Chapter 16
Skin Depth and Waveguides

Abstract The last chapter begins by asking how far electromagnetic waves penetrate into a metal and introducing the concept of skin depth. It is pointed out that this behaviour will depend both on the frequency of the waves, and whether we are considering a near-field or a far-field situation. This is followed by a treatment of waveguides and how these may be used for transporting well defined ("single mode") electromagnetic waves in the microwave and optical region (single-mode optical fibres). The concept of a "cut-off frequency" is introduced.

16.1 Do You Remember ...?

We have previously pointed out in the book that the solution of a wave equation largely depends on the boundary conditions. In Chap. 9, we echoed the same remark in the context of electromagnetic waves. The well-known plane electromagnetic waves are found far from the source and far from structures that can perturb the electrical and/or magnetic field. Plane waves are just one solution of Maxwell's equations, a solution that is only valid in media without free charges, in the remote zone.

What happens if an electromagnetic wave is an incident on a flat metal plate or some other material containing free charges? The charges will be influenced by electromagnetic forces, including the Lorentz force, and will move. The movement will set up a secondary field that will tend to counteract the original field. The free electrons will be able to move over distances amounting to several atomic radii. During their movement, these electrons will collide with atoms and some of their energy will be converted to heat. It is then natural to expect that the electromagnetic field will decrease as it penetrates the material deeper and deeper. The term "skin depth" quantifies this effect and tells us how far into the metal the waves penetrate.

In other situations where the geometry is different, there may sometimes be solutions of the wave equation (or Maxwell's equations) completely different from planar waves. This opens up the possibility to transport waves without significant loss over long distances, and the waves are then transmitted through so-called waveguides. This chapter will deal with skin depths and waveguides.

A. I. Vistnes, *Physics of Oscillations and Waves*, Undergraduate Texts in Physics,
https://doi.org/10.1007/978-3-319-72314-3_16

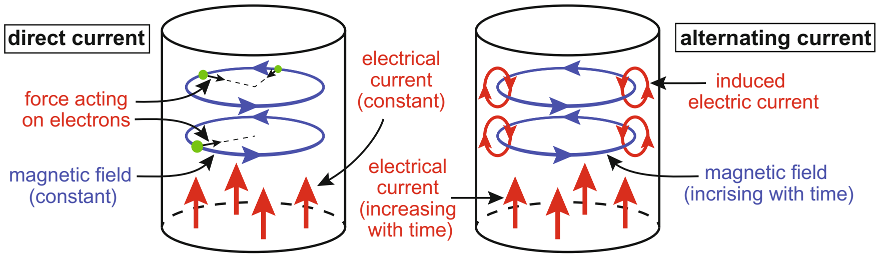

Fig. 16.1 Electric and magnetic fields inside a cylindrical metal conductor with an electrical current. To the left, we have a constant direct current and to the right an alternating current. The directions of the induced currents pertain to the period during which the current grows over time. In a period during which the current decreases, the local induced current loops go in the opposite direction

16.2 Skin Depth

When electromagnetic waves are incident normally on a metal surface, they will be damped as they propagate inside the metal. However, we start with a simpler picture to get the underlying mechanisms.

When we send an alternating electric current through a conductor, the current will not spread evenly over the entire cross section. The current tends to be greatest in the outer parts (or the "skin") of the conductor. The thickness of the layer where the current density is greatest, we call the skin depth.

When we send an alternating current through a cylindrical metal conductor, it is relatively easy to explain the most important mechanism responsible for the skin effect.

A snapshot of the resulting current and fields that this generates is shown in Fig. 16.1. The electrical current will generate circularly oriented magnetic fields perpendicular to and centred in the axis of the conductor. If direct current is flowing through the lead, the electrons will be affected by a force that pulls them towards the centre of the conductor. Called "Hall effect", this phenomenon gives rise to a small potential difference between the outer part of the conductor and the axis of the conductor. The potential difference quickly leads to an electric field that precisely counteracts the transport of electrons towards the centre of the conductor. Aside from this "once and for all" effect that comes into play when power is turned on, the current will be distributed relatively evenly across the cross section with direct current.

