Chapter 12
Geometric Optics

Abstract In this chapter, light is considered as rectilinear rays radiated in all directions from a point at an object. The rays are refracted at air/glass interfaces according to the laws of refraction discussed in a previous chapter. We show that all rays emerging from a point object entering a spherical interface will collect (and use equal time) to a focal point to a good approximation for common conditions. From this treatment, the "lens makers' formula" and "lens formula" are derived, and simple but very useful "ray optics" rules are established. We show how convex and concave lenses can be combined for many different purposes, e.g. for making telescopes, microscopes and loupe (magnifying glass). Light-collecting efficiency, aperture, f-number, depth of view, as well as image quality are discussed. The chapter concludes with a description of the optics of the human eye, defines "near point" and "far point", and shows how spectacles can be used to improve vision in common cases.

12.1 Light Rays

In this chapter, we will see how lenses can be used to make a camera, telescope and microscope, even glasses (spectacles). We will use the term "rays" in the sense of a thin bundle of light that proceeds along a straight line in the air and other homogeneous media. Figure 12.1 illustrates our view of how light rays behave.

The concept of light rays is at variance with the notion that light may be regarded as energy transported by plane electromagnetic waves. Plane waves, in principle, have infinite extent across the direction of propagation of wave. We can reasonably expect that when we make a narrow light beam, it would behave much like a plane wave within the expanse of the beam. A laser beam apparently behaves exactly as we expect light rays to behave.

However, we will see in Chap. 13 that a laser beam does not travel in straight lines all the time. Diffraction (will be treated in a later chapter) can lead to unexpected results. In addition, there is a huge difference between electromagnetic waves in a laser beam and the light from sunlight or lamplight. The laser light usually has a fixed polarization that we find everywhere along the beam, and the intensity is quite stable

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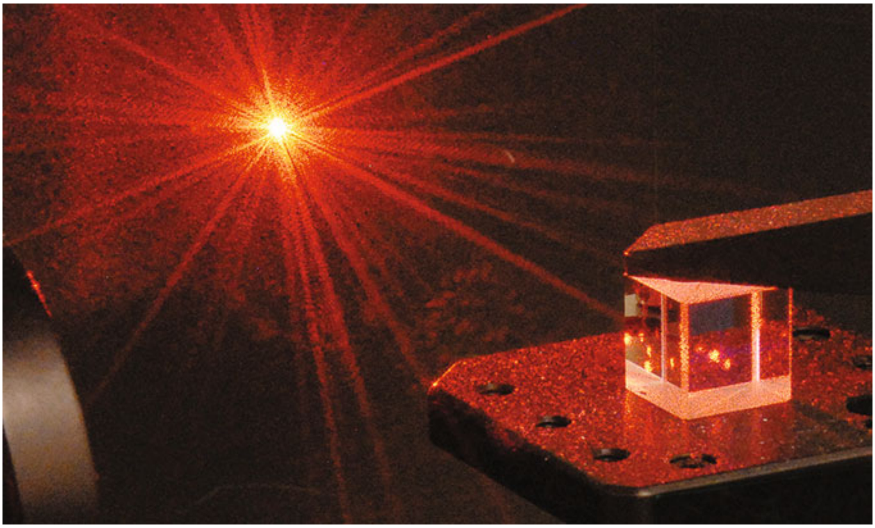


Fig. 12.1 "Light ray" is a useful term in geometric optics, although it is useless in certain other contexts

and varies in a well-defined way across the beam of light. The light from the sun or lamps is far more chaotic in all respects and is much more difficult to describe than laser light. We will return to this in Chap. 15 when we come to treat the phenomenon of "coherence".

Despite the fact that light from the sun and from lamps is very complicated, it appears that the phrase "ray of light" is serviceable and useful as long as we work with lenses and mirrors which are almost flat surfaces over areas which are "several times" (at least ten?) the wavelength. Furthermore, the thickness of the interface between, e.g. air and glass, must be very thin in comparison with the wavelength. You may recognize the criteria we set up in Chap. 10 when we looked at the rules for reflection and transmission of electromagnetic waves at interfaces between two media.

This means, among other things, that Snel's refraction law will apply locally to any area on ordinary lenses. Even for a 5 mm Ø lens, the diameter is 10,000 wavelengths, and most lenses are even larger.

When it comes to water drops, the situation is totally different. For ordinary large water drops, we can apply reflection and refraction laws to calculate, for example, the appearance of the rainbow, but for very small water drops the approximation totally breaks. Then we have to return to Maxwell's equations with curved interfaces, and the calculations will become extremely extensive. The spread of light from such small drops is called Mie scattering. Only after the arrival of powerful computers have we been able to make good calculations for Mie scattering. Prior to that, Mie scattering was seen only as an academic curiosity.

Before we begin to discuss the concept of light rays, I want to point out a challenge. We have previously seen that Maxwell's equations, together with energy conservation, provide the magnitude and direction of reflected and transmitted electric fields after plane electromagnetic waves reach a plane interface between two different dielectric media. From Maxwell's equations, both reflection law and Snel's law of refraction follow. However, we have also seen that both these laws can be derived from the principle of minimum time (Fermat's principle), or more correctly, the principle that the time the light uses along its path has an extreme value. In other words, we have two different explanations. What should we count as more fundamental? It is not particularly helpful to say this, but here is something for you to ponder over.

If we have a light source that emits light in all directions, we can see that the light "chooses" to follow the path that takes the shortest time if we select beforehand the position of the light source and the endpoint. However, if we send a well-defined laser beam in a given direction towards the interface, the beam will be bent in a direction given by Snel's law (derived from Maxwell's equations). If we have chosen an endpoint that does not lie along the refracted ray, the light will not reach the endpoint chosen by us. We must change the incoming beam until the refracted beam reaches the endpoint.

With such a description, the criterion of the shortest possible time from the initial to the final point is rather meaningless. The direction of the refracted beam is fully determined by that of the incident beam. Nevertheless, it remains true that if we choose an endpoint somewhere along the refracted beam, the light path represents the route by following which light uses the least time, but that is somehow an extra bonus, not the primary gain. In geometric optics, in other words, we do not invoke Fermat's principle, but the time aspect still appears in a somewhat related way. [We will come back to contemplation of this type when we discuss diffraction, because then we will also learn about Richard Feynman's thoughts on the foundations of quantum electrodynamics (QED).]

12.2 Light Through a Curved Surface

Imagine a glass sphere in the air and, at a short distance, a luminous point that emits light in all directions (at least the light falls on the sphere). We will now investigate how different light rays envisaged to be emanating from the luminous spot will proceed when they hit different points on the surface of the glass sphere.

In Fig. 12.2 P is the luminous point, and we have chosen a section where both this point and the centre of the sphere C lie. A light ray from P following the line between P and C will hit the spherical surface normally. The part of the light transmitted will travel straight on and continue along the extension of the line PC.

We then choose a ray of light that hits the spherical surface at a point A in the plane we consider. The line CA and its extension will then define the incidence normal, and the incidence plane and the emergence plane are in the plane we consider. The beam will *locally* appear to hit a flat surface, and the usual Snel refraction law applies. The

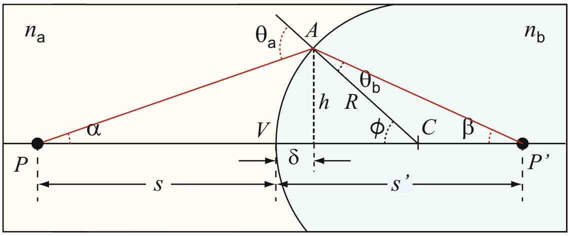


Fig. 12.2 Light rays from a luminous point (object) at P will form an "image" at the point P'. See the text for details

refracted ray is given a certain direction, it will cross the first light beam (which went through the centre) at the point P'.

We need some geometry to determine where the intersection point P' is located. Let R be the radius of the sphere and C its centre. The line that goes through P, C and P' will be called the optical axis. The point V, where the optical axis intersects the spherical surface, is called the vertex. The distance from P to V is denoted by s and that from V to P' by s'. The vertical distance from the point A and the optical axis is denoted by h, and δ denotes the distance between the vertex V and the point where the normal from A meets the optical axis. The symbols for various angles are indicated in the figure.

To make the calculations as general as possible, we will denote by n_a the refractive index of light in the medium where the light source is located (left in the figure), and n_b will signify the refractive index of the sphere (to the right of the figure). We will assume that $n_b > n_a$.

Snel's law gives:

$$n_a \sin\theta_a = n_b \sin\theta_b \ .$$

We also have:

$$\tan\alpha = \frac{h}{s+\delta}, \qquad \tan\beta = \frac{h}{s'-\delta}, \qquad \tan\phi = \frac{h}{R-\delta} \ .$$

Furthermore, we know that an exterior angle of a triangle is equal to the sum of the opposite interior angles:

$$\theta_a = \alpha + \phi, \qquad \phi = \beta + \theta_b \ . \tag{12.1}$$

We will avail ourselves of a simplification that is frequently made in dealing with geometrical optics, namely the so-called *paraxial approximation*. This means that we confine ourselves to situations wherein the angles α and β are so small that both sine and tangent can be replaced by the angle itself (in radians). Under the same approximation, δ will be small compared to s, s' and R. The equations above then take the simplified forms shown below:

$$n_a\theta_a = n_b\theta_b \tag{12.2}$$

and

$$\alpha = \frac{h}{s}, \qquad \beta = \frac{h}{s'}, \qquad \phi = \frac{h}{R} \ . \tag{12.3}$$

Upon combining the first equality in Eq. (12.1) with Eq. (12.2), we obtain:

$$n_a\alpha + n_a\phi = n_b\theta_b \ .$$

Using the second equality in Eq. (12.1), one finds:

$$n_a\alpha + n_a\phi = n_b\phi - n_b\beta$$

which can be transformed into:

$$n_a\alpha + n_b\beta = \phi(n_b - n_a)$$

Upon inserting the expressions for α, β and ϕ from Eq. (12.3) and cancelling the common factor h, one finally gets:

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \ . \tag{12.4}$$

This formula is quite important. It should be observed that the relationship applies independently of the angle α so long as the paraxial approximation holds (small angles). *All* light rays from a light source that makes a small angle with the optical axis will cross the optical axis at the point P'. The luminous point P is called the *object point*, and the intersection point P' is called the *image point*.

So far so good. But what of it? Is it something special that light rays cross each other? Will the different light bundles together give a special result, or might a light beam extinguish another, or that nothing extraordinary will happen at the crossing point anyway?

Figure 12.3 shows the same two light beams as in the previous figure, but we have now turned our attention to something else, namely the *time* light uses in going from the object point to the image point. For the straight ray that goes along the optical axis, the velocity of light will be c/n_a up to the vertex, and c/n_b afterwards. The velocity in glass is smaller. Similar reasoning applies to the ray that goes in the other direction (via A). We see that the distance from the object point to A is longer by γ than that from the object point to the vertex. Thus, light will take a longer time to cover the path PA than PV. On the other hand, we see that the distance AP' is shorter than VP' by an amount equalling ε. We notice that $\varepsilon < \gamma$. If we carry out a thorough analysis (not attempted here), we will find that the *time* light uses for

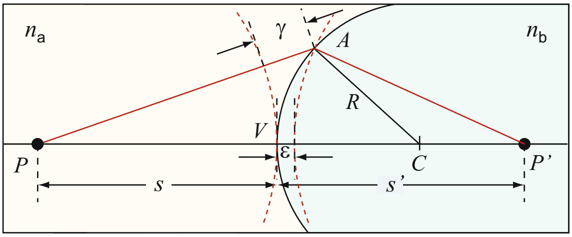


Fig. 12.3 How long will two light beams take in going from a luminous point at P to an "image" point at P'? Note the two dashed circle sectors with centre in P and P', respectively. See the text for details

covering the distance γ in air equals the time it takes for travelling the distance ε in glass (but this is true only if the paraxial approximation holds).

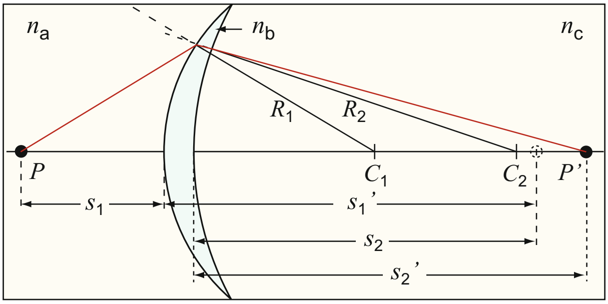
In other words, the light uses the same time from the light source (the object) to the intersection point (image) regardless of the direction of the light ray (within the paraxial approach). Since the light has the same frequency, regardless of whether it is air or glass, this means that there are exactly as many wavelengths along a refracted ray as along the straight one. Consequently, the light coming to the intersection will always be in phase with each other. Consequently, their amplitudes are added. If we could place a screen across the optical axis at P', we would be able to confirm this by seeing a bright spot just there. The word "image" can be used because we can form a real picture of the light source at this place.

