Chapter 11
Measurements of Light, Dispersion, Colours

Abstract Light and sound are the two most important wave phenomena people experience in daily life. We start this chapter by defining various concepts and units used for characterizing light and light sources, both in a purely physical setting and as judged by human vision. We then give important characteristics of the wavelength ranges of the various receptors in the human eye and how our colour vision makes the foundation of additive colour mixing in modern colour picture technology. We also point out how dispersion leads to colour spectra and boundary colours for specific boundary conditions. A few curious phenomena found in human colour vision are also mentioned.

11.1 Photometry

When discussing sound, we mentioned the dB scales used for specifying sound intensity and the like. We saw then that the sound volume could be given in terms of a purely physical unit, such as the amplitude of the average oscillations of air molecules, or as the local pressure amplitude while a sound wave passes, or as dB(SPL). Such measurements have a limited utility since oscillations with a frequency of less than about 20 Hz or higher than about 20 kHz do not matter, no matter how powerful such pressure oscillations in air is. Therefore, we had to introduce our own volume scale related to how sensitive the human ear is to sound with different frequencies. We weighed contributions at different frequencies according to the sensitivity curves for our hearing and devised scales such as dB(A) and dB(C).

Similarly, two parallel measuring systems are employed when we want to specify light levels/light intensities. A "radiometric" measurement system is based on physical energy measurements in units related to watt in one way or another. On the other hand, the "photometric" measuring system is based on visual impression, i.e. on the sensitivity curve for the human eye. Here the basic unit of measurement is lumen (which is itself based on the SI unit candela).

There is no simple proportionality between radiometric values and brightness as perceived by the eye. A pure infrared source may have a high radiometric intensity in

© Springer Nature Switzerland AG 2018

A. I. Vistnes, *Physics of Oscillations and Waves*, Undergraduate Texts in Physics,
https://doi.org/10.1007/978-3-319-72314-3_11

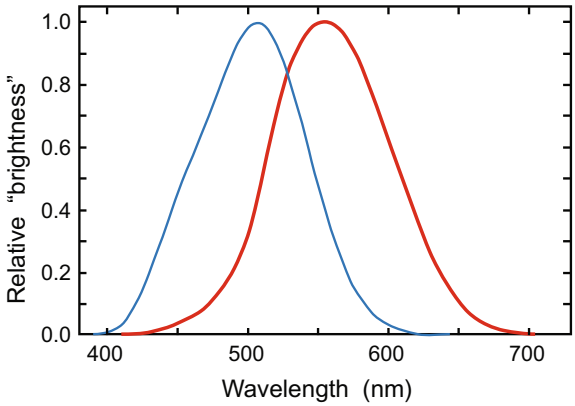

Fig. 11.1 Curves showing how "bright" the human eye perceives different spectral colours to be when the radiance to the eye is kept constant. The red curve (*on the right*) gives the "brightness curve" for colour vision (cones), while the blue curve gives the corresponding information for night vision (rods). The first has a peak at 555 nm (green), while the other has a peak at about 505 nm. Curves of this type vary slightly from person to person, and a standard curve can be made from many measurements. Dickyon, Public Domain, Modified from original [1]

W/m^2, yet the eye will not perceive almost any light from such a source. Hence, we need also units of measurement that are linked to human perception of light levels.

Figure 11.1 shows the sensitivity curve of the eye, for both colour vision (the cone cells) and night vision (the rod cells). It is only the shape of the curve shown, not absolute sensitivity (the peaks are normalized to 1.0).

The changeover between radiometry and photometry is completely analogous to the changeover between sound intensity in W/m^2 or dB(SPL) on one side and dB(A) (or dB(C)) on the other (see Chap. 7 on sound and hearing). We must also integrate physical measurements in W/m^2 and weigh the contributions for different wavelengths with the eye's sensitivity to brightness (Fig. 11.1) for the same wavelength. For colour vision, this means, to take an example, that it takes twice the intensity of light at 510 nm (or 620 nm) to contribute as much to the perceived brightness as light at 555 nm (see Fig. 11.1).

Absolute scale in photometric contexts is determined by setting a correlation between radiometry and photometry for the peak of the curve. The context exists in the definition of the base unit of photometry in the SI system, namely *candela* (abbreviated *cd*):

Monochromatic light with frequency 540×10^{12} *Hz (wavelength about 555 nm) and radiant intensity 1/683 W/sr, by definition, has a brightness of 1.0 candela (cd).*

We will define more precisely what we mean by radiation intensity. At the moment, it is enough to note that the definition connects a radiometric quantity with a photometric quantity for just a single wavelength.

The number 1/683 seems strange, but is related to the fact that the unit used previously was "normal light", which corresponded approximately to the light from a candle. Candela is chosen to match the old unit.

It may be noted that in the SI system there are only seven basic units, so candela really plays a central role. It is basically rather odd when we notice that the unit is so closely linked to human perception. On the other hand, the definition of candela given in the SI system is approximately the same as if it were a radiometric quantity, since the light source has only one wavelength.

Why do we see wavelengths around 500 nm?

As we saw in Chap. 9, visual light is restricted to a very narrow wavelength interval compared to the very impressive range of electromagnetic waves. One can wonder why this is so.

From an evolution perspective for life on earth, the explanation is rather obvious. Electromagnetic waves are strongly absorbed in water for a broad range of wavelengths (Fig. 11.2). Only for a relative narrow range around 500 nm, the absorption is much less than for electromagnetic waves in the infrared and ultraviolet range. The absorption minimum is actually near 500 nm. This number should be compared with the sensitivity curves in Fig. 11.1.

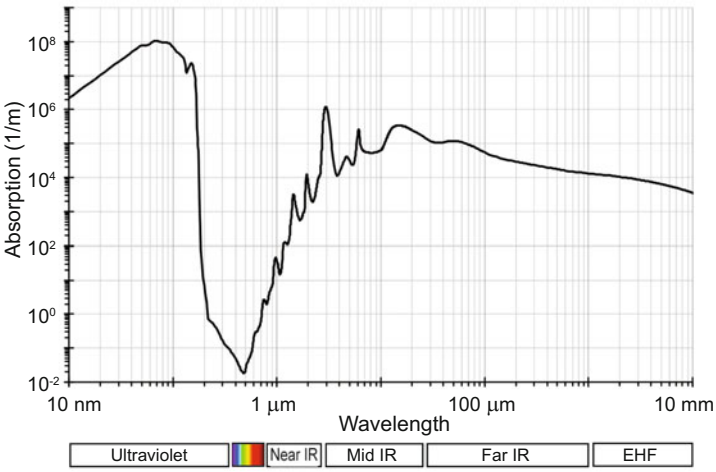

Fig. 11.2 Electromagnetic waves are strongly absorbed in water for a broad range of wavelengths except for the relatively narrow range 200–800 nm. The range of visual light is marked with the spectrum box at the bottom of this figure. See text for details. Kebes, CC BY-SA 3.0 GNU Free Documentation License 1.2. Modified from original [2]

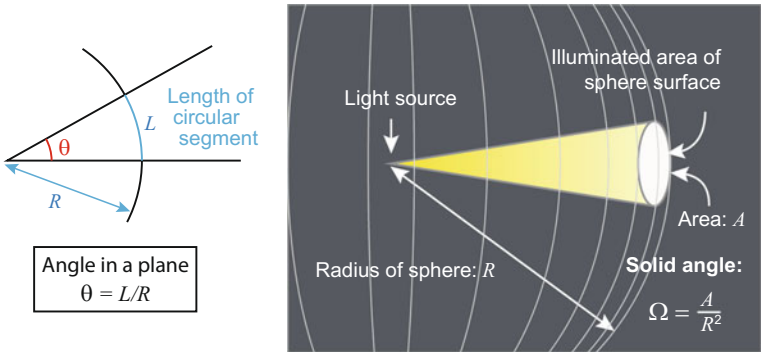

Fig. 11.3 An angle between two lines in a plane is defined as the length of L to an arc (of a circle with centre at the intersection of the lines) divided by the radius R to the circle. The unit of measurement is *radian*. A *solid angle* is defined as an area of A of a spherical segment (part of a sphere with a centre at the vertex of a conical surface that delimits the solid angle) divided by the square of the radius R^2. The unit for the measurement of a solid angle is *steradian* (sr)

Since all life on earth is based on aqueous media, vision and reflection of colour pigments from within plants and animals would be severely hampered if our vision was based on electromagnetic waves outside the 200–800 nm.

The solid angle

We will shortly go through the most common concepts related to the measurement of light, but we start by defining the concept called "solid angle", since this is included in several quantities in terms of which light is measured.

Figure 11.3 illustrates that the definition of the solid angle is a natural extension of the definition of the plane angle. A plane angle is given as the length of a circular arc divided by the radius of the circle. The angle is then independent of the chosen radius.

A solid angle is the area of a spherical segment limited by the conical surface, divided by the square of the radius of the shell.

$$\Omega = \frac{A}{R^2} \ .$$

With this definition, the solid angle turns out to be independent of the choice of the radius, which is what we want. A solid angle that includes a hemispherical shell will have the value

$$\Omega_{\text{hemispherical shell}} = \frac{4\pi R^2/2}{R^2} = 2\pi \ .$$

A solid angle that includes the *entire* space (all possible directions from a point) will then be 4π. The SI unit for the solid angle is a *steradian* (abbreviated *sr*).

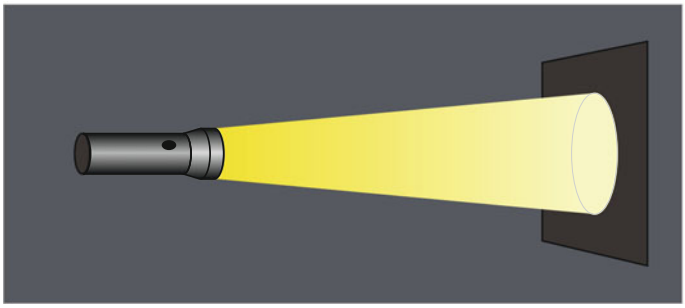

Fig. 11.4 A pocket torch emits light rays in the form of a divergent beam with a certain solid angle. The beam hits a surface, the illuminated part of which then acts as a secondary light source emitting light more or less in all directions in front of the surface (but only a matt or Lambertian surface will become an isotropic source). The figure forms a starting point for defining different photometric quantities

Measurements based on solid angles are usually interesting only when they are conducted far from the source compared with the physical size of the source.