With an alternating current, the situation is different. In addition to the effects we have for direct current, *change* in current with time will lead to local current loops that will try to counteract the magnetic field increase ("Lenz's law"). The local current loops cause the current density in the central parts of the conductor to be counteracted while the current density in the outer part of the conductor increases (see Fig. 16.1). However, the local current loops are phase shifted in relation to how current changes over time. Therefore, the overall picture becomes rather complex when we take into account phase shifts, the sum of more contributions to the electron motion, and geometry. As a result, we get a skin effect that causes the alternating current to be greater in the outer parts of the conductor than in the central ones. Therefore, the alternating current does not utilize the entire cross section of the conductor equally efficiently. This means that the resistance of the conductor for AC is different from that for DC.

The induced current loops influence the local current density more and more effectively as the frequency increases. As a result, the layer where the current is flowing becomes thinner with increasing frequency. Skin depth is frequency dependent.

We will shortly derive an expression for skin depth, but can already mention that for aluminium, which is often used in power lines, the skin depth is 11–12 mm at 50 Hz. This means that for thick power lines with a diameter of about 3 cm, most of the flow will involve an outer layer about 1 cm thick and to a lesser degree the central parts of the wire. Occasionally, such power lines are made hollow because the central part does not contribute significantly to the overall conductivity anyway. On other occasion, a steel wire is used as the central core with an aluminium sleeve around it. The steel core provides increased strength to the lead, and the poorer conductivity of steel compared to aluminium plays little role since the current density in the centre is still quite modest.

Instead of one wire that is extra thick when transferring large amounts of power (high current), one sometimes chooses to add two ("duplex") or three ("triplex") lines within each of the three phases of one power line. The two or three wires are then kept at a constant mutual distance of 10–20 cm for, among other reasons to reduce the overall skin depth effect.

16.2.1 Electromagnetic Waves Incident on a Metal Surface

What will happen if an electromagnetic wave in the radio frequency range falls normally onto a metal surface?

In Chap. 9, we showed how Maxwell's equations lead under certain conditions to the following wave equation:

$$\frac{\partial^2 \vec{E}}{\partial t^2} = c^2 \frac{\partial^2 \vec{E}}{\partial z^2} \tag{16.1}$$

where

$$c = \frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_r \mu_0}} \equiv \frac{1}{\sqrt{\varepsilon \mu}} \; . \tag{16.2}$$

The reader is supposed to be familiar with the symbols.

When the wave is perpendicular to a medium where the conductivity $\sigma \neq 0$ (e.g. a metal), the current density is also different from zero. It can be shown that the wave equation under these conditions gets the form:

$$\frac{\partial^2 \vec{E}}{\partial z^2} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \,. \tag{16.3}$$

We can guess a solution in which the fields decrease exponentially in the metal:

$$E = E_0 e^{i(kz-\omega t)} \tag{16.4}$$

where k now can be complex.

If we substitute this trial solution in Eq. (16.3), we get:

$$k = \sqrt{\mu\omega}\sqrt{i\sigma + \varepsilon\omega} \,.$$

We see that the wavenumber k in this expression is a complex quantity. It is in line with the fact that the exponent on the right-hand side of Eq. (16.4) has an exponentially decreasing term, as expected.

If the conductivity is large, or more precisely: if $\sigma \gg \varepsilon\omega$, the k expression can be simplified to:

$$k = \sqrt{i}\,\sqrt{\mu\sigma\omega} \,.$$

Since $(1 + i)^2 = 1 + 2i - 1$, it follows that

$$\sqrt{i} = \frac{1}{\sqrt{2}}(1 + i) \,.$$

Consequently k can be expressed as:

$$k = \sqrt{\frac{\mu\sigma\omega}{2}}(1 + i) \equiv \frac{1}{\delta}(1 + i)$$

where δ is the skin depth. By inserting this expression in Eq. (16.4), we obtain:

$$E = E_0\, e^{i(z/\delta - \omega t)}\; e^{-z/\delta} \,.$$

The physical solution is the real value of the expression, which is:

$$E(z, t) = E_0 \cos\left(\frac{z}{\delta} - \omega t\right) e^{-z/\delta} \,. \tag{16.5}$$

The question is, however, whether this is too simple a solution. We assumed above $\sigma \gg \varepsilon\omega$. If we set the current sizes for copper, we will:

$$\frac{\sigma}{\varepsilon\omega} = \frac{6.4 \times 10^{18}}{\omega} \, \mathrm{F}^{-1}\,\Omega^{-1} \, .$$

It turns out that the approximation we made holds for all electromagnetic waves from about the X-ray region and longer wavelengths. However, the formula is only valid for frequencies that are far from significant atomic or molecular resonance frequencies, and also from the normal collision frequency of electrons in their migration through the metal under consideration. For nonmetals, a somewhat more complicated correlation between skin depth and electromagnetic properties is derived from the material, but we do not deal with these details here.