12.3 Lens Makers' Formula

In the previous section, we saw how the light rays from a light source (object point) outside a glass sphere converged at the image point inside the sphere. However, such a system is of rather limited interest. We will now see how we can put together two curved surfaces, for example, from air to glass, and then from glass back to air, to obtain rules applicable to lenses. Suppose we have an arrangement as indicated in Fig. 12.4. To find out how Eq. (12.4) is used, we choose to operate with three different refractive indices and let the lens be "thin"; that is, the thickness of the lens is small compared to the distances from the object and image as well the radii of the two interfaces. Under these conditions (and still within the paraxial approach), we get:

$$\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1} ,$$

$$\frac{n_b}{s_2} + \frac{n_c}{s_2'} = \frac{n_c - n_b}{R_2} .$$



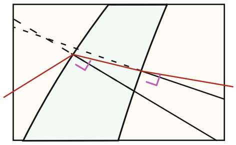


Fig. 12.4 A lens may be composed of two curved interfaces between air and glass. The image of P, if we had only the first interface, is marked with a dashed circle. *To the right*, detailed ray paths through the lens, first with refraction towards the normal (air to glass) and then away from the normal (glass to air). See the text for details

For a glass lens in air, $n_a = n_c = 1$ and $n_b = n$. Furthermore, the image point of other boundaries will be opposite to what we used in deriving Eq. (12.4). It can be shown that we can implement this in our equations by setting $s_2 = -s_1'$. We then make an approximation by ignoring the thickness of the lens; i.e. the lens is supposed to be "thin". Consequently, the equation pair above can be written as:

$$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1}$$

$$-\frac{n}{s_1'} + \frac{1}{s_2'} = \frac{1-n}{R_2} \; .$$

Addition of the two equations gives:

$$\frac{1}{s_1} + \frac{1}{s_2'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \; .$$

If the lens is regarded as an excessively thin element, it is natural to talk about object distance and image distance relative to the (centre of the) lens itself, instead of working with distances from the surfaces. We are then led to the equation that is called the *lens makers' formula*

$$\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \; . \tag{12.5}$$

A special case is that when the object point is "infinitely far away" (s' very much larger than the radii R_1 and R_2). In this case, $1/s \approx 0$, and

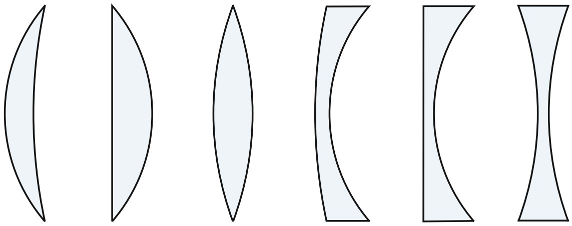


Fig. 12.5 Section through a variety of lens shapes. From left to right: positive meniscus, planoconvex and biconvex; negative meniscus, planoconcave and biconcave

$$\frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) .$$

The image distance for this special case (when the object point is "infinitely far away") is called the "focal distance" or "focal length" of the lens and denoted by the symbol f. The image is then located at the *focal point* of the lens (one focal length from the centre of the lens). With the given definition of focal length f, we end up with:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \tag{12.6}$$

where the focal distance f is defined as

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) . \tag{12.7}$$

The first of these formulas, called the **lens formula**, will be used in the rest of this chapter.

Before going further, let us see how lenses will look for different choices of R_1 and R_2 in the lens makers' formula. These radii can be positive or negative, finite or infinite. Different variants are given in Fig. 12.5.

Lenses with the largest thickness at the optical axis are called *convex*, whereas those with smallest thickness at the optical axis are called *concave*.

We pause to remind ourselves of what has been done so far.

The above derivations involved a number of approximations, and the ensuing formulas are necessarily approximate. This is typical of geometrical optics. The simple formulas are only approximate, and all calculations based on them are so elementary that they could easily have been done in high school. One might feel that, at university level, one would be able to deal with more complicated problems

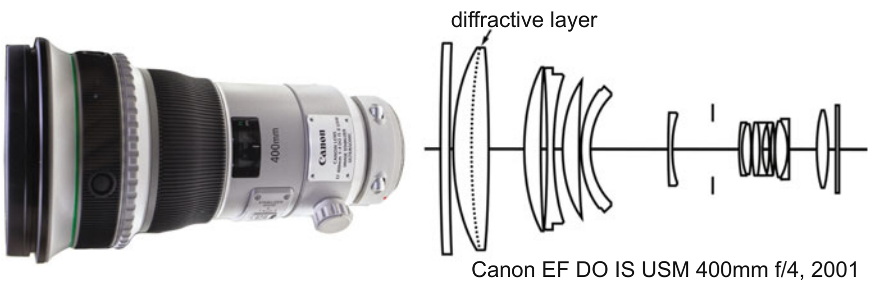


Fig. 12.6 Example of a modern lens for photography: Canon EF 400 mm f/4 DO IS USM objective. Instead of a simple thin lens, as we think of an objective in our treatment of geometrical optics, the Canon lens has 12 groups with a total of 18 elements (lenses) that function together as one. In modern camera lenses, most elements have spherical surfaces, but some have aspherical surfaces. Some elements are made of extra dispersive glass; that is, the refractive index has a different variation with the wavelength than in other most common glass types. Photograph: GodeNehler, Wikimedia Commons, CC BY-SA 4.0, Modified from original [1]. Drawing: Paul Chin (paul1513), GNU Free Documentation License, CC BY-SA 3.0, [2].

and reach more accurate descriptions, but the advanced problems are in fact too complicated to be appropriate for a general book like this. Today, numerical methods are used for the more advanced calculations. It has been found that making a perfect lens is not possible. We have to make a trade-off, and a lens to be used mostly at short distances will have to be designed in a different way than a lens that is meant primarily for long distances.

We have based our treatment on spherical interfaces. This is because until recently it was much easier to fabricate lenses with spherical surfaces than with other shapes. In recent years, it has become more common to fabricate lenses with aspherical shapes, and then, the problem arising from the paraxial approximation is made less severe. We can reduce the so-called spherical aberration by designing aspherical surfaces.

Looking back, we see that Eq. (12.4) contains the refractive indices. Now, we know that the refractive index depends on the wavelength, and this means that the image point P' will have a different position for red light than for blue light. Using multiple lenses with different types of glass (different refractive indices), we can partly compensate for this type of error (called chromatic aberration). Overall, however, it is a very challenging task to make a good lens (Fig. 12.6). It is not hard to understand why enthusiasts are examining new objectives from Nikon, Canon, Leitz, etc., with great interest just after they appear on the market. Have the specialists been able to make something special this time, and if so, in what respect? The perfect lens does not exist!

12.4 Light Ray Optics

We are now going to jump into that part of optics which deals with glasses (spectacles), cameras, loupes (jeweller's magnifying glasses), binoculars, microscopes, etc. The term "ray optics" is distinct from "beam optics", which focuses more on how a laser beam changes with distance (where diffraction is absolutely essential).

There are three main rules to which we will appeal continually.

1. For a *convex* lens, incoming light parallel to the optical axis will go through the focal point after the light has gone through the lens. For a *concave* lens, incoming light parallel to the optical axis will be refracted away from the optical axis after the light has gone through the lens. The direction is such that the light beam appears to come from the focal point on the opposite side of the lens (focal point on the same side as the object).
2. Light passing through the centre of the lens (where the optical axis intersects the lens) will move in the direction along which it came in.
3. Rays passing the front focal point of a *convex* lens will go parallel to the optical axis *after* the lens. For *concave* lenses, rays entering the lens along a direction that passes the rear (or back) focal point will continue parallel to the optical axis *after* the lens.
4. Rule 3 is identical to Rule 1 if we imagine that the ray is travelling in a direction opposite to the actual direction, for both convex and concave lenses.

Light rays drawn according to rule 1, 2 and 3 are coloured red, blue and green, respectively, in a number of the remaining figures in this chapter.

The rules originate in part from the lens formula. We saw that if the object distance s was made infinitely large, the image would be at a distance equal to the focal length of the lens. If multiple light rays are drawn from the object in this case, the rays will enter (approximately) parallel to the optical axis, and all such rays shall pass through the focal point. Whence follows the first rule.

However, the lens formula can be "read" in either direction, so to say. If we place a very small light source on the optical axis at a distance equal to the focal length in front of the lens, the light from the source will go to the lens in many different directions, but the image will then be at a distance of $s' = \infty$. This means that, irrespective of where they go through the lens, the rays will continue almost parallel to the optical axis after the lens.

The middle rule may be even easier to understand. At the centre of the lens (where the optical axis cuts through the lens), the two surfaces are approximately parallel. If a beam of light is transmitted through a piece of plane glass, the light beam will be refracted at the first interface, but refracted back to the original direction as it passes through the other interface. The emergent ray will be slightly offset relative to the

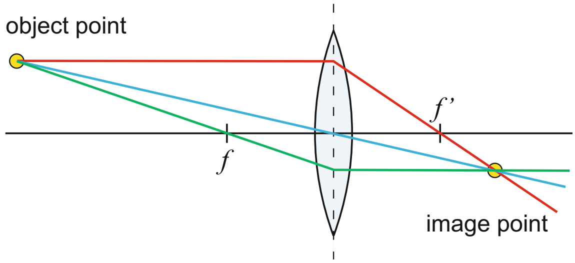


Fig. 12.7 A luminous object point not on the optical axis will be imaged at a point opposite to a convex lens. The image is not on the optical axis. Three guides are used to find the location of the pixel

incoming light beam, but if the angle is not too large and the lens thin, the parallel offset will be so small that we can neglect it in our calculations.

Object beyond the focal point

If these rules are applied to a luminous object point that is not on the optical axis, we get the result illustrated in Fig. 12.7. The three special light rays, specified by our general rules, meet exactly in one image point. The image point is on the opposite side of the optical axis in relation to the object point.

If the object is no longer a luminous point, but an extended body, such as an arrow, we find something interesting (see Fig. 12.8). From each point of the body, light is emitted, and for each point in the object, there is a corresponding image point. Our simplified rules of ray optics imply that, for all points in the object lying in a plane perpendicular to the optical axis, the corresponding image points will lie in a plane perpendicular to the optical axis on the opposite side of the lens (under conditions indicated in the figure). That is, we can image an object (such as the front page of a newspaper) into an image that can be captured on a screen. The image will then be a true copy of the object (newspaper page), except that it will have a magnification or reduction compared with the original, and the image will be upside down (but not mirrored).

The magnification is simply dependent on s and s'. If $s = s'$, the object and image will be of the same size. If $s' > s$, the image will be larger than the object (original), and vice versa. The *linear magnification* or *real magnification* is simply given by:

$$M = -\frac{s'}{s} \; .$$

The minus sign is included only to indicate that the image is upside down in relation to the object.

It is also possible to define a magnification in area. In that case, the square of the expression will be given. (The minus sign is then often irrelevant.)

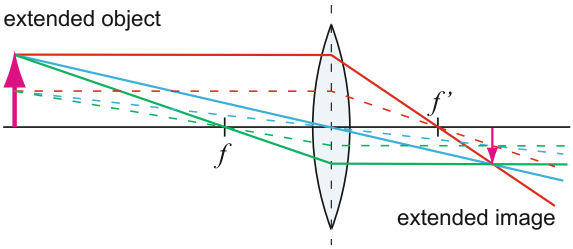


Fig. 12.8 An extended object may be thought of as a plurality of object points, and each point is imaged in a corresponding point on the opposite side of a convex lens. As a result, the object as such is depicted as an image. The image is upside down and has a different size than the object

Object within the focal distance?
So far, there have been relatively easily comprehensible connections between object and image, and we have been able to capture the image on a screen and see it there. But what would happen if we place the object closer to the lens than the focal length? Figure 12.9 shows how the three reference rays now go. They diverge after traversing the lens! There is no point where the light rays meets and where we can collect the light and look at it. On the other hand, the rays of light appear to come from one and the same point, a point *on the same side of the lens as the object*, but at a different place.

> In cases like this, we still talk about an image, but refer to it as a "*virtual image*" as opposed to a "real image" like that discussed earlier. A virtual image cannot be collected on a screen. On the other hand, we can look at the virtual image if we bring in another lens in such a way that the whole thing, after going through the new lens, creates a real image.

For example, if we look at the light coming through the lens in Fig. 12.9, using our eyes, our eye lens will gather the light to project a real image on the retina. Then we see the image. The image is formed on the retina as a result of the light from the object passing through the free-standing lens and subsequently through the eye lens.

However, we can get exactly the same image on the retina if we remove the outer lens and replace the real object with an imaginary magnified object placed as indicated by the "virtual image" in the figure. This is why we speak of a "virtual" image.

Concave lens
Using a concave lens alone, we cannot form a real image for any position of the object (see Fig. 12.10). Concave lenses on their own always provide virtual images, and it is somewhat unusual and demanding to work with ray diagrams for concave lenses. If we decide to use the lens formula, we say that the focal length is negative for concave lenses. We also need to operate with negative object distances and negative image

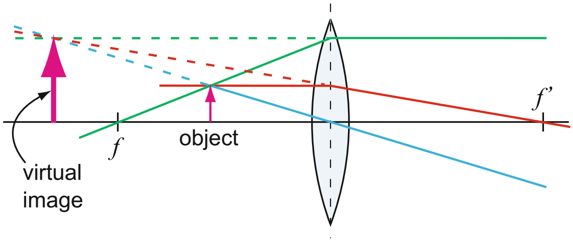


Fig. 12.9 When an extended object is placed within the focal length of a convex lens, no image is formed on the opposite side of the lens. On the contrary, the dashed lines indicate that the object and lens appear to be replaced by an enlarged object on the same side of the lens as the real object. This apparently enlarged object is called a virtual image

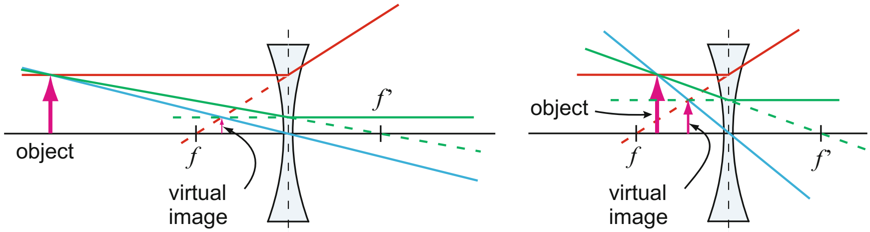


Fig. 12.10 A concave lens alone can never form a real image. If we consider an object through a concave lens, the virtual image looks smaller than the real object

distances depending on whether or not the object and/or image are on the "regular" side of the lens. There are a set of rules for how to treat s, s' and f in the formula for all combinations of cases.