The simple units of measurements

Measurement of light involves many more concepts than measurement of sound does, possibly because the output of a light source can be everything from a narrow beam to light travelling along almost all directions. A laser can produce a very narrow beam of light, while an old-fashioned Edison-type filament lamp emits with approximately equal brightness in any direction. It is not easy to generate narrow sound beams, unlike the case of light beams, simply because the wavelength of sound waves is much greater than the wavelength of light.

The Wikipedia article on "SI radiometry units" lists altogether 36 different radiometric quantities that are in use. We will content ourselves with the six most common quantities used. We choose to provide both the radiometric and photometric quantities at the same time for each type of measurement, using the abbreviation "RAD" and "PHOT" to distinguish what is what.

We choose to refer to Fig. 11.4 that shows a light beam from a pocket torch (flashlight) that strikes a surface a short distance away.

We often wish to send a powerful beam of light from a torch. When companies advertise that a flashlight reaches for example 800 m, the beam must have high power and it must not be too wide. In order for us to see what the beam hits, the light must have not only a high power, but a significant part of this power must lie in the wavelength range that is perceivable to the eye.

Light sources, light intensities, illuminations, etc., are described through a number of concepts and units. Let us describe the most important ones:

How powerful light do we have within a defined solid angle?

- RAD: *Radiant intensity* $I_{e,\Omega}$ is measured in W/sr. Characterizes radiation per unit solid angle in a given direction.
- FOT: *Luminous intensity* I_v (brightness/light intensity) is measured in candela: cd = lm/sr. Characterizes visible light intensity from a light source (per unit solid angle) in a given direction.

On a dark winter night, we wish to light our living room to make it a cosy place. We would evidently use lamps that spread the light a lot. Lights with narrow beams will not provide a pleasant lighting. We are then interested in how much light we get from a light source, and in this case, we do not care how effective the lamp is (how much current it draws). In the past, we switched from a 40 W to a 60 W bulb when we wanted more light. Today, we look at how many lumens the lamp gives since different types of lamps have different efficiencies in generating visible light. To get more light, for example, we change from 500 to 800 lm, and then, following consideration becomes important:

Total amount of light, summed over all directions:

- RAD: *Radiant flux* Φ_e is measured in W. Characterizes the power of *light* from sources (note: not how much power is drawn from the supply!).
- FOT: *Luminous flux/luminous current* Φ_v is measured in lumen: lm $\equiv$ cd sr. Characterizes the flow of visible light coming from the source.

Here it is worth noting that the watt in radiometry corresponds to the lumen within photometry!

The relationship between lumen and candela is that the lumen integrates light in all directions, while candela indicates how much light goes within a chosen solid angle. For a light source that emits approximately the same light in all directions, the number of lumen will be 4π times the number of candela. For light sources with a very narrow light beam, the candela value will be much greater than the lumen value.

A light bulb does not last forever. For example, some last for 500 h, and others are claimed to last for several thousand hours. The cost of using different types of light bulbs, of course, depends on the power consumption, but it also depends on how much light we can get out of a bulb before it is exhausted. Therefore, the total amount of light energy may be of interest in certain contexts:

Total amount of energy/light, summed up over time:

- RAD: *Radiant energy* Q_e is measured in J. Characterizes the source.
- FOT: *Luminous energy* Q_v is measured in cd sr s = lm s (candela steradian second = lumen second). Characterizes the source.

You may have seen that the old Edison light bulbs often had a milky coating on the inner surface of the outer glass envelope. The same is found on many modern LED bulbs as well. There are several reasons for such a coating, but one of the reasons is that the light bulb should not be too uncomfortable to look at directly. It is more comfortable if a light source emits light from a large surface than a small one.

We cannot look directly at the sun since the intensity is so large compared to its size. But the moon can be viewed directly without hurting the eyes. However, if all the light from the moon had come from a tiny area in the sky, our eyes would feel uncomfortable when looking directly at the moon.

In other words, it is not just the amount of light that comes from a light source that matters, but also how large the area (seen by the observer) the light comes from. This takes into account the following criterion:

How powerful is the light within a given solid angle, per unit surface of the source:

- RAD: *Radiance* $L_{e,\Omega}$ is measured in W/(sr m^2). Characterizes radiated power per square metre of projected surface per steradian in a given direction.
- FOT: *Luminance* L_v is measured in cd/m^2 (equivalent to lm/(sr m^2).) Characterizes radiated visible light intensity from each (projected) square metre of the source.

When we use a flashlight for, say, reading a map in the dark or when we have a reading lamp and want to read a book, we need to ask for the amount of light that illuminates the map or book. This is one of the simpler light measurement quantities, and here are the definitions:

Light intensity that falls upon a surface:

- RAD: *Irradiance* E_e is measured in W/m^2. Characterizes radiation intensity towards a surface.
- FOT: *Illuminance* E_v is measured in lm/m^2 $\equiv$ lux. Characterizes luminous flux towards a surface.

A book usually consists of black letters on a white paper. This is not accidental, because the paper itself does not emit light, but it "reflects" light that falls on the paper. In order for us to read a book, there is a requirement for how much light per unit surface comes from the paper to our eyes.

There is a similar requirement for a mobile phone screen or a computer screen or a TV. The quality of the image and how comfortable it is to view the screen depend on how much light per unit area comes from the screen:

Light intensity radiating from a surface:

- RAD: *Radiant exitance* M_e is measured in W/m^2. Characterizes radiation intensity from a surface.
- FOT: *Luminous emittance* M_v is measured in lm/m^2 $\equiv$ lux. Characterizes Luminous flux that radiates from a surface.

Now we are done with reviewing the quantities that are commonly used in light measuring. In addition, there is another item that falls in the list of the photometric quantities and is frequently used in connection with energy efficiency. This last quantity is:

Efficiency to convert electric power into visible light:

- FOT: *Luminous efficiency* η is measured in lm/W. Characterizes how effective a light source is to convert physical power into visible brightness. In this context, the number of watts is simply electrical power drawn from the mains.

Some additional comments

Experience has shown that it is difficult for many people to grasp the difference between radiation intensity and radiance. We will therefore try to concretize. Radiation intensity tells us how much intensity emanates from a light source in a certain direction, as compared to other directions. A spotlight bulb will provide significantly higher intensity in the direction of the spot than in a direction beyond the cone where most of the light is confined. We can describe this distribution by, for example, a graph that shows the radiation intensity as a function of angle from the axis of the luminous cone. The intensity (measured, e.g. as irradiance) at different distances from the cone will vary with the distance, but the radiation intensity, which is the power per unit solid angle, will be independent of the distance from the source.

Radiance, a quantity that closely resembles radiation intensity, is used in cases where the light source is extended so that it is meaningful to specify the amount of radiation intensity that comes from different parts of the source. How much radiation intensity there is, for example, from areas approximately in the middle of the sun (seen from the earth) and how much radiation intensity do we get from areas further out to the edge of sun (seen from us)? We normalize the intensity of radiation from the *projected* surface and arrive at the radiance. Areas near the edge of the sun (seen from the earth) are inclined in relation to the view from the earth. Then we divide the radiation intensity with an apparent area seen from us, that is, an imaginary area that is perpendicular to the direction of view from us that covers the area we calculate the radiation intensity from. For areas at the centre of the solar disc, we do not need to make such a correction since the solar surface in the middle of the solar panel is perpendicular to the direction of vision for us. Then projected surface will be

identical to real surface, which simplifies the calculation of the radiance (radiation intensity per (projected) surface of the light source).

Lumen (lm) is a derived unit: candela multiplied by the solid angle. Lumen indicates how much visible luminous intensity a source emits (integrated across all directions). A source that provides many lumens will provide more visible light than a source with few lumens (see Fig. 11.6).

Let us try to put place of the quantities in context:

Suppose that we have a spotlight source that has a light flux of 4π lumen in all. This is an integrated brightness for the source (including all directions.)

The light intensity from this source is 1 candela in all directions (assuming a dot light source). The brightness is 1 candela no matter how far away from the light source we are, because the brightness characterizes the source and not the light at a given location.

However, if we set up a screen across the light direction, the illumination density/illuminance on the screen will decrease with the distance from the source. Our spotlight source with light flux 4π lumen will have an illuminance of 1 lux on the inside of a spherical shell with radius 1 m centred on the light source. If we double the radius of the spherical shell, the illuminance decreases to 1/4 lux.

Our eye can adapt to an enormous range of light intensities. The ratio of the largest and smallest intensity it can handle is about 10^{10}–10^{12} (depending on what source we consult). However, the eye cannot handle such a large light intensity range at one and the same time. The eye adapts to the average level to which it is exposed. After such adaptation, the ratio of the largest and smallest intensity which the eye can handle *at the same time* is about 10^4. Within this range, the eye can distinguish between about 50–100 brightness levels (grey shades).

Figure 11.5 shows an overview of luminance in our surroundings under different conditions. For many years, it has been argued that the faintest light the human eye can perceive is a few photons per accumulation time in the rod cells (i.e. five to ten photons per second within a tiny area of the retina). However, this is an opinion based on photons as indivisible particles, a view about which there is no consensus (see

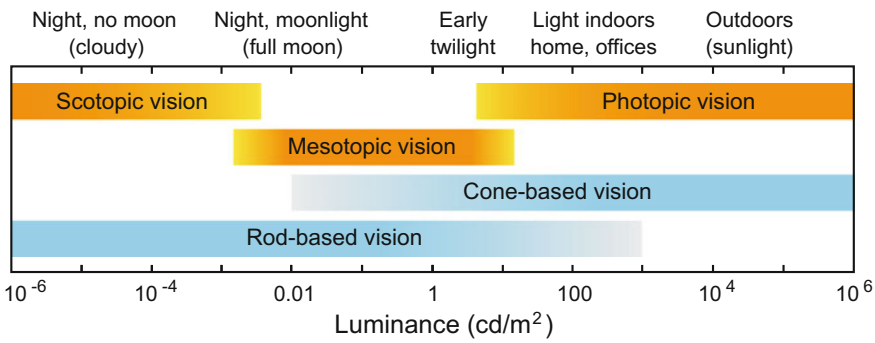

Fig. 11.5 Approximate light level (luminance) in the environment where our visual sense works. The figure is inspired by Fig. 16.2 in [3]

the reference list at the end of the chapter). Regardless of the exact level, it is an incredibly low intensity compared with full sunlight.