Equation (16.5) seems to be adequate for the chosen geometry. The equation shows that the electromagnetic wave continues inside the metal, but its amplitude decreases exponentially, the attenuation factor for each distance δ (the skin depth) being $1/e$. We put in the data for copper in the expression of the skin depth:

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} \tag{16.6}$$

we find the skin depth to be

- 9 mm at 50 Hz
- 66 µm at 1 MHz
- 100 nm at 30 GHz (radar)

This means that the waves at radio frequencies and higher are severely attenuated in the outer part of a metal. For low frequencies, the damping is far less pronounced.

Figure 16.2 shows the relationship between the skin depth δ and the frequency f for five different metals or alloys in a log–log plot.

From the figure, we see that at 1 MHz the skin depth of aluminium is 90 µm, and for 0.9–1.8 GHz mobile phone frequencies the aluminium skin depth has decreased to about 3 µm! This means that, so far as resistance is concerned, at such high frequencies there is little to be gained by making the wires much thicker. A large surface is more important than total cross section. The word “skin depth” seems to be a good choice!

Skin depth lies also at the basis of induction cookers. The commonly used frequency here is around 24 kHz. Using steel pots, in which the conductivity is not particularly high and the relative magnetic permeability is close to 1 (nonmagnetic material), the skin depth becomes so large that large portions of the electromagnetic field from the stove passes straight through the bottom of the pots. Only when we have materials that have a high relative magnetic permeability (containing magnetizable iron), almost all energy in the fields from the oven will be deposited as heat in the bottom of the pan.

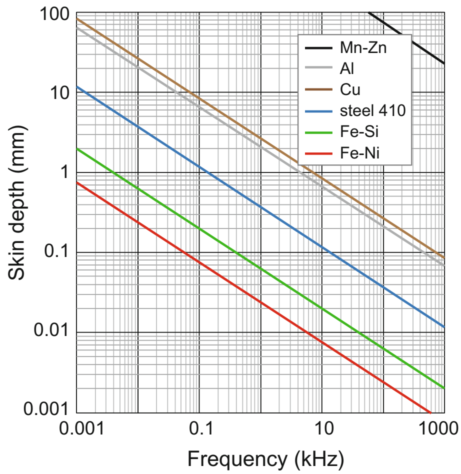

Fig. 16.2 Skin depth as a function of frequency for different metals (idealized). A log–log plot is chosen to cover many decades. Zereks, Wikipedia Commons, CC0 1.0, Modified from original [1]

In pots and pans intended for induction cookers, magnetic steel, such as carbon steel 1010 or stainless steel 432, is used, both of which have a relative magnetic permeability of about 200. From Eq. (16.6), we see that the skin depth then drops considerably compared to nonmagnetic material. The skin depth at 24 kHz will only be 0.1–0.2 mm, and accordingly virtually all the energy from the stove will be deposited as heat in the bottom of the pot.

Comments

The derivation of the expression for the skin depth must be put in perspective. We have shown that Eq. (16.5) is one possible solution of Maxwell's equations. It has not been said that the solution in a concrete case actually *is* this solution! Far from that! We pretended that the solution could be written as a plane wave meaning that the solution does not depend on x and y. To be applicable, the physics must be such that there are no boundary conditions that affect the wave in the x- and y-direction near the place we consider.

This means that Eq. (16.5) must be used with great caution. Geometry in specific situations is often much more important than skin depths calculated blindly from Eq. (16.5).

16.2.2 Skin Depth at Near Field

The mathematical derivation in the previous section was based on electromagnetic waves in the remote zone. That is a situation involving basically electrodynamic conditions in which time variation in electric field creates a magnetic field, and time variation in magnetic field in turn again creates an electric field.