Note that in Fig. 12.10 the object is outside (inside) the focal point in the figure on the left (right). There is no significant difference in the image formation by a concave lens when the object distance is equal to the focal length. This is contrary to what happens with a convex lens.

It is strongly recommended that you study *all* details in how the light rays are drawn in Figs. 12.9 and 12.10 and compare them with the rules in the first part of Sect. 12.4. You will need this for understanding later figures.

12.4.1 Sign Rules for the Lens Formula

The lens makers' formula and lens formula can be used for both convex and concave lenses and mirrors, but sometimes we have to operate with negative values for positions, radii of curvature radii and focal lengths for the formulas to function.

The rules for light coming against lenses or mirrors are as follows:

- Object distance $s > 0$ if the object is a real object; otherwise, $s < 0$.
- Image distance $s' > 0$ if the image is real (real rays meet in the image), $s' < 0$ otherwise.
- Focal distance $f > 0$ for convex lenses, $f < 0$ for concave lenses.
- Focal distance $f > 0$ for concave mirror, $f < 0$ for convex mirror.

In addition, the following convention applies:

- Magnification m is taken to be positive when the image has the same direction as the object, $m < 0$ when the image is upside down.

It is nice to have these rules for signs, but experience shows that they sometimes are more confusing than useful. For that reason, some people choose to decide the signs by drawing the ray diagram, obtaining an approximate value for the image distance relative to object distance, and checking whether the image is real or imaginary. Thereby the sign comes out on its own. The procedure nevertheless means that we know the rules for the focal distance for convex and concave lenses and mirrors.

Slavish use of the lens formula and sign rules without simultaneous drawings based on ray optics will almost certainly lead to silly errors sooner or later!

12.5 Description of Wavefront

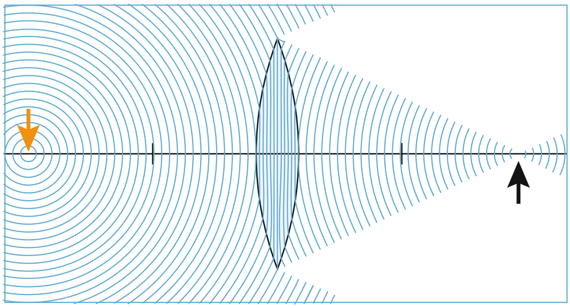
Ray optics is useful for finding the size and position of the image of an object after light has passed through a lens. However, we initially mentioned in this chapter that we usually describe light as electromagnetic waves and that the concept of "light rays" does not square with a wave description.

However, it is relatively easy to go from a wave description to a ray description. The link between them is the fact that when a wave propagates, it moves perpendicular to any wavefront. It may therefore be interesting to see how wavefronts of light from a source develop as they pass through a lens.

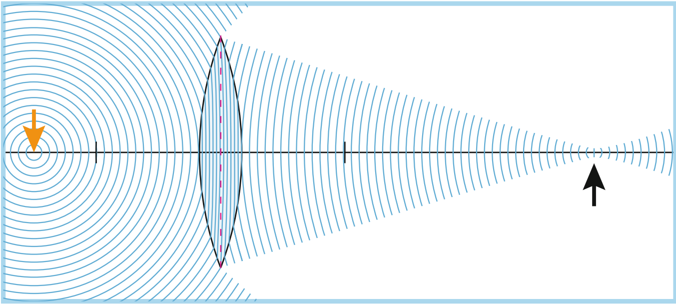
In Fig. 12.11, we have drawn a light source as a luminous point that emits spherical waves of a definite wavelength. The wavelength of light is very small in relation to the lens size, so the distance between the drawn wavefronts is in the range of a few hundred wavelengths.

In Fig. 12.11a, we have chosen to place the object point on the optical axis at a distance of $2f$ from the lens centre plane, where f is the focal length of the lens. The wavefront hits the lens and continues through the lens and continues after the light has passed the lens. The wavefront must be continuous, and since the light goes at a lower speed through the glass than in air, the wavelength inside the glass is less than in air. The wavefronts lie closer together.

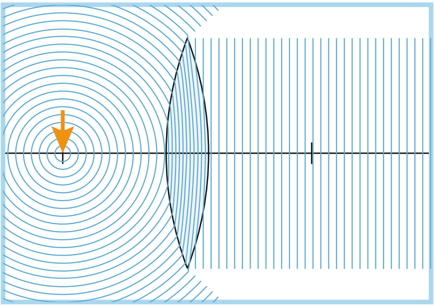
(a)



(b)



(c)



(d)

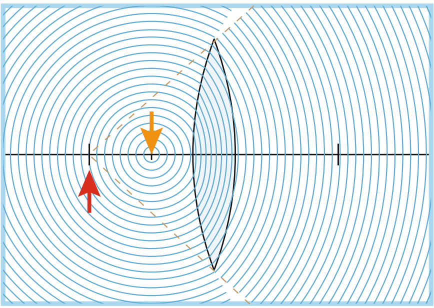


Fig. 12.11 An object point (vertical orange arrow) emits light in all directions as spherical wavefronts. The figure shows how the wavefront changes as they pass through a biconvex lens when the object distance changes. See text for details

In this case, the image point according to the lens formula will be a distance of $2f$ behind the lens. Then the wavefront will appear as shown in the figure, and we also see that the actual light beam narrows and gets a minimum value just at the image point (marked with black arrow). In this case, wavefront within the lens itself is flat, but it is also the only region of the wavefront that is flat in this case.

In Fig. 12.11b, the object point is located at a distance of $\frac{3}{2}f$ from the lens. Now the curvature of the wavefront is larger (the radius of curvature is small) than in the previous case, and the curved glass surface does not manage to align the wavefront so that they get flat inside the glass. After the light has passed the lens, the wavefront has too little curvature (too large radius of curvature) to form the image at a distance of $2f$ behind the lens. According to the formula, the image is now at a distance of $3f$.

In Fig. 12.11c, the object point is located at a distance of f from the lens. The curvature of the wavefront that hits the lens is now so great that the lens as a whole does not manage to gather the light at an image point on the opposite side. In this case, the wavefront becomes flat after the light has passed the lens. The light continues as a cylindrical light beam with unchanged diameter.

In Fig. 12.11d, the object point is located at a distance of $f/2$ from the lens. Now the wavefront of the light after it has passed the lens will be curved the opposite of what we had in the first two cases. There is no image point at which the light is gathered. On the other hand, we see that the wavefront after the light has passed the lens has a curvature that corresponds to the lens removed and that we had put the object at a distance f in *front* of the lens (marked with red arrow). It is this geometry we previously referred to as the "virtual" image.

Since waves in principle can go as well as backwards, Fig. 12.11 can be used to some extent also for light moving the opposite way. However, there are significant differences in the regions in which the light is located.

12.6 Optical Instruments

Several lens combinations were used for making optical instruments in the early 1600s. The telescope opened the heavens to Galilei, and he could see the four largest moons of Jupiter, an observation which had a decisive influence on the development of our world view. The microscope (only a single-lens microscope at that time) enabled Leeuwenhoek to see bacteria and cells and paved the way for a rapid development and increased understanding of biological systems. Optical instruments have played an important role and still have a huge impact on our exploration of nature.

We will presently look at how we can build a telescope and microscope using two lenses. First of all, however, we will take on a simple lens used as loupe, since this construction is classically included in both telescope and microscope.

12.6.1 Loupe

The simplest version of a loupe (“magnifying glass”) is a simple convex lens. The ray path of a loupe is somewhat different from what we have indicated in the figures so far.

An object is placed in Fig. 12.12a one focal length away from a convex lens (loupe). The light from an object on the optical axis (red) will appear, after passing the lens, as rays parallel to the optical axis (plane wavefront perpendicular to the optical axis). Light from a point in the object, which is at a distance d from optical axis (green), will also appear almost like parallel rays, but now at an angle θ with the optical axis.

If we place the eye somewhere behind the loupe, the eye lens will form an image of the object on the retina. Since the light rays coming into the eye (from each object point) are almost parallel (the wavefront is approximately plane), the eye must adjust the eye lens as if one is looking at an object far away. The focal length of the eye lens is then the distance between the eye lens and the retina (see later).

We often use a loupe to get an enlarged image of an item we can get close to. The best image we can achieve without a loupe is obtained when the object is as close to the eye as possible without sacrificing the sharpness of the image (see Fig. 12.12b). This distance to the eye is a limiting value s_{min}. A “normal eye” (see Sect. 12.8) cannot

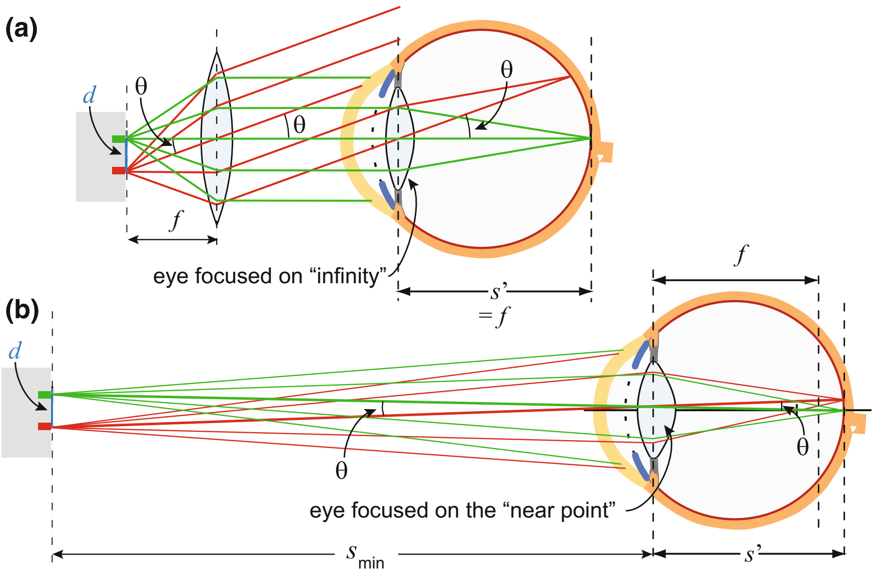


Fig. 12.12 Ray diagram when we consider an object with and without a loupe. In part **a**, an object is placed in the focal plane of a convex lens used as a loupe. The rays are caught by the eye and form a real image on the retina. Part **b** shows the ray path when we keep the object as close to the eye as possible, while at the same time maintaining a sharp image. See text for further information

focus on objects closer than about 25 cm. Therefore, we choose to set $s_{\min} = 25$ cm. The curvature of the eye lens is maximized and the focal length f in this case is slightly shorter than s'. Thus, also in this case s' fits with the size of the eye.

We see from Fig. 12.12 that the image *on the retina* becomes larger when we use a loupe as compared to looking at the object with the unaided eye. The loupe gives us a magnification.

Since the size of the eye itself is unchanged in the two cases, the size of the exposed part of retina will be proportional to the *angle* θ between the incident red and green light rays in Fig. 12.12.

Let θ be the angle subtended by light rays from two object points (as judged from the centre of the eye lens). In Fig. 12.12, these points are shown in red and green. The magnification of a loupe is defined as the ratio of the tangent of the subtended angle when the light reaches the eye via a loupe and the tangent of the angle when the light comes directly (without the loupe) from the object when it is at a distance $s_{\min} = 25$ cm. We often refer to this as "*angular magnification*", as opposed to magnification we have mentioned earlier, where we found the ratio of the magnitudes of an image to the object. For a loupe with a focal length f, the angular magnification becomes:

$$M = \frac{d/f}{d/s_{\min}} = \frac{s_{\min}}{f} \; .$$

Here d is the distance between the two selected points on the object perpendicular to the viewing direction (the distance between red and green object points in Fig. 12.12). The focal length of the loupe is f.

A loupe with a focal length of 5 cm will then have an magnification of 25 cm/5 cm = 5. We usually write 5 X (fivefold magnification). Note that the magnification here is positive because the image we see through the loupe is upright in relation to the object (the image does not turn upside down).

In short, we can say that the loupe has only the function that the object can be moved closer to our eye than can be achieved without the loupe. The effective distance is simply the focal length. If we have a loupe with a focal length of 2.5 cm, we will examine a butterfly wing at an effective distance of 2.5 cm instead of having to move the butterfly 25 cm away from the eye to get a sharp image. The result is a real image on the retina that is about ten times as large as that seen without the loupe.

In a microscope or telescope, a loupe is used with another lens (an objective). Loupes may have focal lengths down to about 3 mm. It automatically gives an almost 100-fold magnification compared to whether we had not used the loupe.

Note that the distance between the loupe and the eye has no bearing on the distance between the red and the green image points on the retina in Fig. 12.12a. However, if the eye is drawn too far away from the loupe, we will not be able to see both the red and the green points at the same time. The field of vision is thus greatest when the eye is closest to the loupe, but the magnification is independent of the distance between the loupe and the eye.