Modern light-sensitive detectors (so-called single-photon detectors) are somewhat more sensitive than the eye, but can suffer permanent damage if the light intensity becomes more than about 10^6 times the lowest light intensity they can detect. However, there are many factors that easily make a comparison with the eye misleading. Sensitive detectors are expensive!

11.1.1 Lumen Versus Watt

In recent years, much attention has been paid to the efficiency of light sources. An Edison light bulb, to coin a phrase, converts about 80% of the energy to radiated energy. The rest is spent hot in heating the filament and the terminals, etc. Of the radiated energy, most is in the infrared region, which is not visible. When using such lamps in environments where we need heating, the low efficiency of making visible light is not a disadvantage. In environments where, on the contrary, we need to remove heat (in warm climates), such lamps are clearly undesirable. We get too little visible light out of the energy used.

In recent years, there have been a lot of different light bulbs on the market with many different sockets and operating voltages (see Fig. 11.6). There are classical filament bulb, halogen bulb, fluorescent lamp and the so-called energy-saving bulb based on the fluorescent lamp, and a number of different LED (light-emitting diodes)-based bulbs.

Fig. 11.6 In part **a**, some examples of today's light bulbs are shown. A particular interesting source of illumination is organic light-emitting diodes. Picture **b** shows one of the first commercial OLED lamps available in the market. The OLED light panel is extremely thin, as seen in panel **c**, and the light is radiating evenly from the total area of the film (**e**) providing a pleasant illumination. The colour spectrum reveals that this OLED radiates light relatively evenly from the visible spectrum (**d**). It is not as good as for a classical filament bulb, but far better than some fluorescent lamps

There are big differences in how efficient the light bulbs are, even when they are of the same type. Fortunately, the manufacturer often indicates the light flux in the number of lumens, but even this is not sufficient. Some bulbs collect most of the light into a relatively narrow beam while others provide light in almost any direction. It has become quite difficult to find one's bearings.

On the Web, we can find different overviews of the light output for different types of light sources. These overviews change several times a year due to developments that take place. From my own observations in shops, and a table on Wikipedia, April 2018, under the term " luminous efficacy ", the following overview for different light sources can be given (light source, light output given in lm/W):

- Stearin candle: 0.3
- Classical incandescent lamp: 5–18
- Halogen bulb: 17–24
- Fluorescent tube (including energy savers): 46–104
- White LED (light-emitting diodes): 45–170
- Theoretical limit for white LED: ca 260–300
- Theoretical limit for an optimal blackbody radiating source (5800 K): 251.

It is worth noting that the light output from, for example, LED lamps differs considerably from one type of LED to another (a factor of four in difference according to actual observations and the overview in Wikipedia). If we want to save energy and think about the environment, we should take a close look at the specifications before choosing new lamps and bulbs!

There is currently intense research and development on new types of light sources. The fact that the Nobel Prize in Physics in 2014 went to the pioneers who found out how we can make blue LED sources shows how important lighting is for the world community. That UNESCO pointed out 2015 as "The International Year of Light" reinforces the sense of importance attached to light.

11.2 Dispersion

We have previously found that dispersion causes light with different wavelength to have different velocities through glass. This means that the refractive index varies with wavelength.

In Fig. 11.7, a diagram shows how the refractive index of light changes with wavelength of a regular type of optical glass (Schott BK7). The curve varies considerably for different types of glass, so if we want to demonstrate the different colour phenomena mentioned in this chapter, a type of glass should be used which gives a large dispersion (often associated with high refractive index). In binoculars, we prefer to search for materials with the least possible dispersion in order to minimize the so-called chromatic aberration (more about chromatic aberrations in Chap. 12). This applies primarily to wavelengths in the visible area.

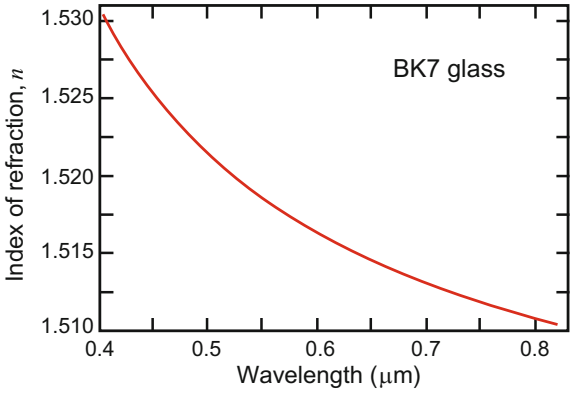

Fig. 11.7 Example of dispersion for one type of optical glass (BK7)

Newton's classic colour experiment is usually described in more or less the following words: "When light is sent through a glass prism, we get a spectrum". In practice, more is needed to ensure that the spectrum has the quality we expect and Fig. 11.8 indicates this. We must send light through a *narrow* slit, and need a lens to make a sharp image of the slit on a screen (as described in the next chapter). We may mimic this situation if we, for example, send sunlight through a narrow slit, without a lens, to make a somewhat blurred image of the slit on the screen. The quality of the spectrum is then rather poor.

Only when these conditions are satisfied, can we put the prism in the light path with a side edge parallel to the slit. The light beam will then deflect, but will form a shifted image of the slit on the screen. We may need to rearrange the lens's location so that the image on the screen becomes as sharp as possible.

The resulting spectrum can be described as a *multitude* of images of the slit, slightly offset from one another. If the light source contains a continuous range of wavelength components throughout the visible region, red light would appear in one place, green somewhere else, and blue on yet another place. The constellation of all these images constitutes a visible "spectrum" on the screen.

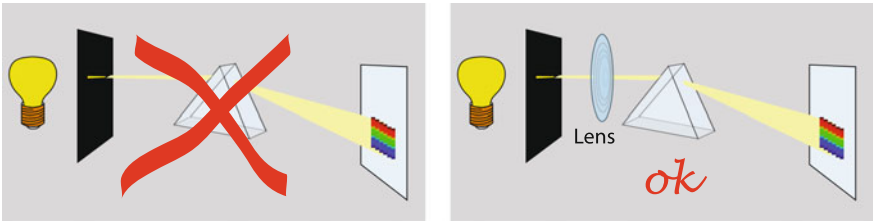

Fig. 11.8 Newton obtained a coloured spectrum when he imaged a slit on a screen and let the light on its way pass through a glass prism. The geometry of the layout is crucial to the result

11.3 "Colour". What Is It?

A number of details do not appear in such a simple description of Newton's spectrum as the one given above. Firstly, what do we mean by "colour"? Many people have a very inadequate impression of colours.

Colour is something we *experience*, a sense impression. The colour sensation has a complicated connection with the physical stimuli that affect our eye. The light is partially absorbed into special proteins in the retinal rods and cones, cells called "photoreceptors". The rods are the most photosensitive receptors and are responsible for sight in the dark. The rods cannot provide colour information and will no longer be discussed here.

The cones, on the other hand, provide colour information. There are three types of cones that can initially be called blue-sensitive, green-sensitive and red-sensitive. These are also referred to as S-, M- and L-cones where the letters stand for "short", "medium" and "long" wavelength for the peak in their sensitivity curves.

In short, the sensitivity and the most sensitive area of the three cones are as follows:

- S-cones, 380–560 nm, peak 440 nm
- M-cones, 420–660 nm, peak 540 nm
- L-cones, 460–700 nm, peak 580 nm.

Quite different numbers are reported in different studies because of individual differences from person to person, and partly because determination of a sensitivity curve is not a trivial task so the values are somewhat dependent on the measurement method used. CIE (see reference list) has adopted a standard that gives the average value for each of the peaks.

Figure 11.9 shows the sensitivity curves for the three types of cones. The figure must be so understood that if we send *monochromatic light* (light with only one wavelength), the curves show the sensitivity of each of the three cone types. At

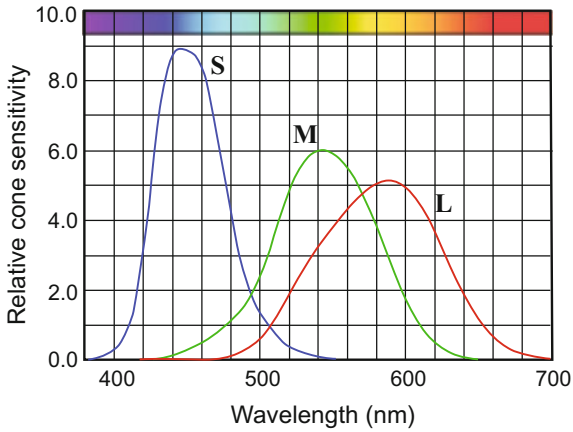

Fig. 11.9 Relative sensitivity curves for the three types of cones in our eye

about 444 nm, the "blue-sensitive" cones (S-cones in the figure) are about twice as sensitive as at about 480 nm (8.8 and 4.2 along the vertical axis, respectively, in the plot). This means that one must double the intensely of light at 480 nm to get the same response from this cone as light at 444 nm. Put in another way: if a person is born with defective green-sensitive (M) and red-sensitive (L) cones, he or she will have exactly the same viewing experience for monochromatic light at 480 nm as for monochromatic light at 444 nm, but at half the intensity. For monochromatic light with wavelength less than 380 nm and greater than 560 nm, this type of cone gives a negligible response.

The special thing is that the three curves in Fig. 11.9 overlap one another, in part very substantially! This means that monochromatic light with a wavelength of approximately 570 nm will stimulate (excite) both the red-sensitive and the green-sensitive cones about equally! We may gather then that the expressions "red-sensitive" and "green-sensitive" are really misleading, and we therefore proceed to mention only cones of types S, M and L (shortcuts for, respectively, "short", "medium" and "long" wavelengths).

In summary: The only signal that a visual cell (one cone) can send is a train of *identical neural pulses*, no matter what light stimulus excites the cell. However, the amount of light required to excite a cone is wavelength-dependent and the sensitivity curve differs for S-, M- and L-cones. The number of pulses per second coming from a cone is approximately proportional to the intensity of the light (assuming that the wavelength distribution of the light is kept constant when the intensity increases). These characteristic features apply to all three types of cones. There is no difference in the shape of the nerve pulses from S-, M- and L-cones.