When we are dealing with near field, the situation is different. For example, we might have a power line with strong electric fields without any particular magnetic

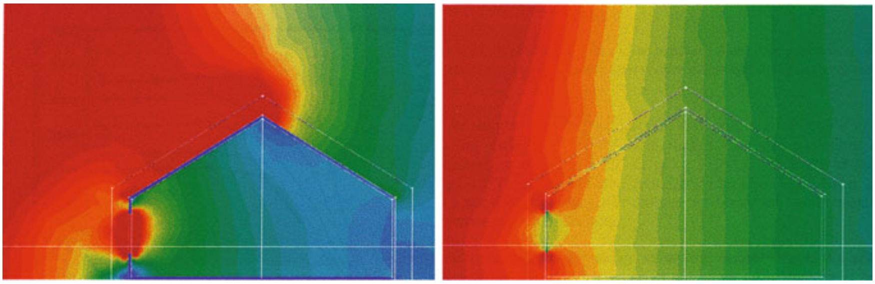

Fig. 16.3 Electric field from a power line (50 Hz) is damped strongly by new woodwork (*left part*: conductivity $\sigma = 1 \times 10^{-6}\,\Omega^{-1}\,\mathrm{m}^{-1}$) while old crushed wood does not dampen the electric field (*right part*: conductivity $\sigma = 1 \times 10^{-9}\,\Omega^{-1}\,\mathrm{m}^{-1}$). The 50 Hz magnetic field went through the walls without noticeable cushioning in both cases. The figure is derived from a master thesis in physics at University of Oslo: Ellen Røhne: Electrical Fields in Houses Near Power Lines—Measurements and Element Method Calculations, 1997

field, and the converse. In such situations, it is often meaningless to talk about skin depth.

A static magnetic field is not noticeably damped, for example, by an aluminium plate, even if it is thick. At 50 Hz, the induced currents will be so small that the induced magnetic field only causes moderate attenuation of an outer magnetic field. The effect is also highly dependent on geometry. If aluminium plates are used to dampen a 50 Hz magnetic field, they must be fully welded so that the induced currents should flow as freely as possible. At the outer edge of the area the aluminium plates cover, the magnetic field is often stronger than if there were no plates there.

It is completely different with electric fields. Static electric field is shielded very efficiently by having a conductive screen connected to earth. Then charges will be drawn to the screen and will neutralize the field on the opposite side of the source. Even at 50 Hz it is easy to remove, for example, electrical fields in homes near power lines. Even a chicken wire under the roof and connected to the ground provides a very effective damping. In fact, even the small electrical conductivity found in relatively new wood is often sufficient to provide a good damping of electric fields from a power line inside a wooden house close to a power line (see Fig. 16.3). The time period of a 50 Hz period is so long (10 ms for each half-period) that there is sufficient time to draw enough charges through the wood to get a good neutralization of the outer electric field. For old, very dry wooden houses, however, the conductivity of the wood is not good enough to provide a good damping.

Summary: In houses near power lines, the magnetic field from the power lines suffers little damping as it permeates through the walls, while the electric field often becomes quite efficiently damped. This is the reason that in the 1980s and 1990s, there was much focus on magnetic fields from power lines and possible health damage, while the electric fields did not attract similar attention. This difference between electric and magnetic fields shows that skin depth is often an inappropriate term in cases where near fields dominate.

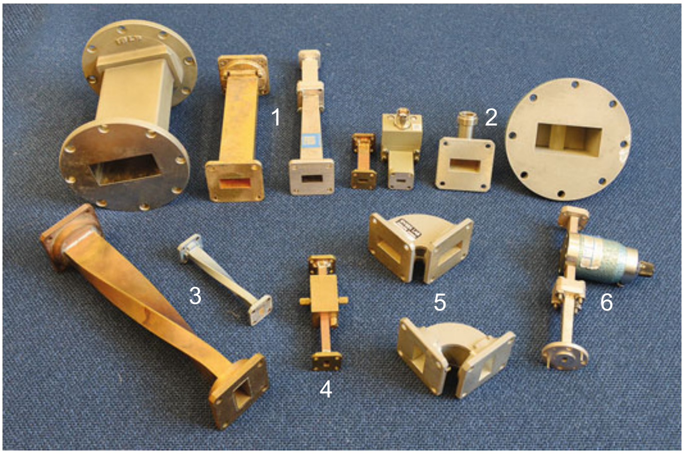

Fig. 16.4 Photograph of some microwave components where waveguides are involved. **1** Straight waveguides, **2** Waveguides with a semiconductor diode working as a detector, **3** Twisted straight waveguides to rotate the polarization 90°, **4** A "phase shifter" for microwaves, **5** 90° bend, one conserving the direction of E, the other conserving the direction of H, **6** A "wave metre", a resonance cavity, to determine the microwave frequency accurately

16.3 Waveguides

A waveguide is a mechanical structure that directs waves from one place to another. In old boats, there was usually a metal pipe from the wheelhouse to the engine room. Someone talking into one end of the pipe could be heard by those at the opposite end several metres away.