12.6.2 *The Telescope*

A telescope consists of at least two lenses (or at least one curved mirror and one lens). The lens (or mirror) closest to the object (along the light path) is called *objective*, whereas the lens closest to the eye is called *eyepiece* or *ocular*. The purpose of the objective is to create a *local image* of the object (in a way moving the object much closer to us than it actually is). The eyepiece is used as a loupe to view the local image.

Although the local image is almost always *much* smaller than the object, it is also much closer to the eye than the object itself. Once we can use a loupe when viewing the local image, we can get an (angular) magnification of up to several hundred times. However, a regular prismatic binocular has a limited magnitude of about 5–10 X. Bigger binoculars require a steady stand so that the image does not flitter annoyingly in the field of view.

Figure 12.13 shows a schematic illustration of the optical arrangement for a telescope. We use the default selection of light rays from the object's largest angular distance from the optical axis (from the top of the arrow). Points in the object on the optical axis will be imaged on optical axis, and usually these lines are not shown.

We notice that the objective provides a real inverted image a little further away from the lens than the focal plane. Objects that are very far away will be depicted

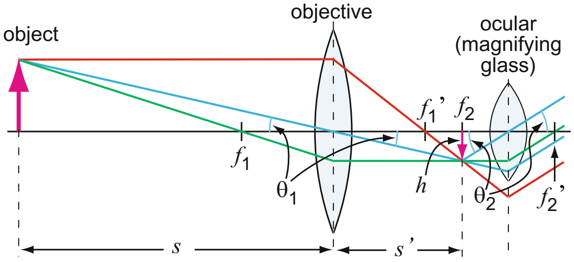


Fig. 12.13 For a telescope, the object is far away compared to the focal length. The lens creates a real, diminished "local" image just behind the focal length. This picture is then considered with a loupe. Total magnification is measured as angular magnification to the object viewed through binoculars compared to no binoculars. The picture in this type of telescope appears inverted (upside down)

fairly close to the focal plane. Objects that are closer fall further and beyond the focal point of the lens.

The eyepiece is positioned so that the image from the lens falls into the eyepiece's focal plane. Then all the light rays from a selected point in the object, after they have gone through the eyepiece, will appear parallel. The eye will focus on infinity and a real image will be formed on the retina.

Magnification

The magnification of the telescope is given as the ratio of the tangents of the angles between the optical axis and the light rays passing through the centre of the lenses.

From Fig. 12.13, we see that the angular magnification can be defined as:

$$M = -\frac{\tan\theta_2}{\tan\theta_1} = -\frac{h/f_2}{h/s'} \, .$$

In other words, the magnification varies with the distance from the objective to the real local image (between the objective and ocular). This distance will vary depending on how close the object is to the objective. It is more appropriate to specify the magnification as a number. It is achieved by selecting the magnification when the object is infinitely far away. Then s is infinite and s' becomes equal to the focal length of the lens f_1. The magnification can then be written as:

$$M = -\frac{h/f_2}{h/f_1} = -\frac{f_1}{f_2} \, .$$

In other words, the angle magnification equals the ratio between the focal lengths of the lens and the eyepiece.

For a telescope with focal length $f_1 = 820\,\text{mm}$ and eyepiece with focal length $f_2 = 15\,\text{mm}$, the angular magnification becomes:

$$M = \frac{820}{15} = 54.7 \approx 55\,\text{X} \, .$$

Note that since there are so many approximations made in the simple variant of geometrical optics that there is no point in specifying magnification with more than two significant figures.

Eyepiece projection *

Before leaving the telescope, we will mention a useful small detail. It is good to look through a telescope or a microscope, but today we often want to record the observed details in such a way that others may also see them. Since the eyepiece usually works like a loupe, we cannot capture a real image by placing, for example, a CMOS image

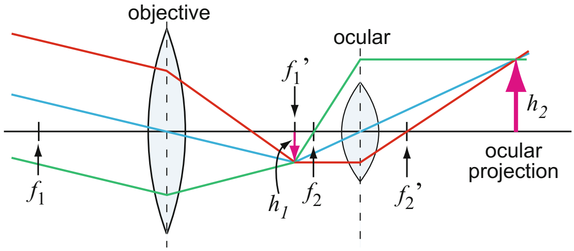


Fig. 12.14 In the case of ocular projection, the eyepiece is not used as a loupe, but as a second imaging lens. See text for details

sensor (photo chip) or an old-fashioned photographic film somewhere behind the loupe. One possibility is to remove the entire eyepiece and place the CMOS sensor exactly where the real image is formed. It is quite common today. The CMOS chip may be the internal chip in a digital single-lens reflex camera (DSLR camera) with interchangeable lenses. We then remove the regular camera lens and use the telescope lens instead.

Let us take an example: we have a telescope with an 820 mm focal length objective. Suppose we want to take pictures of the moon. The angular diameter of the moon is about half a degree. The size of the real image formed by a telescope lens will then be:

$$h = 820 \text{ mm} \times \tan(0.5°) = 7.16 \text{ mm} .$$

If the CMOS chip is 24 mm wide, the moon will cover $7.2/24 = 0.3$ of this dimension. The image of the entire moon has a diameter of only 30% of the smallest dimension of the image ("height"). It will be impossible to get nice details of the moon surface even though the exposure of the image is optimal.

Is there a possibility of enlarging (blowing up) the moon image on the CMOS chip? Yes, it is possible by use of "ocular projection".

The principle is quite simple. Normally the eyepiece is used as a loupe, and then, the image from the objective is placed at the focus of the eyepiece. If we push the eyepiece further away from the objective, the real image will be outside the focal plane, and then, the eyepiece can actually create a new real image using the first real image as its own object. Figure 12.14 shows the principle. In order for the new real image to be larger than the first real image, the eyepiece must be pushed only *slightly* farther from the objective than its normal position. In principle, we can get as large a real image as we want, but the distance from the eyepiece to this last real image is in proportion to the size of the image. We must then have a suitable holder to keep the CMOS piece a proper distance behind the eyepiece. With such a technique, we can easily take pictures of details on the surface of the moon with the 820 mm focal length telescope.

However, there is a catch in the method. The lens captures as much light regardless of whether or not we use eyepiece projection. When the light is spread over a larger surface, it means that the brightness per pixel on the CMOS chip decreases. The exposure must then take place over a longer period of time to get a useful image.

It should also be added that eyepieces are usually optimized for normal use. Lens defects may show up in eyepiece projections that would otherwise go unnoticed.

Ocular projection can also be used in microscopy, and the method is very useful in special cases.

12.6.3 Reflecting Telescope

Large astronomical telescopes usually use curved mirrors as objectives. The main reason for this is that reflection laws for a mirror are not wavelength-dependent. Long wavelength light behaves approximately the same as short wavelength light, which eliminates the chromatic deviation due to the wavelength dependence of the refractive index of glass.

As with lenses, it is easiest to make curved mirrors when the surface is spherical. However, this shape is not good because parallel light rays will be focused at different locations depending on how far from the axis the light rays come in. Mathematics shows that it would be far better to choose a surface that has the shape of a paraboloid. In the left part of Fig. 12.15, there are examples of three different light rays coming against a parabolic mirror parallel to the optical axis. The rays are reflected according to the reflection laws and end up in exactly the same point (focal point). A telescope with such a parabolic mirror as an objective can get very sharp images and at the same time very high brightness.

Unfortunately, it is complicated to make telescopes with parabolic surface with the precision needed for light waves (since the wavelength is so small). It is therefore common for low-cost telescopes to use mirrors with spherical surface, but with an *opening* small relative to the *radius* (see the right part of Fig. 12.15). The difference

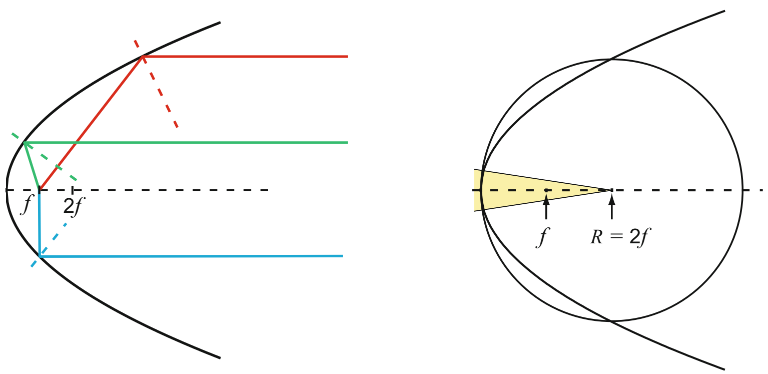


Fig. 12.15 *Left part*: A parabolic mirror ensures that all light rays coming in parallel to the optical axis are focused at the same spot, irrespective of whether the rays are near or farther away from the optical axis. *Right part*: A parabolic mirror and a spherical mirror have the same shape provided that the "opening angle" is small, i.e. the mirror diameter is small compared with the focal length.

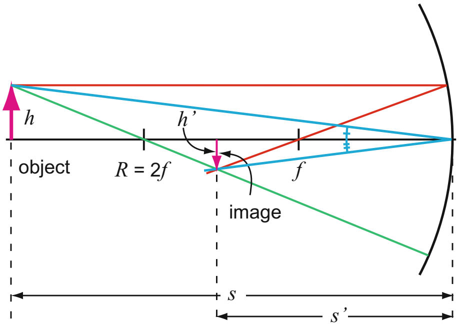


Fig. 12.16 Example of construction of image formation for a concave mirror

between parabolic and spherical shape is then not so great. Alternatively, we can combine a spherical mirror with a glass correction lens ("Schmidt corrector plate") to get a good overall result at a lower price than if we were to make a near-perfect parabolic mirror.

Construction rules for mirror

We can study image formation by a curved mirror in a manner rather similar to that used for thin lenses. We combine the properties of spherical and parabolic shapes to make the rules as simple as possible and get:

1. Any incident ray travelling parallel to the optical axis on the way to the mirror will pass through the focal point upon reflection.
2. For any incident ray that hits the centre of the mirror (where the optical axis intersects the mirror), the ray's incident and reflected angles are identical.
3. Any incident ray passing through the focal point on the way to the mirror will travel parallel to the principal axis upon reflection.

In Fig. 12.16, the image formation of a concave mirror is shown where the object is slightly beyond twice the focal length. Take note of all the details concerning how the three ray-tracing lines are drawn.

We can use the lens formula also for a mirror, but then be extra careful to consider the sign to get it right.

A concave mirror (concave mirror) will form a real image of the object, provided that the object is placed farther away from the mirror than one focal length.

A problem with a mirror is that the image forms in the same area as the incident light passes through. If we set up a screen to capture the image, it will firstly remove the light that reaches the mirror, and secondly, we get diffraction effects due to the edge between light and shadow (see Chap. 13). There are several tricks to mitigate these drawbacks. One of the classic tricks is to insert an oblique flat mirror for reflecting part of the beam away from the area the light enters (see Fig. 12.17). A telescope of this type is called a *Newtonian reflector*.

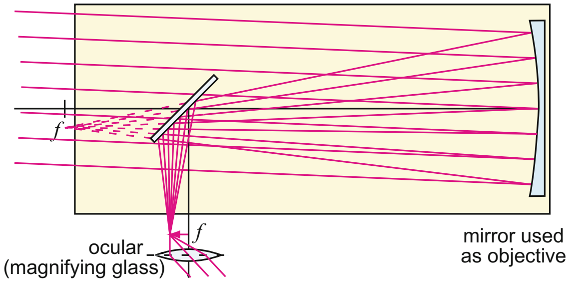


Fig. 12.17 In a Newtonian reflector, a slanting mirror is used to bend the light beam from the main mirror so that we can use an eyepiece and look at the stars without significantly blocking the incoming light. The slanting mirror does remove a small part of this light

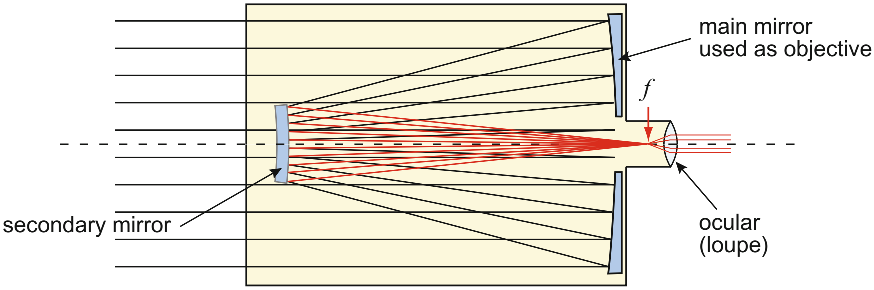


Fig. 12.18 In a Cassegrain telescope, a secondary mirror is used to reflect the light beam from the main mirror back through a circular hole in the main mirror. The secondary mirror does remove a small part of the incoming light. Light rays are drawn in black before the secondary mirror and in red thereafter

Another choice of construction is to use a curved mirror to reflect the light from the main mirror back through a circular hole in the main mirror (see Fig. 12.18). This design, called a *Cassegrain telescope*, makes it possible to make compact telescopes (short compared to their focal length). Schmidt correction plate is often used also for Cassegrain telescopes.

12.6.4 The Microscope

In the telescope, we used the objective to create a local image of the object, and this image was viewed through a loupe. The strategy works well when the object is so far away that we cannot get close to it. It is precisely in such situations that we need a telescope.