The response is in a way "digital"; either we have a pulse or there is no pulse. The brain deduces colour information because it keeps track of what type of visual cell (cone) each nerve fibre comes from. If the brain receives, for example, about the same number of neural pulses per second sent by M- and L-cones from a region of the retina, and little from the S-cones in the same area, the brain gives us a greenish-coloured colour experience of the light that reaches this part of the retina (compare with Fig. 11.9).

A small digression:
When you see curves such as those in Fig. 11.9, you hopefully associate them with a fundamental phenomenon from the book's first chapters. You encountered curves that looked very similar to each of the curves in Fig. 11.9 when we discussed forced oscillations and resonance. Then, the resonance curves had frequency along the x-axis and some amplitude along the y-axis, while in Fig. 11.9 we

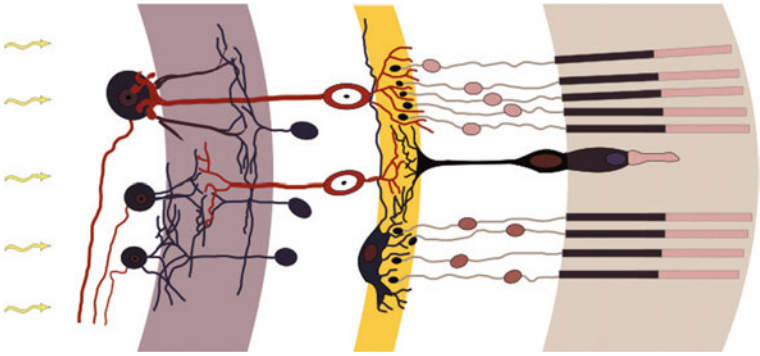

Fig. 11.10 Signals from the cones (to the right) are processed by many types of cells in our eye and on our way to and in the brain itself. The vision process is therefore very complicated. (Light is actually coming in from the left.) Cajal/Chris, Wikimedia Commons, CC BY-SA 3.0. The figure is based on [4]

have wavelength along the x-axis. However, since frequency and wavelength are linked via $f\lambda = c$, the wavelength axis can easily be converted to a frequency axis. According to semi-classical theory, there is a correlation between forced oscillations and absorption in the visual receptors.

Our vision is very advanced and the raw image we get from the actual visual cells (cones) is further processed in many different ways. Already in the retina we have four types of cells that process the response from the photoreceptors (cones) (Fig. 11.10). These are so-called horizontal cells, bipolar cells, amacrine cells and ganglion cells. Each cell has its own function, including contrast enhancement, or reacting specifically to temporal changes in brightness (e.g. when the subject is moving). The cells are also involved in the signal processing related to colour discrimination. There is also extensive processing of the signals from the retina in certain relay hubs in the visual field, and even more in the brain's visual centre. What an impressive machinery lies behind our visual perception!

We will first focus on the simplest principles of colour (chromatic) experience, and the *main rule* in this context is that the colour is determined by the interrelation between the amount of light absorbed in the three types of cones. Then we will mention other factors that also affect our colour experience.

11.3.1 Colourimetry

If we consider only monochromatic waves in the visible region, we find that light absorption in the three types of cones will change in a unique manner as the wavelength of light is varied. Monochromatic light provokes visual sensations known as "prismatic colours". These colours are in a class by themselves and are perceived as "saturated" colours. We cannot make a spectral red any redder than it already is (at least not with the colour index assigned to it).

If we let in light with *many* wavelengths, the response from the cones will equal (almost) the *sum* of the responses due to the individual monochromatic contributions. The summation rule can be traced to the superposition principle. Needless to say, this description holds only within a limited intensity range, but it will be adopted here for the sake of simplicity.

Energy absorption in M-cones can be expressed mathematically as follows:

$$M = \int \phi(\lambda) M(\lambda) d\lambda \tag{11.1}$$

where $\phi(\lambda)$ is the spectral intensity distribution of the incident light (formally called colour stimulus function). $M(\lambda)$ is the spectral energy sensitivity for M-cones corresponding to the middle curve in Fig. 11.9.

Energy absorption in the two other types of cones can be described in an analogous manner. The three expressions so obtained describe only relative absorption (because the expressions do not take calibration into account).

It is not hard to see that monochromatic light of ca 570 nm plus monochromatic light of ca 420 nm will provide the same stimulation of the three types of cones as will a mixture of monochromatic light at 660, 500 and 410 nm. The only proviso that must be met is:

$$M = \phi_1(570) M(570) + \phi_1(420) M(420) =$$

$$\phi_2(660) M(660) + \phi_2(500) M(500) + \phi_2(410) M(410)$$

and likewise for L and S. We obtain three equations with three unknowns (assuming that the two ϕ_1-values are known).

The upshot of the foregoing analysis is that *we can get the same colour sensation from widely different physical stimuli*. By "stimulus" we mean specific distribution of intensity for various wavelengths—in other words, the spectral distribution of the stimulating light. The spectral distribution of the light that appears to us as a special green colour can, as a matter of fact, be rather different from the spectral distribution of another light source, although we perceive the latter as exactly the same green colour as the first (termed "metamerism"). It is by no means to be supposed that "colour" is synonymous with the spectral colour defined above!

How fortunate for us that this happens to be so! We exploit it extensively today, especially in photography and coloured visual displays on a TV screen or computer monitor. In all these cases, we usually start out with three colours and mix them with each other in different proportions to form "all other colours". But there are some limitations.

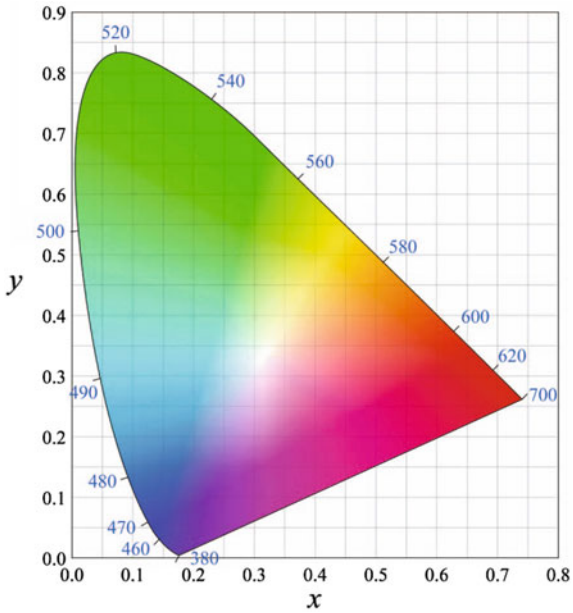

Fig. 11.11 "Colour Horseshoe" defined by CIE in 1931. Detailed description of this "xy colour space diagram" is in the text. NRC, Public Domain [5]

CIE 1931

Figure 11.11 shows a so-called colour horseshoe, which is devised according to a special scheme. Along its curved edge lie the "pure" spectral colours from red to violet. On the straight line between red and violet, called the "line of purples", lie the nonspectral magenta and purple-red mixtures. The centre of the horseshoe corresponds to light that is perceived as white.

Along the axes, so-called x and y coordinates are determined from a transformation of (S, M, L). How may a colour stimulus be determined by *three* parameters: (S, M, L) while the colour horseshoe reproduces colours only in a *two*-dimensional plot?

The three stimuli indicate *both* colour and light intensity information. For a given light intensity (or rather *luminance*), the three parameters will not be independent of each other. Only two can be chosen freely. By using an appropriate transformation of the cone absorbances, we can transform into two independent parameters x and y which indicate the colour irrespective of the light intensity (luminance). The inverse transformation is *not* unambiguous! The colour horseshoe in principle indicates all the colours we can experience at a given luminance and can therefore be regarded as a general "colour chart". It is therefore called a *chromaticity chart*.

The mathematics behind the current transformations has been developed over many years. The colour horseshoe was adopted in 1931 as a standard for colour measurement by CIE (Commission International de l'Eclairage in French, The International Commission on Illumination in English). The transformations used are still

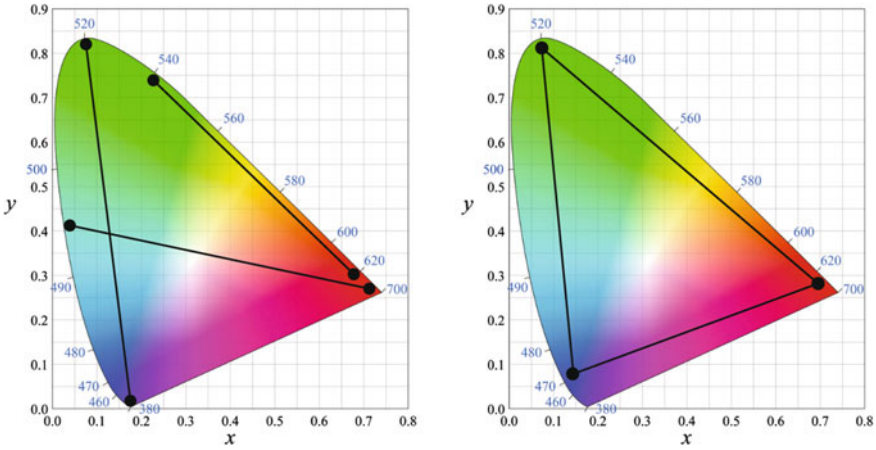

Fig. 11.12 Finding the colour for additive colour mixing corresponds to the "centre of gravity" for the colour coordinates that are included in the mixture. To the left are given three examples of the colours that can be obtained by mixing two colours, marked with straight lines. Further information is in the text. *Right part:* The colour scope of a computer screen using three types of coloured pixels lies within the triangle spanned by the colour coordinates of the pixels. The amount of colour within the resulting triangle is significantly less than the amount of colour that the entire colour horseshoe represents

being discussed, and several Norwegian physicists (such as Arne Valberg, Knut Kvaal and Jan Henrik Wold) have worked on this issue for many years.

The colour horseshoe is useful in many ways. If we start with two colours (two coordinates in the colour horseshoe) and mix them with equal weight (defined appropriately), our colour scheme will correspond approximately to the point in the colour horseshoe that is in the middle of the two points we started with. This is indicated in the left part of Fig. 11.12. Starting with equal amounts of near-spectral stimuli at 540 and 620 nm, the colour we perceive will be quite similar to the colour of a wavelength of 572 nm wavelength (most would denote it as yellow). However, if we mix in approximately equal amounts of near-spectral stimuli at 495 and 680 nm, we will perceive the colour mixture as approximately "white" (a light grey without colour).