An even more well-known waveguide is the doctor's stethoscope. Sound from the heart and lungs is caught in a small funnel held against the skin, and the sound is directed to the ears of the doctor. There is more physics involved in a stethoscope than many are aware of!

In our context, we will concentrate on waveguides for electromagnetic waves. At the bottom lie Maxwell's equations and the wave equation derived in Chap. 9, but now the differential equations must be solved with a set of boundary conditions completely different from what we had in the far field and representing plane electromagnetic waves.

Waveguides for electromagnetic waves are common in the microwave range, that is, frequencies between 2 and 40 GHz (wavelengths from 15 to 0.67 cm). [The range is actually even wider.] The most commonly used forms are hollow rectangular metal pipes, like those shown in Fig. 16.4.

When Maxwell's equations are to be solved for such geometry, the boundary conditions are as follows:

- Electromagnetic waves do not pass through the metal, but are reflected.
- Any electrical field that meets a metal surface must be (approximately) perpendicular to this surface.
- Any magnetic field that meets a metal surface must be (approximately) parallel to the surface.

The electric and magnetic field can of course have other directions towards the metal than those we just listed. However, the above listed boundary conditions are chosen to find a solution of Maxwell's equations that cause as small currents as possible in the metal. It is necessary that the wave does not lose too much energy per unit length as it moves through the waveguide.

There are generally a number of different solutions of Maxwell's equations for a waveguide with a rectangular cross section. Electric and magnetic fields have very different distributions and direction in space compared with the planar wave solution in the remote field zone discussed in Chap. 9.

However, for a given frequency there are only a finite number of possible solutions, and if the wider dimension of the waveguide cavity is less than half the wavelength, it is actually no solution. When the wider dimension in the cavity is between a half and an entire wavelength, and the shortest dimension is only half the longest, there is only one possible solution of Maxwell's equation that corresponds to a wave. The wave pattern we obtain in the waveguide is uniquely determined. We say we have *single-mode transmission*. The lowest frequency that can be sent through a waveguide is called "cut-off frequency".

If we increase the frequency of the electromagnetic waves so that the wider dimension in the cavity of the waveguide is larger than a wavelength, there are at least two different solutions of Maxwell's equations. Then the wave can go through the waveguide in (at least) two different ways. We get a multimode propagation.

In a rectangular waveguide, the smallest dimension is usually half the size of the wider dimension. This ensures that the polarization of the electromagnetic waves can be one way only.

Referring to Fig. 16.5, the following list state the names of frequency band, approximate dimensions for waveguides, cut-off frequencies and optimal frequency ranges:

Frequency band	Wider dimension (mm)	Cut-off frequency (GHz)	Optimal frequency range (GHz)
G	58	2.6	3.2–4.9
X	27	5.6	6.9–10.5
Ka	8.3	19	22–34
Q	5.9	25	32–48

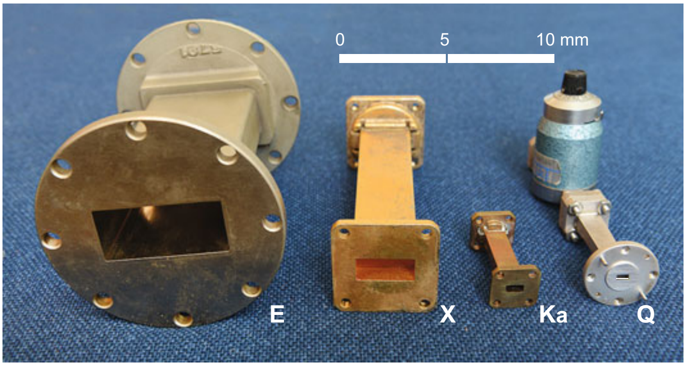

Fig. 16.5 Photograph of waveguides for four different frequency bands. The dimension varies with the frequency so that only one mode will be present for the signal of interest

16.3.1 Wave Patterns in a Rectangular Waveguide

Figure 16.6 shows a schematic representation for the field distribution in a so-called TE10 waveguide. TE stands for "transverse electric". The electric field is perpendicular to the wider surface of the waveguide with rectangular cross section. The field distribution is not the same as a plane electromagnetic wave. Where does the difference lie?