When we look at, for example, the cells in a plant stem, we have the object right in front of us. We do not need to create any local image, because we have the original. Then we use another strategy to see an enlarged image. The strategy is really exactly

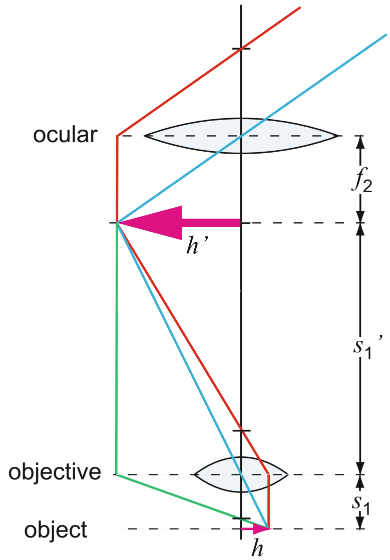


Fig. 12.19 Ray path in a microscope. The object can be placed arbitrarily close to the focal point of the objective, which, consequently, forms a real enlarged image of the object. This image is then viewed with the eyepiece that acts as a loupe

the same as for ocular projection. We place the object just outside the focal point of the lens (which now has a small focal length) to form a real-life image well behind the lens. This enlarged image of the object is then viewed by a loupe. The ray diagram of a microscope is illustrated in Fig. 12.19.

The magnification due to the objective alone is:

$$M_1 = \frac{s_1'}{s_1} \, .$$

This magnification can, in principle, be arbitrarily large, but then the actual image will move far from the lens, and the microscope would become unmanageable. By using a very short focal length lens, preferably only a few mm, we can achieve a significant magnification even for a tube length (distance between the objective and eyepiece) of 20–30 cm.

In addition, the loupe gives, as always, an (angular) magnification of:

$$M_2 = \frac{25\ \text{cm}}{f_2} \, .$$

The total magnification of the microscope comes out to be:

$$M_{\text{tot}} = \frac{25\ (\text{cm})\ s_1'}{f_2 s_1} \, .$$

In digital microscopes, a CMOS chip can be put directly in the plane where the real image from the objective is formed. The picture is then viewed on a computer screen and not through an eyepiece (ocular). It is difficult to define a magnification in this case since the digital picture can be displayed in any size.

For an 8 mm objective and a 30 cm tube length, and an eyepiece with focal length 10 mm, the total magnification (expressed in mm in the middle equality) is:

$$M = \frac{25\ (\mathrm{cm})\ s_1'}{f_2 s_1} \approx \frac{250 \cdot (300 - 10)}{10 \cdot 8} = 906 \approx 900\ \mathrm{X}\ .$$

Note:
Lately, it has been an extreme revolution within microscopy. By use of advanced optics, different kinds of illumination of the object, use of fluorescent probes, optical filters, scanning techniques, extensive digital image processing, and more, it is today possible to take still pictures and videos of real cells moving around, showing details we a few years ago thought would be impossible to catch by a microscope.

The physical principles behind these methods are exciting and take the wave properties of light into account to the extreme. For the interested reader, we recommend to start with the Wikipedia article on microscopy.

12.7 Optical Quality

12.7.1 Image Quality

Here is a word of warning. If we want to buy a microscope (or telescope for that matter), we can in fact get inexpensive microscopes with the same magnification as that provided by costlier instruments. The magnification itself is really far less important than the image quality. Heretofore, there has been no well-established system for specifying the image quality. Accordingly, there has been room for considerable trickery, and many have bought both microscopes and binoculars, which was no better than throwing money out of the window, because the image quality was too bad. Hitherto, one could rely only going to an optician for buying binoculars and getting a vague subjective sales talk on quality. So frustrating!

Fortunately, this is about to change. The practice of specifying optical quality in terms of measurements based on a "Modulation Transfer Function" (MTF) appears to have gained a firm foothold now. This is primarily a method of determining how sharp and contrasting images we can get. Colour reproduction does not come within the purview of this method.

To put it briefly, the MTF values tell us how close the lines in a black- and white-striped pattern can be, before the differently coloured stripes begin to blend into each

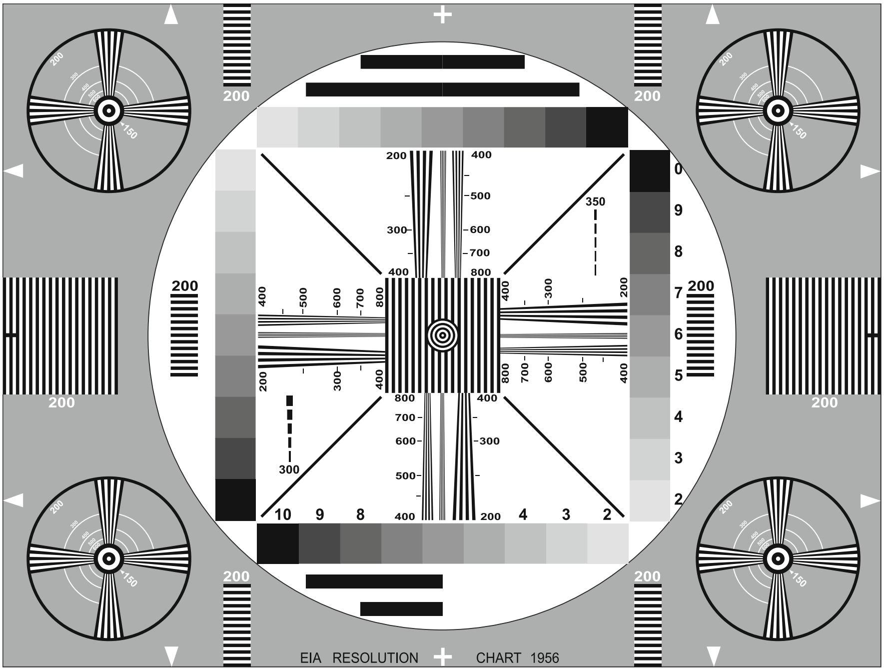


Fig. 12.20 One of several popular test objects “EIA Resolution Chart 1956” to measure the quality of an optical system. The striped patterns are used when testing the resolution, while the grey-toned steps at the edges can be used to test whether optics and CMOS chips provide a good reproduction of different brightness levels. BPK, Public Domain, [3]. A newer ISO 12233 Chart may eventually replace the EIA chart as a high-resolution test pattern for testing imaging systems

other. With increasing density, the stripes at first become more and more grey at the edges, but eventually the stripes disappear altogether.

Several test images have been developed that can be used to determine MTF values, thus telling a little about the optical system’s quality in terms of contrast and resolution. In Fig. 12.20, there is given an example of a widely used test plate with various striped patterns with gradual change in the number of stripes per mm. For example, if we have such a high-quality test circuit board (high resolution), we can, for example, look at the pattern through a telescope, camcorder or camera and see how nice stripe details we can detect in the final images/pictures. We will return to this issue later in the book, but with a different test object. (See also problems at the end of this chapter.)

The quality of an optical system can be impaired because of many reasons. Diffraction, due to the fact that the light has a wave nature, will always play a role. Diffraction, however, will only provide a limitation for very good optical systems. Most systems have more serious sources of degradation of optical quality than diffraction.

To avoid spherical and chromatic aberrations in lenses, modern objectives and eyepieces are often composed of several (many) lens elements (see Fig. 12.6). We

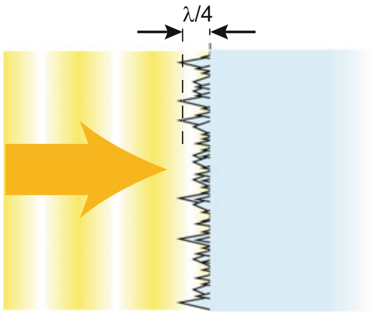


Fig. 12.21 A schematic figure showing the principles of the structures underlying the new type of anti-reflex treatment based on nanotechnology

know from previous chapters that when light goes from air to glass, about 5% of the intensity at the surface is reflected. If the light is inclined towards the surface, the reflection may be even greater (for some polarization, as we saw in Fresnel's equations).

With 5% reflection at every outer and inner glass surface in an objective consisting of say eight elements, quite a bit of light will go back and forth several times between elements, which will tend to diminish sharpness and contrast.

For many years, we have been tackling this problem by applying anti-reflection coatings on glass surfaces (see Chap. 13). Reflection can be reduced substantially by this remedy. The problem is, however, that such treatment depends both on the wavelength and on the angle with which the light hits the surface. Anti-reflection treatment of this type significantly improves image quality, but the treatment is not as good as we would like for systems such as cameras and binoculars where light with many wavelengths is admitted at the same time.

Since about 2008, the situation has changed dramatically for the better, and there is some fun physics behind it! Nikon calls their version the "Nano Crystal Coating", whereas the competitor Canon calls it "Subwavelength Structure Coating". Figure 12.21 shows the main principle.

When in Chap. 10 we calculated how much light would be reflected and transmitted at an interface between air and glass, our use of Maxwell's equations was based on some assumptions. To put it plainly, we said that the interface had to be "infinitely smooth, flat and wide" and "infinitely thin" in relation to the wavelength. Then integration was easy to implement and we got the answers we received. We claimed that the conditions could be fulfilled quite well, for example, on a glass surface, since the atoms are so small in relation to the wavelength.

The new concept that is now being used is based on "nanotechnology", which in our context means that we create and use structures that are slightly smaller than the wavelength of light.

The surface of the glass is covered with a layer that has an uneven topography with elements whose size along the layer is less than the wavelength, and the thickness of the layer is about one-quarter wavelength (not as critical as the traditional anti-reflection coating). From a traditional viewpoint, such a layer seems an absurd idea. One might think that the light would be splintered all over when it meets the randomly slanting surfaces, but this is not right. The light as we treat it is an electromagnetic wave that is extensive in time and space. In a manner of speaking, we can say that the wave sees many of the tiny structures *at the same time*, and details smaller than the wavelength will not be followed separately when the wave has propagated several wavelengths further.

Another way to describe the physics of these new anti-reflection coatings is to say that the transition from air to glass gradually occurs over a distance of about a quarter wavelength. Then the reflection is greatly reduced.

It should be added that since the atoms are small compared to the nanocrystals used, we can still use Maxwell's equations to see what happens when an electromagnetic wave hits a lens with nanocrystals on the surface. For example, if the structures are 100–200 nm large, there are about 1000 atoms in the longitudinal direction of these crystals. All calculations on such systems require the use of advanced numerical methods.

The nanocrystalline coating is so successful that about 99.95% of the light is transmitted and only 0.05% is reflected. This new technology is used nowadays on all expensive lenses from Canon and Nikon and has improved the image quality significantly.

12.7.2 Angle of View

So far, we have drawn the three ray-tracing lines from the object to lens plane to image plane without worrying about whether the lines are going outside or within the lens elements themselves. This is all right as long as we are only interested in finding out where the image is formed and what magnification it has. The light from an object follows all possible angles, and as soon as we have established where the image is placed and how large it is, we can fill in as many extra light rays as we wish. We have enough information about how the additional lines must be drawn.

At this point, it is meaningful to consider which light rays will actually contribute to the final image we see when we look, for example, through a telescope. Based on this type of consideration, we can determine what angle of view a telescope or microscope will provide.

Figure 12.22 gives an indication of how this works in practice. The dashed lines indicate the extremities of which light rays from the arrow's tip are captured by the lens and how they continue onwards. In this case, we see that only half of the light that the lens catches will go through the eyepiece. If the object had an even greater angular extent, we could risk that no light from the outermost parts of the object would reach the eyepiece, although in fact some light passes through the lens. By doing this type of analysis, the maximum image angle of, for example, a telescope can be determined, assuming that we actually know the diameter of the objective as well as the ocular.

In practice, it is not quite so simple, because the lenses and eyepieces that are used are composed of several lenses to reduce spherical and chromatic aberrations,

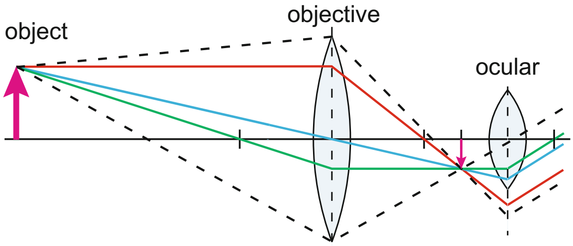


Fig. 12.22 Once we have established the three supporting lines to display image formation, we can fill in with all imaginable light rays that actually pass through a lens (for simple optical layout with a few elements). When multiple lenses are combined, not necessarily all light coming through the first lens will go through the next. This type of consideration can provide an approximate target for what angle of view a telescope or microscope will have

etc. Nevertheless, considerations of this type can provide a rough and ready measure of the field of vision.

It should also be added that different constructions of eyepieces provide quite different experiences when we look through, for example, a telescope. In the old days, we had to keep the eye at a certain distance from the nearest element in the eyepiece to see anything, and what we saw was generally black except a small round field where the subject was. Today, good eyepieces give much more latitude (up to 10 mm) in choosing the position of the eye (in relation to the eyepiece) for viewing an image, and the image we see fills more or less the effective angle of view of the eye. No black area outside is noticed, unless one actively looks for it. As we look through such eyepieces, we get the impression that we are not looking through a telescope at all, but simply *are* at the place shown in the picture. In astronomy, we speak of a sense of "space walk" when such eyepieces are used.

12.7.3 Image Brightness, Aperture, f-Stop

Everyone has handled a binocular where the image is bright and nice, and other binoculars where the image is much darker than expected. What determines the brightness of the image we see through binoculars?