When we consider a computer screen, a mobile phone screen, an iPad, a TV screen or the like, there are three types of light that build up the image: "red", "green" and "blue". These stimuli each have their coordinates (chromatic points) (x, y) in the colour horseshoe. *The colours we can form with these three primary colours are within the triangle that the three coordinate points form in the colour horseshoe.* The amount of all colours we can form with the three primary colours is called *colour scope*, for example, the computer screen.

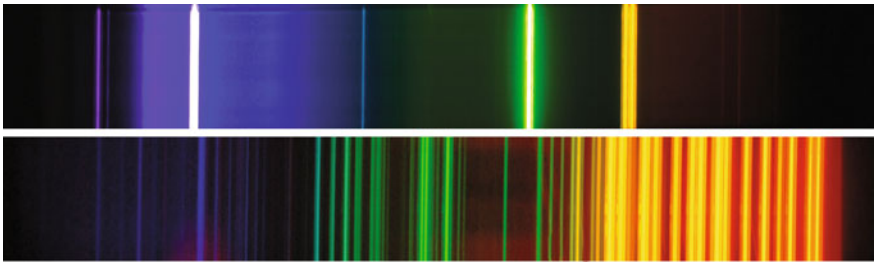

Fig. 11.13 Spectral lines show beautiful saturated colours when viewed directly in the laboratory. After photography and reproduction (as here), the amount of colour becomes much smaller

We can try to select three points and draw lines between them to identify which colours can be produced by the three primary colours, and we will then discover that a variety of colours lie *outside* the triangle as the points expand. An example of such a triangle is given in the right part of Fig. 11.12. Since an inner triangle can never cover the entire colour horseshoe, it means that the colours we can see on a computer screen, etc., are a rather pale image of the amount of colour we can experience in nature. A variety of colours on flowers, for example, are far more saturated when you see the flower in reality than what we can reproduce on a computer screen (or photograph for that matter).

An example of the lack of colour achievable with three-colour reproduction is shown in Fig. 11.13. In the figure, there are two spectra of gases: one with a few spectral lines and one with many more. Spectral lines are in fact the most saturated colours we can have, and when the lines are observed directly in a laboratory, we notice this. The same spectral lines shown in a photograph are just a pale copy of reality (as the figure shows).

In industrial context, different colour systems have been developed than CIE. Colours are included in far more parts of a modern society than we usually think about. For example, food quality is assessed using colours. Two colour systems used in industrial context are NCS and Munsell.

For example, Natural Colour System (NCS) is used when we buy paint in a store. The paint is mixed in the store. There are four basic colours in NCS (in addition to white and black), namely red (R), yellow (Y), green (G) and blue (B). When a colour code is 1020-Y90R, the first two digits give the mixture of white and black. In our case, these numbers are 10, which means that it must be mixed 10% black and 90% white. The two next digits tell us how much we should have of this grey colour in relation to the colour we want. The numbers 20 means that 20% of the grey colour we mixed will be used and 80% colour/colour. Y90R means that it will use 10% yellow and 90% red, and this colour mixture will be mixed with the grey colour in the ratio of 20% grey and 80% red/yellow.

11.3.2 Colours on a Mobile Phone or Computer Display

Let us now make a practical check on how colours are generated on a TV, mobile phone or computer screen. Figure 11.14 shows in the centre a small part of a computer screen with Windows icons. We have taken a picture closer to the screen to see details. A clip from the Google icon has the colours green, white and dark blue. Another snippet has the colours red, yellow, white and light blue.

To the right and to the left, there are selected representative "pixels" that the image is built with. Each pixel on this screen has three vertical fields. These fields can give the colours red, green and blue, respectively, and only these. These three colours correspond to the points shown in the right part of Fig. 11.12. We see here quite clearly that, for example, the colour yellow on the computer screen really is generated only by means of red and green light. The pixels are so small that the light from the red and the green field in a pixel hits the same eye cells.

Additionally note that dark blue or blue-black is generated virtually by using zero red and green light and only weak blue light. Light blue (slightly light blue-green), however, is generated with almost maximum of blue, some green and a little red. White is generated with powerful red, strong green and powerful blue at the same time. It is fascinating that, using only three primary colours, we can generate as many colours as we actually can!

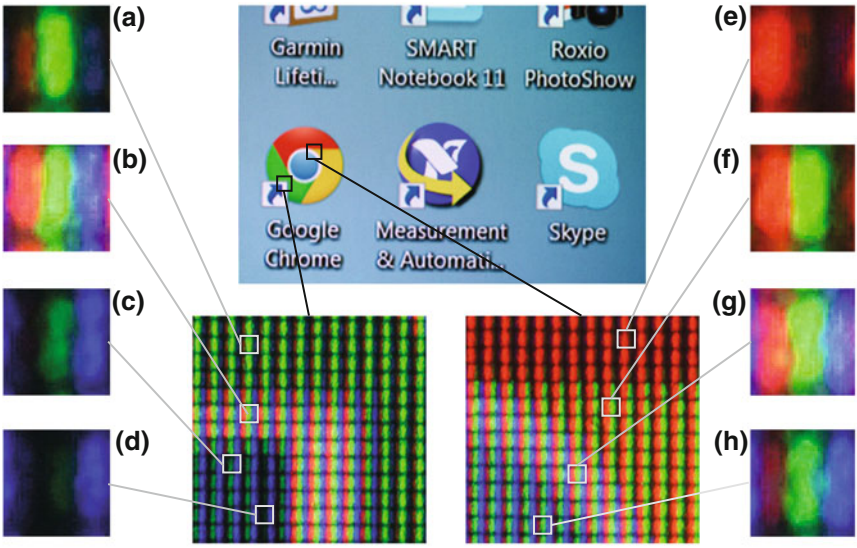

Fig. 11.14 Photographs of a computer screen. A section of icons on the "desktop" on a Windows computer is shown at the top middle. Two small clips from the Google icon are shown underneath. Pixels from different coloured areas are shown to the far left and right of the figure. A particular pixel *can only* emit red, green or blue light (over its entire area). The colours the pixels give us are **a** green, **b** white, **c** dark blue (blue-green), **d** blue-black, **e** red, **f** yellow, **g** white and **h** light blue (blue-green)

11.3.3 Additive Versus Subtractive Colour Mixing

Art posters use, for example, 7-colour, 9-colour, 13-colour printing. One of the reasons for this is that the colour gamut in the final image should be as large as possible. It is natural to draw parallels to the triangle on the right in Fig. 11.12 in this context. However, we must remember that when colours are mixed using pigments that are illuminated by an external light source, all colour blends are far more complicated than those we have mentioned above. We have so far just referred to *additive* colour mixture that occurs when mixing (superposition) of light. In an art poster (or in a painting or print of a photograph), *subtractive* colour mixing is at work. Pigments absorb some of the light that falls on them, and the light sent back to us will cause the pigmented surface to appear with a certain colour when illuminated, for example, with sunlight. If we add more pigments together, e.g. by mixing yellow and blue pigments, the surface will often look green. However, if the pigments are illuminated by light with only a few wavelengths (e.g. light from some diodes (LED) or from fluorescent tubes), there is no guarantee that the mixture of yellow and blue pigments will look green!

The best colour rendering is achieved with light sources that have a continuous spectral distribution, i.e. common old-fashioned light bulbs or halogen bulbs. In art exhibitions and the like, it is therefore important to use such lighting instead of fluorescent lamps, energy-saving light bulbs or simple types of LED lamps.

Incidentally, the word subtractive colour mixture is a little misleading. To find the cone absorption when the stimulus corresponds to light reflected from a mixture of two pigments, the spectral reflection coefficients of the pigments must be *multiplied* with each other.

By the way, Helmholtz was the first to describe the difference between additive and subtractive colour mixing. This happened about 200 years after Newton's colour mixing model based on the superposition of light (additive colour blend).

It is not trivial to make pigments from scratch. Often one uses natural pigments from, for example, plants or minerals. Only a limited number of pigments are available, and when we print an artwork, it may sometimes be useful to use more than three "colours" (pigments) to get as good reproduction as possible, even if the original is only found as RGB (three target figures) from a digital camera. We cannot expand the colour gamut relative to the image (colour gamut spun out of the RGB values), but we can *reproduce on paper* the colour gamut better than by using fewer pigments.

In order to achieve a larger colour gamut, we need to start with more than three stimuli already in the data recording. It helps little to start with a digital camera with only three detectors per pixel and believe that if only we had a good printer, then the overall result would be almost perfect! This is analogous to audio recording: if we are going to process sound at a sampling rate several times that used in CD audio, then there is no point in starting with a low resolution in the processing of sound and raise it later on. We must have the highest sampling rate already at the very first digitization of sound. In studio recording of sound, it is now fairly common to use higher sampling rates than the CD standard. For capturing images, a trend has started to employ cameras with more than three detectors per pixel, and as such, screens with more than red, green and blue luminous points. It is not inconceivable that in the future photography apparatus and computer monitors will be based on technology with more than three basic stimuli.

11.4 Colour Temperature, Adaptation

The formalism given in Eq. (11.1) and the description of mathematical transformations leading to the CIE chart can leave an impression that a certain spectral distribution of light will always give us the same visual impression (colour). That is wrong. Human vision is highly advanced and has incorporated a form of adaptation that is very useful. In short, the colours in a Norwegian flag will be judged to be red, white and blue, whether we view the flag under lamplight in the evening or in the sunshine under a brilliant blue sky during the daytime. This happens despite the fact that the spectral distribution of the light from the flag is quite different in the two cases. The ability to adapt is a consequence of natural selection: it was useful also for the Neanderthals to be able to make out the colours in the light from the fire in the evening as well as in direct sunlight.

The big difference in spectral distribution in these examples is well presented in Planck's description of "black body radiation", which is close to what is emitted by a hot body. In an incandescent lamp, the temperature of the filament is in the range of 2300–2700 K. The surface of the sun has a temperature of about 5500 K. This causes the continuous spectrum of these light sources to be quite different, as shown in Fig. 11.15. In the light bulb, the intensity of the blue spectral colours is much less than for the red ones, while in the spectrum of the sun, blue spectral colours are more prominent than the red ones.

Figure 11.16 illustrates the importance of different lighting if we detect the appearance of an object without adaptation. A digital camera is used to capture images of one and the same object in lamp and sunlight. The colour temperature correction in the camera could be set manually. Photos are taken at three different settings on the camera for each of the light sources. The settings are: "colour temperature 2700, 4000 and 5880 K".

