a road, an undrained analysis may also be sufficient for the analysis of the stability of the embankment itself, see Fig. 24.4. In many cases it can be assumed that the construction of the embankment is one of the most critical phases in its lifetime. If the embankment “survives” the construction, then it will probably will be stable forever. The pore pressures are largest during the construction of the embankment. Later these will be reduced, the effective stresses will increase, and therefore the shear strength will increase. In many cases this additional strength is sufficient to even accept future additional loadings, for instance by water pressures against the slope of the dike, or by traffic, in case of a highway. In some exceptional cases, of very soft soils, with a very low permeability, there may be additional undrained creep deformations, prior to the effect of consolidation, so that the pore pressures may increase in the first few days or weeks after construction. In some cases this has indeed resulted in failure of an embankment a few days after its construction.

Of course it is not sufficient to assume, without further proof, that the reduction of the pore pressures, caused by consolidation, will be sufficient to accommodate the additional pore pressures due to the additional loading. A dyke is built to withstand

**Fig. 24.5** Excavation

the forces of the water during a storm with high water levels, and the behavior of the dyke under these conditions needs careful analysis. Immediately after application of the load, in this case the water pressure against the slope of the dyke, the soil may be considered as undrained, but after some days of high water the dyke must still be stable. During prolonged periods of high water, the pore pressures in the dyke may gradually increase, because of inflow of water into the dyke body, and an unsafe situation may be created by the reduction of the effective stresses in the dyke. An undrained analysis of the dyke stability may be one element in its design, but an effective stress analysis, considering various combinations of loading and drainage, must also be performed.

An undrained analysis is unsafe if it is to be expected that the pore pressures will increase after the construction. As an example one may consider the case of an excavation, see Fig. 24.5. The excavation can be considered as a negative load, which will result in decreasing total stresses, and therefore decreasing pore pressures immediately after the excavation. Due to consolidation, however, the pore pressures later will gradually increase, and they will ultimately be reduced to their original value, as determined by the hydrologic conditions. Thus the effective stresses will be reduced in the consolidation process, so that the shear strength of the soil is reduced. This means that in the course of time the risk of a sliding failure may increase. A trench may be stable for a short time, especially because of the increased strength due to the negative pore pressures created by the excavation, but after some time there may be a collapse of the slopes. This may be very dangerous for the people at work in the excavation, of course.

## 24.2 Undrained Shear Strength

For the comparison of drained and undrained calculations, and for the actual calculation in an undrained analysis, it is often necessary to determine the undrained shear strength  $s_u$  of a soil, from the basic shear strength parameters  $c$  and  $\phi$ . This can be done by noting that in a saturated soil there can be practically no volume change in undrained conditions, so that the isotropic effective stress remains constant. Thus the average effective stress remains constant, and this means that the location of the Mohr circle is constrained. Usually the state of stress in the soil is such that the vertical stresses are reasonably well known, because of the weight of the soil and a possible

load. If the pore water pressure is also known, this means that it can be assumed that the vertical effective stress  $\sigma'_{zz}$  is known. Usually the two horizontal stresses will be equal, and their magnitude may be estimated (or perhaps measured), even while that is not always very easy. Here it is assumed that the horizontal effective stress  $\sigma'_{xx}$  is also known. Thus the average effective stress,  $\frac{1}{3}(\sigma'_{zz} + 2\sigma'_{xx})$  is known. If the soil is loaded this average effective stress will remain constant,

$$\sigma'_0 = \frac{1}{3}(\sigma'_{zz} + 2\sigma'_{xx}) = \text{constant.} \quad (24.4)$$

In case of failure of the soil the combination of the major principal stress  $\sigma'_1$  and the minor principal stress  $\sigma'_3$  must be such that the Mohr-Coulomb failure criterion is satisfied, i.e., with (20.12),

$$\left(\frac{\sigma'_1 - \sigma'_3}{2}\right) - \left(\frac{\sigma'_1 + \sigma'_3}{2}\right) \sin \phi - c \cos \phi = 0. \quad (24.5)$$

Because  $\sigma'_1 + \sigma'_3 = \frac{2}{3}(\sigma'_1 + 2\sigma'_3) + \frac{1}{3}(\sigma'_1 - \sigma'_3)$  this can also be written as

$$\left(1 - \frac{1}{3} \sin \phi\right) \left(\frac{\sigma'_1 - \sigma'_3}{2}\right) - \left(\frac{\sigma'_1 + 2\sigma'_3}{3}\right) \sin \phi - c \cos \phi = 0. \quad (24.6)$$

Because the average effective stress can not change in undrained conditions, we have, before and after the application of the load,

$$\frac{1}{3}(\sigma'_1 + 2\sigma'_3) = \sigma'_0, \quad (24.7)$$

where  $\sigma'_0$  is a given value, determined by the initial stresses, see (24.4).