In Fig. 12.23, a single lens with object far away has been drawn, along with the image that forms approximately in the focal plane. When the total image angle that the object spans is θ, the focal length of the lens f, and the extent of the image in the image plane is h_1, we have:

$$\tan(\theta/2) = \frac{h_1/2}{f}$$

$$h_1 = 2f\tan(\theta/2)\ . \tag{12.8}$$

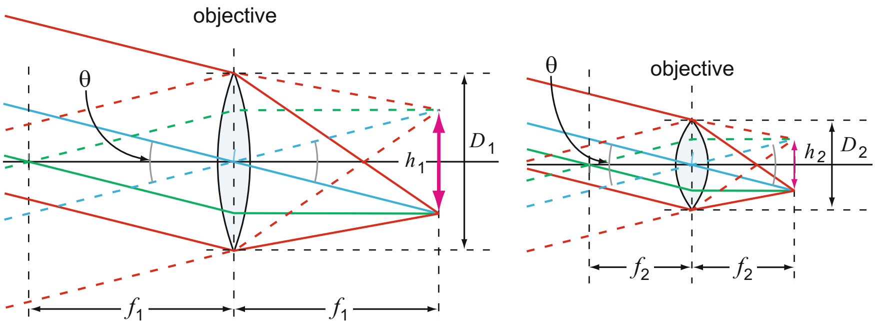


Fig. 12.23 A lens collects a limited amount of light from an object, and this amount of light is spread out over the area of the image being formed. In this figure, the object is assumed to be very far away so that the image is formed in the focal plane. *Comparison between left and right part*: If the lens diameter is reduced to half, while focal length is reduced to half, the light intensity (irradiance) of the image that is formed will remain unchanged. See text for details

For example, if we view the moon, the angular diameter will be about half a degree. If the focal length is 1 m, the image lens will have a diameter of 8.7 mm. How much light is gathered from the image of the moon in the focal plane? It depends on how much light we actually capture from the light emitted from the moon. When the light reaches the lens, it has an irradiance S given, e.g. in the number of microwatt per square metre. Total radiant power collected by a lens of diameter D is $S\pi(D/2)^2$ (in microwatts). The total radiant power will be distributed over the image of the moon in the focal plane, so that:

$$S\pi(D/2)^2 = S_i\pi(h_1/2)^2 .$$

The irradiance S_i in the image plane becomes:

$$S_i = \frac{\pi(D/2)^2}{\pi(h_1/2)^2} S .$$

If we use Eq. (12.8) and rearrange the terms, we get:

$$S_i = \frac{S}{4\tan^2(\theta/2)}\left(\frac{D}{f}\right)^2 \tag{12.9}$$

where θ is the angular diameter of the moon and f and D are, respectively, the focal length and the diameter of the lens.

The first factor is determined by the light source alone; the second, by the lens alone. The greater the ratio D/f, the brighter is the image formed by the lens (and this is the image that may be viewed by an eyepiece or detected by a CMOS chip or a similar device).

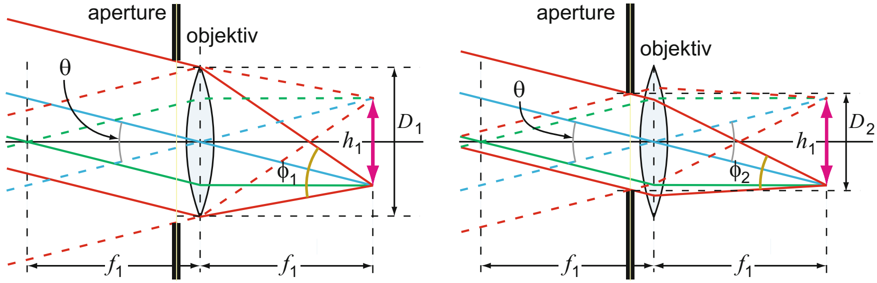


Fig. 12.24 If an aperture is used to reduce the light intensity in the image of an object, the size of the image will always be unchanged—only the brightness goes down. The ratio between focal length and diameter of the light beam that goes to the lens indicates the so-called aperture stop, or f-stop

In Fig. 12.23, two different lenses are drawn, with different radii and different focal lengths. If the focal length goes down to half (right part of the figure), the size of the image (diameter) will be half the size of the left part. But if the lens diameter also drops to half, the area that can catch, for example, light from the moon, will go down to a quarter. However, when the *area* of the image also goes down by a factor of four, it means that the irradiance in the image plane is identical to what we have in the left part of the figure. The ratio D/f is the same in both cases. It has therefore been found that the ratio D/f is a measure of the brightness of the image formed by a lens.

If we insert a photographic film or a CMOS chip into the focal plane and capture the image of, for example, the moon, we will have to collect light for a certain amount of time in order to get a proper exposure. If the lens is referred to as "fast", we will need less time than if the lens is referred to as "slow". A telescope with a large-diameter objective lens (or mirror) will capture much more faint light from distant galaxies than a small-diameter telescope. However, it is not the diameter alone, but the ratio D/f that determines the brightness of the image.

Aperture and f-stops

Cameras use an aperture to change the amount of light that will fall on the film or the CMOS chip. An aperture is simply an almost circular opening whose diameter can be changed by the operator. This is indicated in Fig. 12.24. The image size does not change if we reduce the opening and bring less light onto the CMOS chip, but the irradiance in the image plane will decrease.

The light gathering power of a camera objective is usually indicated by a so-called f-number (also called f-stop or aperture) defined by:

$$\text{f-number} \equiv f/D$$

where f is the lens's focal length and D is the effective diameter of the light beam let through by the aperture and objective. If we compare with the expression in Eq. (12.9), we see that the irradiance of the light that hits the film or the detector chip is inversely proportional to the square of the f-number.

The f-number is usually written in a special way. The most commonly used notation is "f:5.6" or "f/5.6" (f-number = 5.6). Typical f-numbers are 1.4, 2, 2.8, 4, 5.6, 8, 11, 16, 22 corresponding to relative irradiance of about 1/2, 1/4, 1/8, 1/16, 1/32, 1/64, 1/128, 1/256 and 1/512. The higher the f-number, the less light reaches the image plane.

These steps are called "stops", and one stop wider will admit twice as much light by increasing the diameter by a factor of $\sqrt{2} \approx 1.4$. We see that if we change the f-number with one stop, it corresponds to the irradiance in the image plane by a factor of two up or down. Increasing f-numbers correspond to less effective diameter of the lens and thus less light intensity on the light-sensing device. To get the same amount of energy collected per pixel in the CMOS chip, then the exposure time must either be doubled (for each increment in the f-number) or halved (for decreasing f-numbers).

Depth of view

From Fig. 12.24, we can notice another detail. If we move the CMOS bit slightly back or forth in relation to where the image is, a luminous point will be replaced by a luminous disc. The light beam towards the focal plane has a solid angle which decreases when the lens opening is reduced, indicated as ϕ_1 and ϕ_2 in the figure. This means that if we move the sensor chip a little way away from the focal plane, the pixels on the chip will be larger when the aperture is completely open (maximum light intensity in) than when the aperture is smaller (less light emits).

Fig. 12.25 When a luminous point in an object is imaged by a lens, its image will be a point located approximately in the prevailing focal plane. If we are to depict more objects that do not lie at the same distance from the lens, there is no focal plane for the various images that are formed. Then, luminous points in the objects that are at different distances from the lens than the one we have focused on will be depicted as circular discs, and the image will be "blurred". By reducing the aperture, blurring will be reduced (angle ϕ_2 is less than angle ϕ_1 in Fig. 12.24) and we say that we have gained greater depth of field. In the photograph on the left, a lens with f-number 3.5 is used with a shutter speed 1/20 s. In the photograph on the right, the same lens and focus are used, but now with f-number 22 and shutter speed 1.6 s

If we take a picture of a subject where not all objects are the same distance from the lens, we will not be able to get the image of all the objects in focus at the same time. If we have a wide aperture, the blur of the objects that are not in the focal plane will be greater than if the aperture is narrower. The smaller the opening (higher the f-stop number), the less blurred will be the picture. In photography, we say that we have greater "depth of field" (DOF) at small aperture (large f-number) compared to large aperture (small f-number). Figure 12.25 shows an example of this effect.

12.8 Optics of the Eye

Figure 12.26 is a schematic representation of the structure of a human eye. Optically, it is a powerful composite lens that forms a real image on the retina in the back of the eye. The amount of light that reaches the retina can be regulated by adjusting the size of the dark circular opening (pupil) in the iris. The retina has a very large resolution (which allows us to see details), but only in a small area around the point where the optical axis of the eye meets the retina. This area is called *macula lutea*, meaning the "yellow spot" (see Fig. 12.27), and the photoreceptor cells in this region are the so-called cones, which enable us to discriminate between different colours. In other parts of the retina, the density of photoreceptor cells is not so large, and the majority of these cells, called rods, are more sensitive than the cones, but they do not provide any colour information (see Chap. 11). Curiously, the light must pass through several cell layers before it reaches the photoreceptor cells. This may have an evolutionary origin since humans stay outdoors in daytime, when the sunlight is quite powerful. There are species that live in the deep dark layers of the ocean depths where the light reaches the optic cells (only rods) directly without going through other cell layers first.

The major focusing action of the eye comes from the curved surface between the air ($n = 1$) and the cornea. The contribution of the eye lens consists merely in modifying the optical power of the combined refractive system. The refractive indices of aqueous humour and vitreous humour are approximately 1.336 (almost

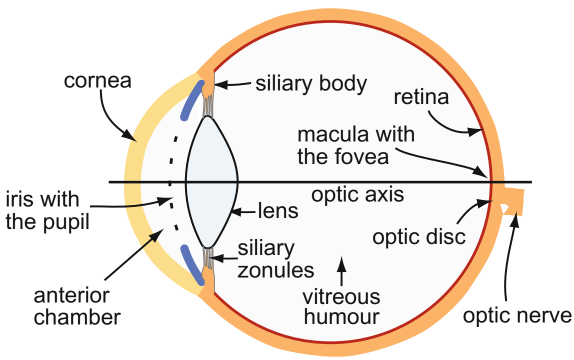


Fig. 12.26 Schematic representation of the human eye

Fig. 12.27 Pictures of retinas taken by a camera through the pupils with a special flash device. The image was taken in a routine check at c)optikk, Lørenskog, 2018. The blind spots are the yellow areas on the image where blood vessels and nerves originate from the eyeball. The so-called yellow spot (*macula lutea* in Latin) is an approximately 1.5 mm diameter area roughly in the middle of the slightly dark oval area somewhat to the left of the centre in the **R** (*right*) eye picture and somewhat to the right of the centre in the **L** (*left*) eye picture. In the macula lutea, the cones are most abundant and the area is responsible for our colour vision and sharp-sightedness. The light must pass through several cell layers before reaching the light-sensitive visual cells

the same as for water), while the eye lens has a refractive index of 1.41. The difference between these refractive indices is much smaller than that between air ($n = 1$) and the cornea ($n = 1.38$), which explains why the total optical power (or refractive power or focusing power) is largely due to the refraction at the surface of the cornea.

The size of the eye remains almost unchanged during use, as indicated in Fig. 12.28. When focusing on objects that are close to us or far from us, it is the shape of the eye lens that is adjusted. The shape is controlled by the ciliary muscle and zonular fibres (which are attached to the lens near its equatorial line). When the ciliary muscle is relaxed, the zonular fibres are stretched, which makes the lens become thinner and reduces its curvature; the converse takes place when the ciliary muscle contracts. For a normal eye, the focal point of the lens will generally be on the retina, and objects that are "infinitely far away" will then form a real upside-down image on the retina. When the muscle contracts, the zonular fibres slacken, allowing the lens to become more spheroidal and gain greater optical power. Then the focal point falls somewhere in the vitreous chamber, and objects that are not so far from the eye could form a real image on the retina. The image spacing s' in the lens formula always stays constant (about 20 mm), but the focal length of the overall lens changes; this process is called *accommodation*.

With advancing age, the eye lens hardens, the tension of the zonular fibres deteriorates, and the activity of the ciliary muscle declines. As a result, ageing leads to what is called *presbyopia*—the continuous loss of the ability of the eye to focus on nearby objects. Specifically, the nearest point on which middle-aged person is able

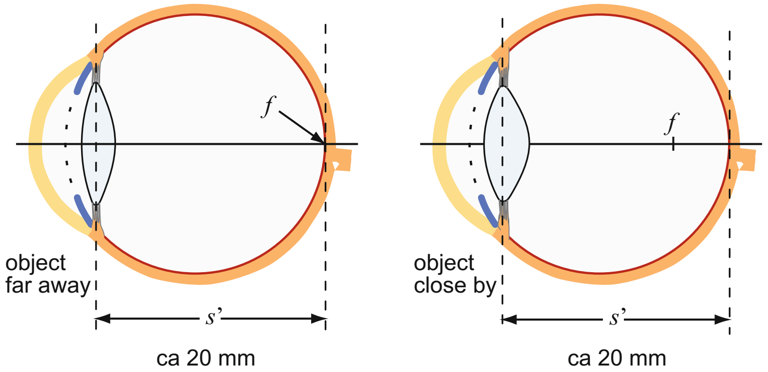


Fig. 12.28 Size of the eye remains almost unchanged, but the focal length of the compound lens can be changed. That way, we can see objects sharply over a whole range of distances

to focus recedes to about 1 m from the eye, whereas younger persons can focus on objects as close as 10–20 cm.

The smallest distance from the eye at which an object may still give rise to a sharp image on the retina is called the *near point*. The largest distance from the eye at which an object can still give a sharp image is called the *far point*. For a normal eye, the far point is at an "infinite" distance.