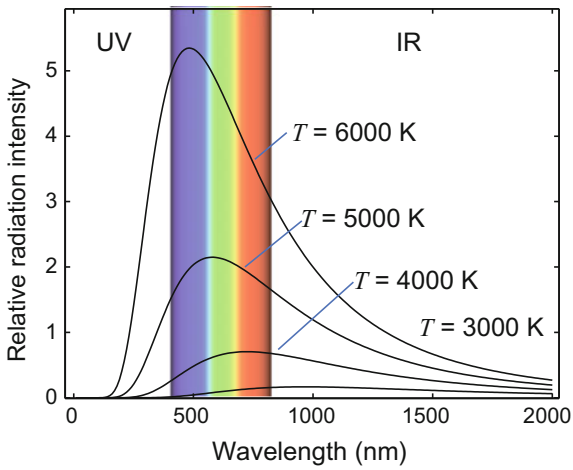

Fig. 11.15 Spectral distribution of electromagnetic waves emitted by hot bodies (blackbody radiation). The curves are values calculated from Planck's radiation law for different temperature of the body

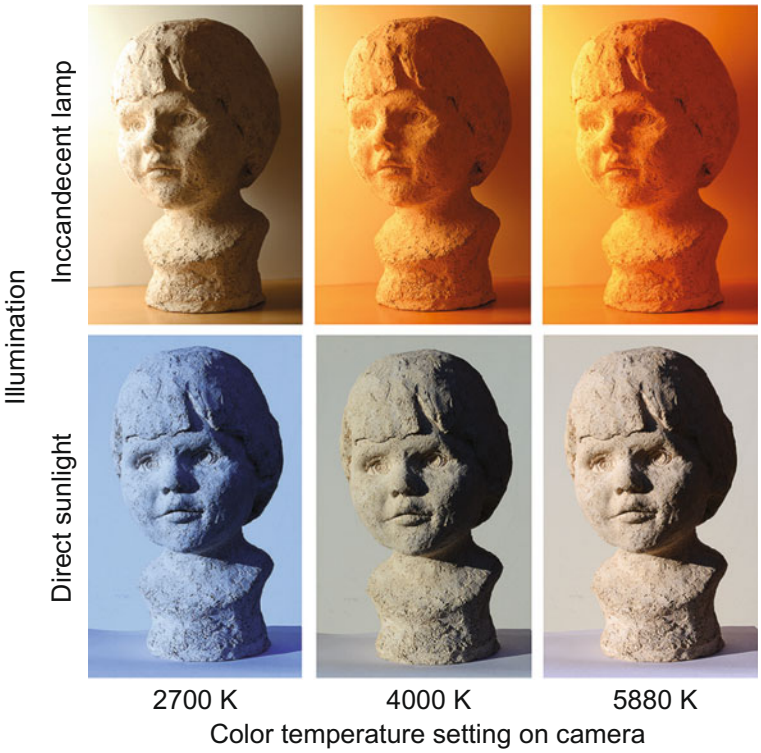

Fig. 11.16 Photographs of one and the same head sculpture in plaster in two different lights and three different manual colour temperature settings on the camera. The figure indicates the actual difference in spectral colour distribution from the object in lamplight and in sunlight. Nevertheless, with our visual sense, we perceive that the head looks almost white regardless of whether it is viewed under lamplight or sunlight. This is due to the visual adaptation capacity

Figure 11.16 shows that when we take pictures of objects in lamplight after setting the camera for approximately 2700 K, the pictures look as we expect them to, but they have a ghastly red appearance for a colour temperature setting of 5880 K. Similarly, images in sunlight often produce correct colour in the pictures if the camera is set to about 5880 K while the image looks very blue if we chose a colour temperature setting of 2700 K.

Most modern digital camera has a built-in form of adaptation similar to that of human vision. If we shoot the pictures in Fig. 11.16 with such a camera, the result would have been relatively "correct" in most cases, but if the object and its surroundings by themselves are very reddish or bluish, the camera could perform unintended corrections that can not easily be avoided.

We have now seen that human vision (the eye plus all further processing even in the brain) has a fabulous capacity for adapting to different spectral distribution of the dominant light source. What we call a red, green or blue surface depends not only on the absorption by the cone (S, M, L) of the light from the surface, but also to a large extent on the surroundings. It has some important implications: if we apply

colour correction of digital images, for example, in Photoshop or similar software, it is important to have a grey surface along with the image where the colours are to be assessed. If we still find that the grey surface looks grey, we have a reasonable guarantee that our eyes have not adapted to the image to be corrected. If we do not check the eye's adaptation mode in relation to a grey surface, we may deceive ourselves, and the final result may become unsightly.

11.4.1 Other Comments

There are also other forms of adaptation in our visual sense. The eye also adapts with respect to intensity. In sunlight, a "grey" flat surface reflects much more light than a "white" surface will reflect at dusk. Nevertheless, we call the first one grey and the other white. What we call white, grey or black surfaces depends not so on the intensity of light from the surface as on *the relative intensity from the surface in relation to the environment*. In a similar way, our colour assessment of a field in the visual image is greatly influenced by the colours in the neighbouring fields of the visual image. An example is given in Fig. 11.17. There are many other delights related to the eye and the rest of the visual system, not least related to contrasts (e.g. read about Mach band in Wikipedia) but we cannot take the time to talk more about this matter than we have already done.

Finally, a short comment about the colour horseshoe: If we look at Fig. 11.11 on several different computers, we will find that the colours look quite different from screen to screen. This is partly due to the fact that the three pixel colours are slightly different from one type of screen to another. Graphic artists often perform a screen calibration and transform colour information using matrices before viewing images on screen or before printing the images. Such a transformation is usually called a "colour profile". In the process of achieving a good colour profile, a standard card (an image) is used, for example, in the subject when shooting. The colour profile can then be designed so that the end result gets as close to the original default as possible. The X-rite colour checker is an example of such a chart. It can be obtained from Edmund Optics (see reference list).

Colour management is one of the issues we have to grapple with when handling today's technology. Colour correction is a profession!

11.5 Prismatic Spectra

Now that we know a little more about how we perceive colours, we are ready to return to Newton's colour spectrum from a prism. Many think of spectral colours as the colours they see in the rainbow: ROYGBIV (or Roy G. Biv): red, orange, yellow, green, blue, indigo and violet. But what do we really see when we examine a Newtonian spectrum produced by a narrow opening? Well, the spectrum looks like the top of Fig. 11.18. The curious thing is that, in fact, we see only red, green, blue

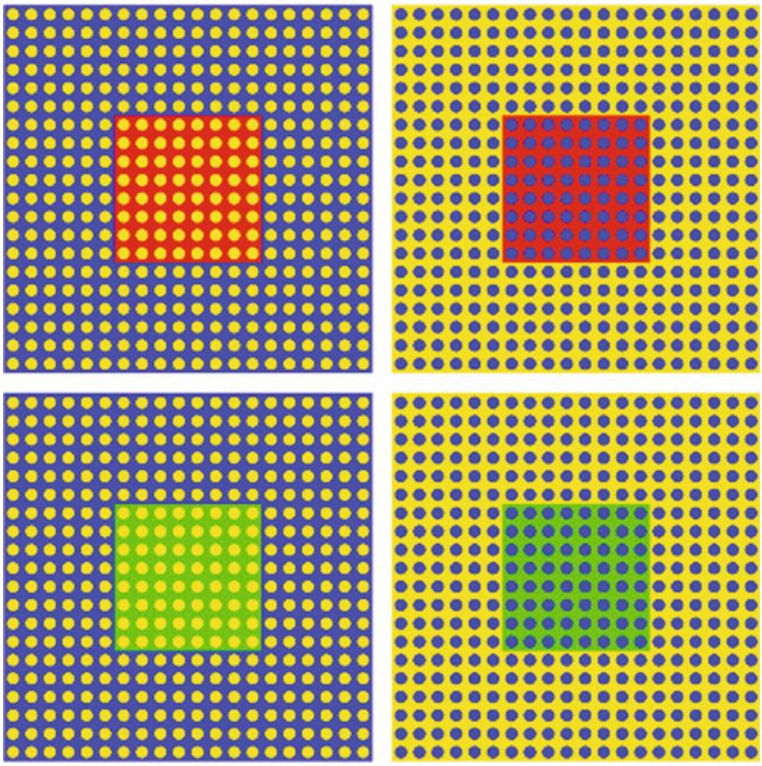

Fig. 11.17 Our colour experience of a surface is affected by colours close by. The red squares in the upper part of the figure have exactly the same spectral distribution, but look different. Similarly, the green squares in the lower part are identical, but look different since the neighbouring areas have a completely different colour

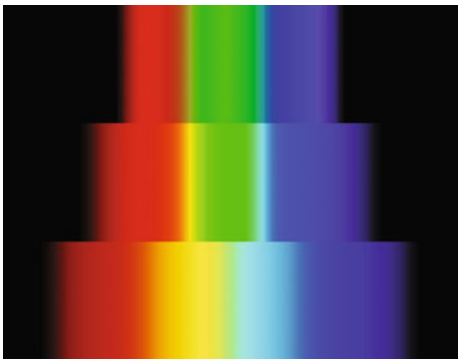

Fig. 11.18 Spectrum from a narrow slit (top) and from increasing slit width below

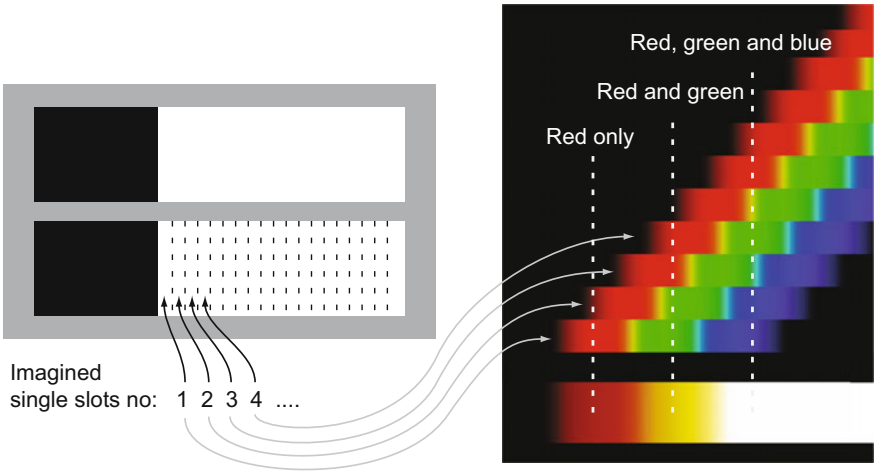

Fig. 11.19 Imaging of an edge can be considered as the sum of images of many slots next to each other. See the text

and partly violet. There is very little yellow and orange, and clearly less yellow than in the rainbow! How can it be explained?