From (24.6) to (24.7) the undrained shear strength  $s_u$  is found to be

$$s_u = \frac{\sigma'_1 - \sigma'_3}{2} = c \frac{\cos \phi}{1 - \frac{1}{3} \sin \phi} + \sigma'_0 \frac{\sin \phi}{1 - \frac{1}{3} \sin \phi}, \quad (24.8)$$

This formula enables to estimate the undrained shear strength if the drained shear strength parameters  $c$  and  $\phi$  are known, as well as the initial average effective stress  $\sigma'_0$ . The relation is illustrated in Fig. 24.6. In this figure a number of Mohr circles for the effective stresses are shown, on the basis of the assumption that the average effective stress  $\sigma'_0$  remains constant. The total stresses always differ from the effective stresses by the (unknown) value of the pore water pressure. The location of the total stress circles is not known, and not relevant. Their magnitude is always equal to the magnitude of the corresponding effective stress circle, as the pore pressure increases all normal stresses, both  $\sigma'_{xx}$  and  $\sigma'_{zz}$ .

Equation (24.8) indicates that  $s_u = c$  if  $\phi = 0$ , as could be expected. If  $\phi > 0$  the undrained shear strength  $s_u$  increases with the average effective stress  $\sigma'_0$ . This

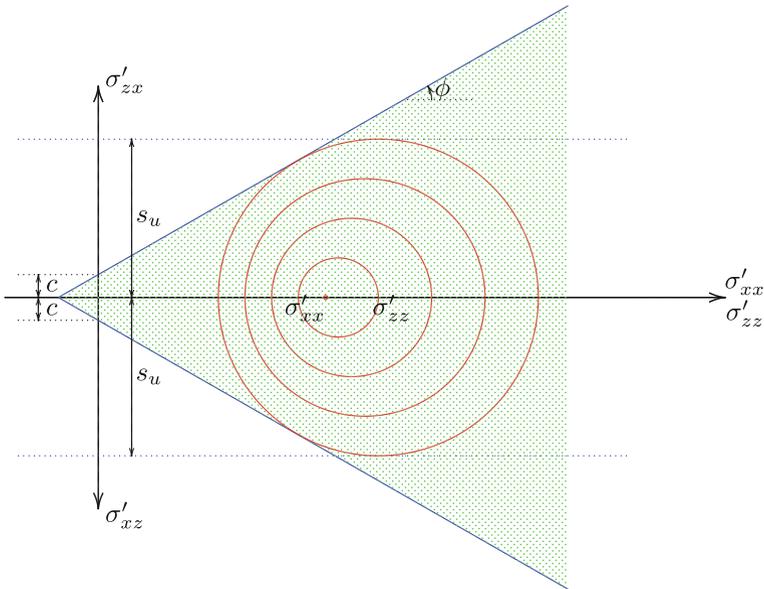


Fig. 24.6 Mohr circles for undrained behavior

means that a preload, followed by consolidation, has a positive effect on the undrained strength of the soil.

It should be noted that in the derivation of Eq. (24.8) it has been assumed that a volume change can be produced only by a change of the average effective stress. This means that effects such as anisotropy, dilatancy and contractancy have been disregarded. That is an important restriction, and it means that the formula is a first approximation only.

*Example 24.1* A consolidated undrained triaxial test is done on a clay sample. The cell pressure is 50 kPa, and the sample is found to fail when the additional axial stress is 170 kPa. What is the undrained shear strength  $s_u$ ?

**Solution**

In this case  $\sigma_3 = 50$  kPa and  $\sigma_1 = 220$  kPa. The undrained shear strength in this case is  $s_u = \frac{1}{2}(\sigma_1 - \sigma_3)$ , or  $s_u = 85$  kPa.

*Example 24.2* Of a certain soil it is known from previous studies that  $c = 20$  kPa and  $\phi = 30^\circ$ . An undrained analysis must be made for a case in which the original vertical effective stresses are 80 kPa, and the horizontal effective stresses are estimated to be 40 kPa. What is the value of  $s_u$  to be used?

**Solution**

The appropriate formula is Eq.(24.8). In this case the average effective stress  $\sigma'_0 = 53.3$  kPa. Using the given values of  $c$  and  $\phi$  the formula gives  $s_u = 52.8$  kPa.

It may be noted that if the original horizontal stress was equal to the vertical effective stress the answer would be  $s_u = 68.8$  kPa. This illustrates the important effect of the horizontal stresses.

It may also be noted that all these results can be verified using a construction of the appropriate Mohr circle, see Fig. 24.6.