The optical power of the eye, spectacles

Now let us use the lens formula for a quantitative analysis of the eye:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \, .$$

For a normal eye focusing on an object really far away, s will be almost infinite, and with s' approximately equal to $20\,\mathrm{mm} = 0.02\,\mathrm{m}$ we get:

$$\frac{1}{f} = \frac{1}{0.02\,\mathrm{m}} = 50\,\mathrm{m}^{-1} \, .$$

The ratio $1/f$, called *optical power* (and by other names, including *dioptric power* and *refractive power*), is specified in *dioptre*, which equals m^{-1}. A normal eye focusing on infinity thus has an optical power of 50 dioptres.

For a normal eye, which has a near point of 25 cm, the optical strength is given by:

$$\frac{1}{f} = \frac{1}{0.25\,\mathrm{m}} + \frac{1}{0.02\,\mathrm{m}} = 54\,\text{dioptre} \, .$$

In other words, the eye lens can only change the total optical power by about ten per cent. The cornea takes care of the rest of the optical power.

Not all eyes are "normal". Some have corneas with excessive curvature, on account of which the optical power becomes too large in relation to the distance between the lens and the retina. When such a person tries to see an object far away, the real image will not hit the retina but lie somewhere inside the vitreous chamber. The person will therefore not have a sharp vision when he/she looks at something far away. Such a person we call *nearsighted*. Nearsightedness can be remedied using glasses or contact lenses. The eye's natural dioptric power must be contraindicated since it is too large, and the glasses or outer eye lenses must be concave (negative eyepiece).

If someone has a cornea with an abnormally small curvature, the optical power of the eye becomes too small. When such a person tries to look at an object 25 cm from the eye (nominal near point), the real image will not fall on the retina but will theoretically fall behind it. The image on the retina will be blurred. Such a person is called *long-sighted*. Again, we can compensate for the error by inserting an extra lens in the form of glasses or external eye lenses until we look sharply also for bodies 25 cm away. In this case, the lens must be convex (positive eyepiece).

When young, a person with an imperfect vision can get, with the help of spectacles, an almost normal vision, with a near point of 25 cm and a far point at infinity. As the age increases, the capacity for *accommodation* decreases, and one pair of spectacles will no longer be able to give a near point at 25 cm and at the same time the far point at infinity. It becomes necessary to continually wear glasses and take them off, maybe even to switch between two sets, to see satisfactorily both at short and long distances. There are also so-called "progressive glasses" where the upper part of the glass has one focal length and lower part another (with a continuous gradation between them).

One type of lens imperfection can be described by saying that the cornea has an asymmetrical shape, almost like an ellipsoid with one meridian being significantly more curved than the meridian perpendicular to it. In such a case, there are different optical powers for the two directions. This lenticular error is called *astigmatism* and can be corrected using lenses that have cylindrical rather than spherical surfaces. Such lenses are called cylinder lenses. They can be combined with spherical surfaces if desired.

Today it is quite common to undertake laser surgery for changing the corneal surface if the lens has significant inborn flaws. In that case, parts of the cornea can be burned and shaped so that the person gets a normal vision and does not have to wear glasses (until age-related defects make it necessary).

Examples

It is rather easy on our own to form a rough impression of what glasses we need in case we are slightly nearsighted or long-sighted. Here are a few examples:

Suppose we can focus sharply only up to 2.0 m, that is, the far point without glasses is 2.0 m. This means that the optical strength of the lens is:

$$\frac{1}{f} = \frac{1}{2.0\ \text{m}} + \frac{1}{0.02\ \text{m}} = 50.5\ \text{dioptre}\ .$$

The optical strength is thus too large by 0.5 dioptre, since it should have been 50.0 dioptre for the far point. The remedy is to use glasses with optical strengths of -0.5 dioptre, at any case for viewing distant objects. The elegance of these calculations is that we need to only add or subtract optical strengths to get the optical strength of the combination.

In the next example, we take a person who is unable to focus on distances closer than 50 cm. This person's optical strength is then:

$$\frac{1}{f} = \frac{1}{0.5\ \text{m}} + \frac{1}{0.02\ \text{m}} = 52.0\ \text{dioptres}\ .$$

In this case, when considering the near point, the optical strength should be 54.0 dioptres, which means that there is a deficit of 2 dioptres. The person thus needs spectacles of optical strength +2.0 dioptres to move the near point from 50 to 25 cm.

12.9 Summary

Geometric optics is based on the thinking that the light from different objects propagates like "light rays" in different directions, where each ray behaves approximately as (limited) plane electromagnetic waves. These light rays will be reflected and refracted at interfaces from one medium to another, and their behaviour is determined by Maxwell's equations and satisfies the laws of reflection and refraction in common materials.

When an interface between two media is curved, light rays incident at the interface will have different inclinations compared with the refracted rays.

For thin lenses, we can define two focal points, one on each side of the lens. Light rays will have infinitely many different directions in practice, but it suffices to use two or three guides to construct how an object is imaged by a lens so that we get an image. The guides are characterized by the fact that light parallel to the optical axis is broken through the focal point on the opposite side of a convex lens, but away from the focal point on the same side as incoming light beam for concave lenses. Light rays through the centre of the lens are not broken. We normally draw only lines to the centre planes of the lenses instead of incorporating detailed refraction on each surface. Guides may go beyond the physical extent of the lens, but only the light rays that actually pass through the lenses contribute to the light intensity of the image.

All light rays that go along different paths from one object point to one image point spend the same time on the trip, whether they go through the central or peripheral parts of the lens. This ensures that different light rays interfere constructively when they meet. When real light rays meet in this way, light can be intercepted by a screen, and we speak of the formation of a real image at the screen. If the real beams diverge from each other after passing through a lens, but all seem to come from a point behind the lens, we say that we have a virtual image at this point. If we consider light rays that diverge from each other with the help of our eye, the light rays will again be collected on the retina and form a real image there. Therefore, we can "see" a virtual image, even though we cannot collect this image on a screen.

The lens formula is a simplification of lens makers' formula, and only object distance, image distance and focal length are included. In the lens formula, the focal length is considered positive for a convex lens and negative for a concave lens. The signs for the object distance and image distance change with how the light rays reach the lens relative to where they exit. We must consider the sign in each case in order not to commit an error. A drawing that shows the beam angle is essential to avoiding mistakes in such cases (must check that the result looks reasonable).

A lens can be used as a loupe. Magnification is then an angle magnification, because placing the object at the lens's focal point, the virtual image will apparently be infinitely far away and be infinite. Different angles that indicate maximum propagation of an object lead to a similar physical extent to the actual image on the retina when we consider the object through the louse. The primary function of the loupe is that we can effectively keep the object much closer to the eye than the eye's near point. That is, we can effectively position the object much closer to the eye and still look sharp, compared to looking at the object as close as possible (sharp) without aids.

Lenses can be assembled for optical instruments such as telescopes and microscopes. For the telescope, an objective is used to create a local image of the object that we can consider with a loupe. The result can be a significant magnification. For the microscope, the object is placed outside, but very close to the focus of the lens. The real image that the lens then makes is significantly larger than the object. Again, a loupe is used to view the real image that the lens makes.

Alternatively, a CMOS chip is inserted into the image plane and the eyepiece (loupe) is removed. This is often the case with many of today's "digital microscopes". In such cases, "magnification" is a very poorly defined term since it will in practice depend on which computer screen size the image is finally displayed.

The human eye has a fixed image distance of approximately 20 mm. The focal length of the optical system is mainly determined by the cornea, but can be slightly adjusted since the eye lens strength can be varied within an interval. A normal eye can focus sharply on objects at a distance of approximately 25 cm to infinity. This corresponds to a total lens strength from 54 to 50 dioptres. If the cornea has a too large or too small curvature, the lens strength is too large or too small. Then we will not be able to focus sharply over the entire range from 25 cm to infinity, and we need glasses to compensate for deficiencies in the optical strength of the eye lens.

12.10 Learning Objectives

After going through this chapter, you should be able to:

- Explain why light from objects can be regarded as "light rays" when the light hits for example a lens.
- Calculate where the image point of a point source is after the light from the source has met the surface of a glass sphere.
- Explain the terms object, image, focal point, object distance, image distance, focal length, radius of curvature, concave, convex, real and virtual image.
- Derive (possibly with some help) the lens makers' formula of a positive meniscus lens and specify the simplifications usually introduced.
- Explain main steps in the derivation of the lens formula under the same conditions as in the previous paragraph.
- State the three main rules used in the construction of the ray path through lenses and mirrors (ray optics) and apply these rules in practice.
- Explain why sometimes we need to change the sign for some quantities when the lens formula is used.
- Explain two different ways to specify the magnification of optical instruments.
- Explain how a loupe is routinely used and what magnification it has.
- Describe how a telescope and microscope are constructed and what magnification they have.
- Describe how a reflecting telescope works and how it avoids undue obstruction of the incoming light.
- Calculate how large an image of a given subject (at a given distance) we can obtain in the image plane for different camera lenses.
- Calculate approximately the angle of view of a camera or binoculars when the relevant geometrical data are specified.
- Explain briefly how nanotechnology has led to better photographic lenses.
- Explain the optical power of a lens/objective and know what the f-numbers tell us.
- Explain the concept "depth of field" and how this changes with the f-number.
- Explain the optics of the eye, and explain what the terms near point, far point and accommodation mean.
- Know the optical strength of the eye and how the optical strength is varied.
- Calculate, from simple measurements of near point and far point, the approximate optical strength of the spectacles a person may need.

12.11 Exercises

Suggested concepts for student active learning activities: Light ray, ray optics, beam optics, wavefront, paraxial approximation, focal point, object/image, lens makers' formula, lens formula, convex, concave, (real) magnification, angular magnification, loupe, magnifying glass, normal eye, ocular, objective, telescope, microscope, reflecting telescope, optical quality, angle of view, image brightness, aperture, f-stop, depth of field. near point, far point, optical strength, dioptre.

Comprehension/discussions questions

1. The laws of reflection and refraction mentioned in this chapter apply to visible light. Light is regarded as electromagnetic waves. Will the same laws apply to electromagnetic waves in general? (As usual, the answer must be justified!)
2. An antenna for satellite TV is shaped like a curved mirror. Where does the antenna element itself be placed? Is this analogous to the use of mirrors in optics? Which wavelength does satellite TV signals have? And how big is the wavelength relative to the size of the antenna disc?
3. An antenna for satellite TV with a diameter of 1 m costs about hundred USD, while a mirror for an optical telescope with a diameter of 1 m would cost an estimated thousand times more. Why is there such a big difference in price?
4. A "burning glass" is a convex lens. If we send sunlight through the lens and hold a piece of paper in the focal plane, the paper can catch fire. If the lens is almost perfect, would we expect, solely on the basis of geometric optics, all the light to be collected at a point with almost no extent?
5. If you have attempted to use a burning glass, you may have discovered that the paper is easier to light if the sunspot hits a black area on the paper compared with a white one. Can you explain why?
6. Cases have been reported that fishbowls with water and spherical vases with water have acted as a burning glass, causing things to catch fire. Is it theoretically possible from the laws we have derived in this chapter? What about "makeup mirrors" with a concave mirror, may such a mirror pose any threat?
7. Based on the lens makers' formula, we see that the effective focal length depends on the wavelength since the refractive index varies with the wavelength of light. Is it possible for a biconvex lens to have a positive focal length for one wavelength and negative focal length for another wavelength?
8. How can you quickly find the approximate focal length of a convex lens (converging lens)? Do you also have a quick test for a concave lens (diverging lens)?
9. Does the focal length change when you immerse a convex lens into water?
10. Does the focal length change when you immerse a concave mirror into water?
11. If you look underwater, things look blurred, but if you are wearing diving goggles, you experience no blurring. Explain! Could you get rid of extra spectacles with no layer of air anywhere? In that case, should the spectacles have concave or convex lenses?
12. A real image (created, e.g. by an objective) can be detected by placing a paper, a photographic film or CMOS chip in the image plane. Is it possible to record a virtual image in one way or another?
13. The laws of reflection and refraction, lens maker's formula and lens formula are all symmetrical with respect to which way the light goes. In other words, we can interchange the object and image. Can you point out mathematically how this reversibility is expressed in the relevant laws? Are there any exceptions to the rule?

14. (a) We have a vertical mirror on a wall. A luminous incandescent lamp is held in front of the mirror so that the light reflected by the mirror hits the floor. However, it is not possible to form an image of the incandescent lamp on the floor. Why? (b) We have a laser pointer and we use it in a similar way to the incandescent lamp, so that the light from the laser pointer is reflected by the mirror and reaches the floor. *Now* it appears that we have formed a picture of the laser pointer (the opening of this) on the floor. Can you explain what is going on?
15. How long must a mirror be and how high must it be placed on a vertical wall so that we can see all of ourselves in the mirror at once? Will the distance to the mirror be important?
16. The two cameras in an iPhone model have, according to Apple, objectives with focal lengths of 28 and 56 mm, respectively. Is it possible that the lenses actually have these focal lengths? How do you think the numbers should be understood? Why does Apple choose to give the numbers this way? (Hint: Prior to the digital revolutions, the picture size on the film was usually 24×36 mm. See also problems 23–24.)