The explanation is found by increasing the width of the opening a little. In the next two examples in Fig. 11.18, we have simulated spectra from slots with increasing width. *Now* we see a yellow area more easily! What is this due to?

This is because when we consider spectral colours, it is only a very narrow wavelength range that gives us the colour impression "yellow". Most of the yellow we perceive is due to the blending of red and green spectral colours (additive colour mixing). We will then get a coordinate point in the colour horseshoe that lies a bit within the edges.

To understand how we think of the colour scheme, refer to Fig. 11.19 which shows how the image would look if we did not image a hole on the screen but instead an edge between a surface without light and a surface with homogeneous "white" light. The light also passes here a glass prism. We can imagine that the bright area is a sum of many single slots adjacent to one another. Each of the slots (if sufficiently narrow) will provide a spectrum that looks red, green and blue. Each column is slightly offset in relation to the neighbouring area, so that the spectra are also slightly shifted in relation to one another.

If we add up the light striking different parts of the screen, we see that at the left end only the red light comes in. The sum is then red. Just to the right of this position, we get a mixture of red and green light, but no blue. The sum of these two lights will be perceived as yellow. Just to the right of this field again, we will get a mixture of red, green and blue. The sum is perceived as white. This is how it continues, and the result will be what we see at the bottom of the figure.

We see then that when we image an edge on a screen but make the light go through a prism, the edge will be coloured with a red and a yellow stripe.

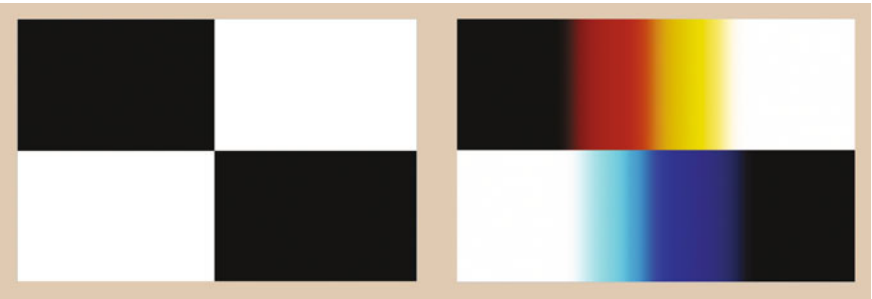

Fig. 11.20 There are two types of border colours, depending on which page is black and which is white in relation to the prism orientation. The left part of the figure shows the black and white distribution of light we start with. The right part shows how the image of the original light distribution would look if the light went through a prism. The colour effect is a simulation customized visual impression from a computer screen. Real reds look prettier, but they must be experienced in vivo!

If we image an edge where the light and dark have exchanged places, the violet/blue area will not blend with the other colours. Next to this, we get an area of cyan (a mixture of green and violet, crudely speaking). Next to this, we get a mix of all colours and we experience this as white.

An edge of this type will have two coloured stripes in the borderland between white and black: blue violet and cyan. The red-yellow and the violet-cyan stripes we call *boundary spectra* (sometimes also called boundary colours). An example of the boundary colours is shown in Fig. 11.20. When you see edge colours in practice, the stripes are much narrower (and nicer) than we can get an impression of in this figure, but it depends of course on distances and many other details.

If you look through a pair of binoculars and choose to look at a sharp borderline between a white and a black area in the periphery of the field of view, you will almost always see boundary colours at the edge. On good binoculars, where attempts have been made for reduction of the dispersion of light through glass (using combinations of different glass types in the lenses), the edge colours are not very clear. In cheaper binoculars, the border colours are significant and destroy the contrast and sharpness of the image we see.

Coming back to the rainbow: why do we see yellow much more clearly in the rainbow than in a Newtonian spectrum where we use a narrow slit (opening)? In the rainbow, the "opening" is in practice raindrops, and the angular extent of each raindrop is compatible with a small opening. But the sun itself has an extent of about half a degree (angular diameter) in the sky! The rainbow, therefore, becomes a summation of many rainbows that lie a little apart (which originates from different zones on the sun surface). It is *this* summation (corresponding to using a wide opening) that gives us a bright yellow in the rainbow!

11.5.1 A Digression: Goethe's Colour Theory

In Newton's spectrum, we have a highly specialized geometry that gives us the usual spectrum. Historically, Goethe responded to Newton's explanation, not least because Newton did not clearly realize that his spectrum only emerged by *imaging* a slit (after the light from the slit passed through a prism). Goethe showed that other geometries gave completely different colours. Among other things, the "spectrum" from the inverse geometry of Newton, namely a narrow black stripe on a white background, has a different colour gradient than the Newtonian spectrum as indicated in Fig. 11.21. Goethe thought that Newton's explanation was much too simple, and we have to draw the chromatic boundary conditions to understand the colours that are perceived in different geometries.

Goethe explored many different geometries and found many symmetries in the phenomena and introduced certain "colour harmonies", but we should not go into detail.

In Norway, the poet André Bjerke was an important disciple of Goethe. He led a discussion group over a number of years, among which the physicists Torger Holtsmark and Sven Oluf Sørensen were keen participants. A book on the subject: "Goethe's Colour Theory. The Selection and Comments of Torger Holtsmark" was published by Ad Notam Gyldendal's publishing house in 1994.

Personally, I have not discovered that Goethe's has a greater power of explanation than our usual physics model (based on light as waves) in terms of colour phenomena. Figure 11.19 shows the main principle of how we can proceed to build how the colours will come out of a variety of geometries.

On the other hand, Goethe's colour theory has had an important historical function because it focused on Newton's spectrum, not merely "the light that went through a prism". It focused on symmetries and geometries in a great way as until then was not as well known as now. In my language suit, I would like to say that the Goethe fans' points remind us that calculations based on Maxwell's equations depend to a large extent on the boundary conditions! Occasionally, we are physicists too sloppy when we describe phenomena and when we provide explanations. In that case, we will lose valuable details and may be left with wrong beliefs.

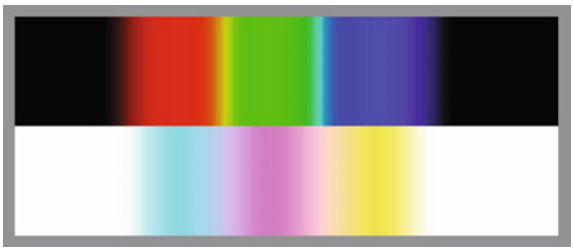

Fig. 11.21 Colour spectrum we get from a Newtonian column and the colour spectrum we get from a "reverse slit", i.e. a black narrow stripe on a light background. Also this picture is the result of a simulation. Real spectra (without going through photography or computer monitors) are far more beautiful to look at!

11.6 References

In Norway, it is perhaps the University College in Gjøvik that has the best competence in colour concentrated in one place. They are gathered under the umbrella The Norwegian colour Research Laboratory (https://www.ntnu.edu/colourlab).

Norsk Lysteknisk Komite is the Norwegian national body of the global lighting organization CIE. More information on the websites of "Lyskultur, Norsk kunnskapscentrum for lys" (https://lysveileder.no/).

Here are a few other links that may be of interest if you are interested in reading more about colours:

- http://www.brucelindbloom.com/ and http://www.efg2.com/ (accessed May 2018)
- International Commission of Illumination, Commission Internationale de L'Eclairage (CIE) is an organization that deals with lighting and visual perception of light. Their website is at http://www.cie.co.at/. In particular, details of the transition between physics and perception are given in the Photometry report. The CIE System of Physical Photometry ISO 23539:2005(E)/CIE S 010/E:2004.
- X-rite colour checker to facilitate colour correction can be purchased including at Edmund Optics: https://www.edmundoptics.com/test-targets/color-gray-level-test-targets/large-x-rite-colorchecker (accessed May 2018).
- A book on the topic: Richard J. D. Tilley: *Colour and Optical Properties of Materials*. John Wiley, 2000.
- On the limit of the absolute sensitivity of the eye: See G. D. Field, A. P. Sampath, F. Rieke. *Annu. Rev. Physiol.* 67 (2005) 491–514.

Note:
Matlab programs used to generate the border colours, and the reverse spectrum (the last four figures in this chapter) are available from the author Arnt Inge Vistnes.

11.7 Learning Objectives

After working through this chapter, you should be able to:

- Explain the need for different quantities such as radiant power, radiation intensity, radiance, irradiance, brightness, light output within radiometry and photometry, and make some calculations where you change from one quantity to another.
- Explain the connection between colour perception (chromatic colour) and physical spectral distribution (which can be obtained from a spectroscope, e.g. by dispersing light with a grating or a prism). For example, you should be able to explain that "yellow light" actually does not need to have any spectral yellow in it whatsoever, and yet to be perceived as "yellow".

- Explain the CIE colour horseshoe and the theory of additive colour mixing, and explain what colours we can and cannot reproduce fully, using, for example, digital pictures on a TV or computer screen.
- Explain the term colour temperature and what it implies for human colour experience and for photography.
- Provide relatively detailed qualitative explanations of how we can achieve a "Newtonian" spectrum, boundary spectrum and reverse spectrum, and point out the importance of boundary conditions.
- Reflect a little over a detector, for example, the eye's visual receptor (cone cell in the retina) has only a limited sensitivity range, and usually associate it with so-called forced oscillations earlier in the book.

11.8 Exercises

Suggested concepts for student active learning activities: Radiometric and photometric quantities, radiant energy, luminous energy, solid angle, radiation intensity, difference between lumen and lux, luminous efficiency, photopic, scotopic, photoreceptors, rods and cones, sensitivity curves, dispersion, colourimetry, chromaticity chart, colour horseshoe, CIE diagram, spectral colour, additive colour mixing, subtractive colour mixing, colour temperature, adaptation in intensity and colour, colour spectrum, border colours, Goethe's colour theory.