Problems

17. Draw a ray diagram for a convex lens for the following object distances: $3f$, $1.5f$, $1.0f$ and $0.5f$. For one of these distances, only two of the usual three standard rays can be used in the construction of the image. Which? State whether we have magnification or demagnification of the image, whether the image is up or down, and whether the image is real or virtual.
18. Determine, by starting from the lens formula and one of the rules for drawing ray diagrams, the smallest and largest magnification (in numerical value) a convex lens may have. Determine the condition that the magnification will be 1.0.
19. Repeat the calculation in the previous task, but now for a concave lens. Determine again the condition that the magnification will be (approximately equal to) 1.0.
20. When we find the image, formed by a convex lens, of an object “infinitely far away”, we cannot use the three standard light rays for the construction of the image. How do we proceed in such a case to find the location of the image in the image plane?
21. We have a convex meniscus lens with faces that correspond to spherical surfaces with radii of 5.00 and 3.50 cm. The refractive index is 1.54. What is the focal length? What will be the image distance if an object is placed 18.0 cm away from the lens?
22. A narrow beam of light from a distant object is sent into a glass sphere of radius 6.00 cm and refractive index 1.54. Where will the beam of light be focused?
23. Suppose that you have a camera and take a picture of a 1.75 m tall friend standing upright 3.5 m away. The camera has an 85 mm lens (focal length). What is the distance between the lens and the image plane when the picture is taken? Are you able to fit the entire person within the image if the image is recorded on an old-fashioned film or a full-size CMOS 24×36 mm image sensor? How much of the person do you get in the picture if it is recorded with a CMOS photo chip of size 15.8×23.6 mm?

24. Repeat the previous task for the camera in a mobile phone. For example, an iPhone model has a true focal length of 3.99 mm, and the photo chip is about 3.99×7.21 mm (the numbers apply to the general purpose camera and not the telephoto variant of this camera).
25. When Mars is closest to the earth, the distance is about 5.58×10^7 km. The diameter of Mars is 6794 km. How large will be the image if we use a convex lens (or concave mirror) with focal length 1000 mm?
26. The old Yerkes telescope at the University of Chicago is the largest single-lens refracting telescope of the world. It has an objective that is 1.02 m in diameter and a f-number of f/19.0. How long is the focal length? Calculate the sizes of the images of Mars and the moon in the focal plane of this lens. (The angular diameter of the moon is about half a degree, and the angular diameter of Mars can be estimated from the information in the previous task.)
27. A telescope has a lens with a focal length of 820 mm and a diameter of 100 mm. The eyepiece has a focal length of 15 mm and a diameter of 6.0 mm. What magnification does the telescope have? How big is the image angle? Can we see the whole moon disc at once?
28. A slide projector (or data projector, for that matter) has a lens with a focal length of 12.0 cm. The slide is 36 mm high. How big will be its image on a screen 6.0 m from the projector (lens)? Is the image upright or inverted?
29. Suppose we have two glasses, one with optical strength $+1.5$ dioptres on both lenses and one with optical strength $+2.5$ dioptres on both glasses. We only find one of the glasses and would like to check if these are the stronger or weaker. Can you provide a procedure on how to determine the optical strength of the glasses we found?
30. (a) Where is the near point of an eye for which an optician prescribes a lens of 2.75 dioptres? (b) Where is the far point of an eye for which an optician prescribes a lens with lens strength -1.30 dioptres (when we look at things a long distance away)?
31. (a) What is the optical strength of spectacles needed by a patient who has a near point of 60 cm. (b) Find the optical strength of the spectacles for a patient who has a far point of 60 cm.
32. Determine the accommodation (in the sense of a change in optical strength) of a person who has a near point at 75 cm and a far point at 3.0 m.
33. In a simplified model of the eye, we see that the cornea, the fluid inside, the lens and the vitreous humour inside the eye have all a refractive index 1.4. The distance between the cornea and the retina is 2.60 cm. How big should the radius of curvature be for an object 40.0 cm from the eye to be focused on the retina?
34. A loupe has a focal length of 4.0 cm. What magnification will it give under "normal" use? Is it possible to get a magnification of 6.5 X using the loupe in a slightly different way than described as a standard (do not think about ocular projection)? If so, tell us where the object we consider must be placed, and say something about how we can now use the eye.

35. A spherical concave mirror will not collect all parallel rays at one point, because the effective focal length will depend on how close the optical axis of the beam hits the mirror.
(a) Try to set up a mathematical expression for effective focal length of a beam that comes in parallel with the optical axis a certain distance from the axis. The radius of curvature of the mirror is set equal to R. As a parameter, we can use the angle θ between the incoming beam and the line that runs between the centre of curvature of the mirror and the point where the beam hits the mirror surface.
(b) For which angle will effective focal length have changed with 2% relative to the focal length of the rays coming in very close to the optical axis?
(c) Can you explain why mirror scanners based on spherical mirrors often have high f-numbers (low brightness)?
36. Suppose you have a removable telephoto lens for a camera. The focal length is 300 mm. Suppose you also have a good quality 5X loupe. You want to make a telescope of these components and have a suitable tube in which the lenses can be held.
(a) What is the focal length of the loupe lens? What is the magnification of the telescope?
(b) State the distance(s) between the objective and loupe (used as the eyepiece) when the telescope is to be used to view objects from 10 m to infinitely far away?
(c) You would like to be able to use the telescope for a distance of 25 cm. Is it possible? (As usual: The answer must be supported by arguments.)
(d) The telephoto lens has a diameter of 60 mm. What is the f-number (corresponding to the largest aperture) this lens can have? (Simplify the discussion by regarding the lens as a simple thin lens.)
37. A lens telescope is to be used by an amateur astronomer. The focal length of the objective is 820 mm, and the diameter 10.0 cm. The objective is located at one end of the telescopic tube and the eyepiece holder at the opposite end. The eyepiece holder can be adjusted so that we get a clear picture of the starry sky and planets. In order to use slightly different magnification on different objects, the amateur astronomer has four different eyepieces with focal lengths 30, 15, 7.5 and 3.0 mm. The diameter of the lens in these eyepieces is 48, 20, 11 and 3.7 mm, respectively. We treat all lenses as if they were "thin".
(a) How long should the telescope tube be (the distance between the objective and ocular)?
(b) How much change in position must the eyepiece holder allow?
(c) How much longer should the eyepiece move if we want also to use the telescope as field glasses with the minimum object distance equal to 20 m?
(d) What is the f-number of the objective?
(e) What do we understand by the "magnification" of a telescope?
(f) Estimate how much magnification we get for the four different eyepieces.
(g) Estimate the approximate image angle we receive for the 30 and 3.0 mm eyepiece.

(h) Compare this with the image angle of the moon, which is about 0.5°.
(i) How big will Jupiter look under conditions best suited for observations, when we view it through our telescope with the 3.0 mm eyepiece? (Approximate radius of earth's orbit is 1.50×10^{11} m and of Jupiter's orbit 7.78×10^{11} m. Jupiter's diameter is about 1.38×10^{9} m.)

38. The telescope constructed by Galileo consisted of a convex lens and a concave eyepiece. Such a telescope is called today a Galilean telescope, and a principle sketch is shown in Fig. 12.29 for a case where the object is far away.
The image from the objective (red arrow in the figure) is for this configuration placed "behind" the eyepiece (the eyepiece is closer to the objective than the image formed by the objective). The Galilean telescope therefore becomes shorter than for a telescope where both objective and ocular were convex (positive focal width).
Let us analyse the light rays in the figure.
(a) We have assumed, for the sake of simplicity, that the objects we are looking at are "infinitely far away" and that the eyes focus as if the objects were placed infinitely far away. How are these assumptions manifested in the way we have drawn the rays in the figure?
(b) Explain in particular which construction rules for light ray optics we have used for the red and violet ray in the figure.
(c) Show that the (angular) magnification (numerical value) of the Galilean telescope is given by the relation $M = f_1/f_2$, where f_1 and f_2 are the numerical values of the focal lengths of the objective and eyepiece, respectively.
(d) (Somewhat difficult) Would it be possible to use eyepiece projection enabling this telescope to be used for recording pictures directly on an image sensor or film? Explain.

39. A laboratory microscope has an objective with focal length 8.0 mm, and an eyepiece with focal length 18 mm. The distance between the lens and the eyepiece is 19.7 cm. We use the microscope so that the eyes focus as if the object is placed infinitely far away. We treat the lenses as if they are "thin".
(a) What distance should there be between the object and the objective when using the microscope?

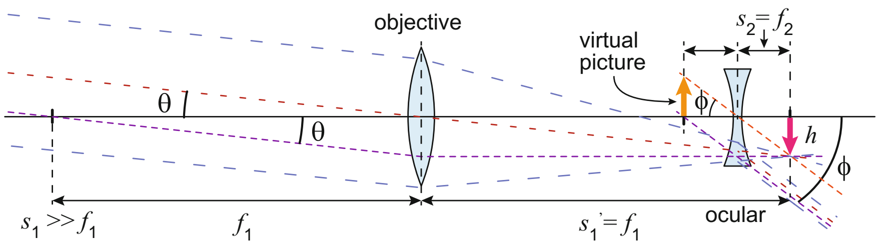


Fig. 12.29 Galilean telescope

(b) How large is the linear/real magnification provided by the objective (when used alone)?
(c) How large is the magnification provided the eyepiece alone?
(d) How is magnification defined for a microscope?
(e) What is the magnification of this microscope?

40. Show that when two thin lenses are in contact, the focal length f of the two lenses together will be given by:

$$1/f = 1/f_1 + 1/f_2$$

where f_1 and f_2 are the focal lengths of the individual lenses. We have a converging meniscus-shaped lens with refractive index 1.55 and radii of curvature 4.50 and 9.00 cm. The concave surface is turned vertically upwards, and we fill the "pit" with a liquid having a refractive index n = 1.46. What is the total focal length of lens plus liquid lens?

41. In this task, we will compare the cameras in an iPhone 5S and a Nikon D600 SLR camera with "normal lens". The following data are provided:
iPhone 5S: The lens has a focal length of 4.12 mm and aperture 2.2. The sensor chip has 3264 × 2448 pixels and is 4.54 × 3.42 mm in size. Nikon D600 with normal lens: The lens has a focal length of 50 mm and aperture 1.4. The sensor chip has 6016 × 4016 pixels and is 35.9 × 24 mm in size.
(a) Determine the effective diameter of the two lenses.
(b) Determine relative irradiance that falls on the photo chip in each camera. We ignore the influence of different image angles. (iPhone has a maximum image angle of 56° while the 50 mm lens on a Nikon D600 has a maximum image angle of 46°.)
(c) Fig. 12.30 shows identical snippets from photographs taken under approximately the same conditions with an iPhone device and a Nikon D600 device. Try to describe the difference in the quality of the pictures, and try to explain why there may be such a difference.
(d) Would the graininess in the iPhone image be larger or smaller if we increased the number of pixels in the iPhone to the same level as the D600?

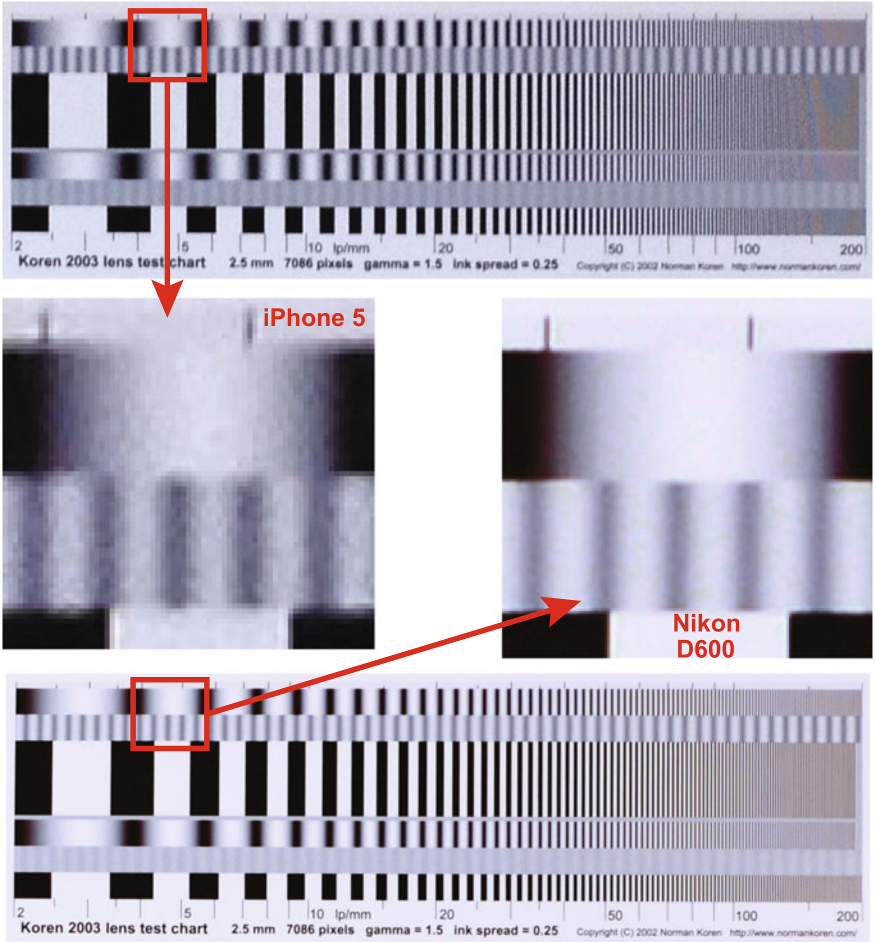


Fig. 12.30 Parts of photographs taken with the iPhone and Nikon D600. The length of Kohren's test chart was approximately 1/4 of the longer dimension of the picture in both cases. Notice not only how close the lines can lie and yet be distinct from each other, but also the graininess in the grey parts of the pictures. Details are likely to be better when the images are viewed on screen with some enlargement, than when they are viewed on A4-size paper. (Effective ISO values are not necessarily comparable.)

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