Comprehension/discussion questions

1. From Fig. 11.9, you can find a few spectral regions where only one of the cones absorbs light. What does it mean for the colour experience we can have for different spectral colours within each of these ranges?
2. Consider the colour horseshoe in Fig. 11.11, and especially the wavelengths that lie along the edge of the horseshoe. Attempt to estimate the lengths of the rim that correspond to spectral colours in the range 400–500 nm, 500–600 nm and 600–700 nm, respectively. This is related to how easily we can detect *changes* in chromaticity when we assess colours. What would you expect this relationship to be (qualitatively)?
3. Do you find any clues in Fig. 11.9 for the relation you found in the previous task? Attempt to write down your argument in as precise and easily intelligible form as possible! (This is an exercise in being able to argue clearly within physics.)
4. Why are wavelengths not listed along the straight edge of the colour horseshoe?
5. Try to see boundary colours by looking, through binoculars or a lens, at a sharp edge between a bright and a dark area. Make a sketch that shows approximately what you see. Point out how the edge should lie in order to see red-yellow

Fig. 11.22 Photograph of various structures seen through Fresnel lenses in glass (objects are out of focus). Boundary colours are visible

and violet-cyan boundary colours, respectively. Alternatively, you can point out boundary colours in Fig. 11.22.

6. Digitalization circuits like those used for digitizing sound usually have 12–24 bit resolution. How many decades of sound intensity can we cover with such equipment? For photographic apparatus, 8–16-bit resolution is commonly used for specifying light intensity. How many decades of light intensity can we cover with such a camera? Compare with the intensity range that human hearing and vision can handle. Why do sound recordings and photographs still work satisfactorily despite the limited bit resolution?
7. How much variation in intensity, in powers of tens, can be handled by our auditory and visual senses (see Fig. 11.5)? Explain that senses that function over such a large "dynamic range" must actually be based on providing a logarithmic response (at least not a linear response).
8. In tests where we want to determine how sensitive the eye is to light (absolute threshold), it has been found that the responses from many visual cells within a certain area of the retina are summed. These visual cells are believed to be linked to the same nerve fibre ("spatial summation area"). Such a summation area corresponds to the light source having an extent of about 10 arcs of minute ($1/6°$) seen from the eye's position. Discuss which of the photometric quantities must be under our control in this type of experiment. Also discuss why some of the other quantities are not relevant.
9. White glass beads are often used in light bulbs (domes) to get an even and pleasant lighting without sharp shadows. Despite the fact that the dome is spherical, the radiance from the edge of the dome seems to be approximately equal to the radiance of the central areas (see Fig. 11.23). Discuss the reason for this is and

Fig. 11.23 Photograph of the light from a spherical dome of white glass

explain why the radiometric unit *radiance* is suitable for coping with this property. Can you see other dome constructions or shapes that *not* would give the same result? (Hint: Read about "Lambert's cosine law" on Wikipedia.)

Problems

10. Suppose, for the sake of simplicity, that light can be described as a stream of photons with an energy equal to $E = hf$, where h is Planck's constant and f is the frequency of the photons. An ordinary laser pointer usually has a power of a few milliwatts. What attenuation is needed to bring the intensity of such a laser beam to a level that corresponds to the limit of what our eye can perceive (assuming the limit is equivalent to about "500 photons per second") when the laser has a wavelength of 532 nm?
11. A table in Wikipedia with physical data about the sun states that the sun's "luminosity" is 3.846×10^{26} W.
(a) The word "luminosity" (within astronomy) indicates that this is a photopic unit (a unit related to the human visual sense). What quantity is it *really*, judging from the tables of radiometric and photometric quantities given in Sect. 11.1?
(b) The same table contains the following information: "Mean intensity" = $2.009 \times 10^7\,\mathrm{W\,m^{-2}\,sr^{-1}}$. The sun has a diameter of 1.392×10^9 m. Describe in words what "mean intensity" tells us and what term should actually be used for this quantity. Show by calculation that you actually obtain the expected relationship between the "mean intensity" and "luminosity" given here in the task text.
(c) We will calculate theoretically how much power can be captured from sunlight at the surface of the earth, e.g. in a solar collector or solar panel. What

radiometric or photometric quantities are then of interest? Find the value of this quantity, given that the distance between the earth and the sun is 1.496×10^{11} m and that about 30% of the sunlight coming into the outer atmosphere is reflected or absorbed there. [Note: In addition to the straightforward calculations, it is in addition important in this problem to give the result with an appropriate number significant figures, taking *all* the given information into consideration.]

12. Eigerøy's Lighthouse occupies a pride of place among the coastal lighthouses of Norway (see Fig. 11.24). Built in 1854, it was the first lighthouse with a cast iron tower in Norway. The lighthouse is 33 m high, and the light comes out 46.5 m above the sea level. The right part of Fig. 11.24 shows a part of the impressive lens system along with the light bulb (plus a spare bulb). The lighthouse emits light with brightness 3.9×10^6 cd in three light beams 90° apart (three lens sets 90° apart and the entire lens set rotates around the light bulb with a revolution time of 4 s). The lighthouse is one of the strongest along our coast, and it is claimed that "the light reaches 18.8 nautical miles beyond the shore".
(a) How could it be that a light bulb with a few hundred watts can be seen 18.8 nautical miles away?
(b) Would the light from the lighthouse be seen even further than 18.8 nautical miles if, for example, we doubled the power? (Hint: Check if the distance of 18.8 nautical miles has any bearing on the curvature of the earth.)
(c) Assume that the light bulb is 500 W and has about the same luminous efficiency as incandescent lamps. Estimate the solid angle of the rays.
13. In this exercise, we will compare two very different light sources: (1) an ordinary, old-fashioned 60 W light bulb (incandescent lamp) that shines freely in the room (no screen) and (2) a 4 mW green laser pointer wavelength 532 nm. The

Fig. 11.24 Eigerøy Lighthouse is a powerful monument from a time before the GPS made its entry. *To the right*: the lens system that surrounds the light bulb is impressive. There are three equivalent lens systems that are located 90° apart. The last "wall" is open to permit entry. The lens systems are over two metres tall

laser beam is circular and has a diameter of 9.0 mm at a distance of 10.38 m from the pointer (the beam is actually the most intense in the middle, but we can use an approximation that the light intensity is equally large within the entire specified diameter of the beam). Remember, "60 W" for the incandescent lamp is the power that the lamp draws from our mains, while "4 mW" indicates the power of the actual light emitted by the laser pointer.
(a) Specify the radiation flux and radiation intensity of each light sources.
(b) Estimate luminous flux and luminous intensity for both light sources for the direction the light is most powerful. (Hint: Use Fig. 11.1 and the information given in Sect. 11.1.)
(c) Compare the brightness (luminous intensity) of the light beam from the Eigerøy Lighthouse (previous task) with the brightness of the laser pointer specified in this task. Do you think it might be appropriate to replace the old light sources in the coastal lighthouse with a revolving laser?

14. Find out about how wide the rainbow is (angular width from red to violet). Compare this with the angular diameter of the sum. Does it seem to be in accord with the assertion that the extent of sun may have some bearing on the colours we observe in the rainbow (compared to a common Newtonian spectrum obtained by using a very narrow slit)?
15. Pick up a few colour cards from a paint shop and try to determine which colour system the colours are listed under; if it is the NCS coding, you might be able to decipher the codes?
16. Figure 11.25 shows parts of the wrapping of an LED bulb. Make sure you understand all the information provided on the package, and answer specifically the following questions:

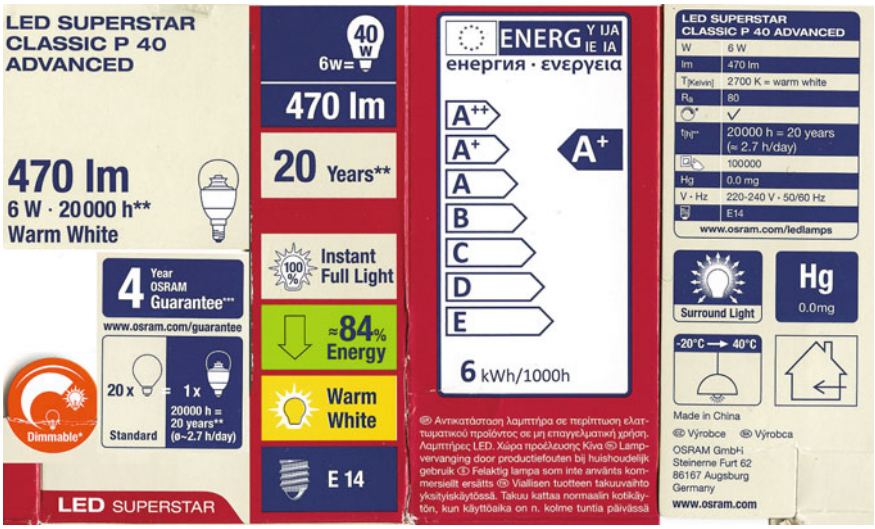

Fig. 11.25 Parts of the package of an LED bulb. See the task text for details

(a) What is the luminous flux and about how much luminous energy can this light bulb give?
(b) Determine the approximate brightness (luminous intensity) of the bulb and the approximate illuminance we can expect on a white A4 sheet held at a distance of 1 m from the light bulb. What assumptions did you make to reach the numbers you specified?
(c) What is the luminous efficiency of this LED bulb?

Acknowledgements I would like to extend my heartfelt thanks to Jan Henrik Wold (at the University of South-Eastern Norway, Drammen), and Knut Kvaal (at the Norwegian University of Life Sciences, Ås for useful comments on this chapter. Any errors and omissions in this part of the chapter are entirely my responsibility (AIV) and not theirs.

References

1. Dickyon, https://en.wikipedia.org/wiki/Luminosity_function. Accessed April 2018
2. Kebes, https://commons.wikimedia.org/wiki/File:Absorption_spectrum_of_liquid_water.png. Accessed April 2018
3. E.F. Schubert, *Light-emitting diodes*, 2nd edn, Figure 16.2 page 276. (Cambridge University Press, 2006). ISBN-13:978-0-511-34476-3
4. S.R.Y. Cajal, Histologie Du Système Nerveux de l'Homme et Des Vertébrés, Maloine, Paris (1911), https://en.wikipedia.org/wiki/Retina#/media/File:Retina-diagram.svg. Accessed April 2018
5. BenRG, https://en.wikipedia.org/wiki/CIE_1931_color_space#/media/File:CIE1931xy_blank.svg. Accessed April 2018