Imagine a plane electromagnetic wave as we discussed it in Chap. 9. If we had such a field distribution within the rectangular waveguide, the electric field would be parallel to two side edges. Such a field would cause large currents of electrons in the metal wall of the waveguide, and thereby a large loss.

In a waveguide, initial conditions and boundary conditions force a solution of Maxwell's equations that can be at least as "beautiful" as the planar wave solution. The field distribution in a TE10 waveguide is such that the electric field is always perpendicular to the larger internal surface, but the field decreases towards zero as we approach the side surfaces. As a result, there will be far weaker electrical currents in the side surfaces than would be with a plane wave.

Occasionally, it is said that the wave pattern of a waveguide corresponds to a planar wave being reflected back and forth between the walls of the waveguide. This is a misleading description. The waves are solutions of Maxwell's equations under the given boundary conditions and are a distinctive solution. However, when the waveguide dimension becomes large relative to the wavelength, there are many different solutions of Maxwell's equations. In such cases, it makes sense to compare solutions with reflected plane waves through the waveguide.

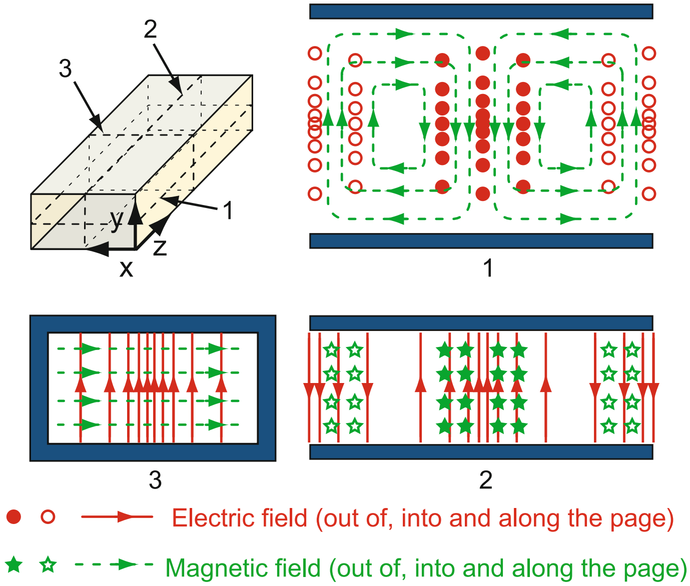

Fig. 16.6 Field distribution of a TE10 mode for the electric field inside a rectangular waveguide. To fit the dimensions of the waveguide relative to the wavelength, only the TE10 mode survives. The wave moves at close to the speed of light in vacuum in the z-direction (right in sections 1 and 2)

However, the electrical field lines across the waveguide start and end in electrical charges on the surface inside the waveguide. Since the wave moves along the waveguide, these charges must also move. This causes induced currents in the inner surface of the waveguide. This is unfortunately not shown in our figure. The inner surface of waveguides is usually coated with silver or gold in order to make the conductivity as large as possible. Then the loss will be minimal. The silver or gold need only be a few microns thick since the skin depth at these frequencies happens to be so small.

Electromagnetic waves with frequencies in the range of 2–60 GHz have traditionally been used for radar, but now these frequencies are also used for mobile telephony and data transmission. Particularly for radar purposes, large powers on the signal transmitted from a transmitter to the radar antenna are often used. It is problematic to send such signals through common wires and coaxial cables—waveguides can often withstand higher powers in the transmission. The microwaves then follow the tube system up to several metres from the generator (preferably so-called klystron) to the antenna where the microwaves are transmitted.

The waveguides are usually made as tubes with rectangular cross sections and flanges to unscrew different pieces. Some pieces can turn the field 90°, and other pieces can make a 90° break on the waveguide itself (see Fig. 16.4).

One attempts to avoid cracks in the waveguides, which prevents currents in the surface. The currents go along the wide walls. To avoid interrupting these currents, we can only make long slots *along* waveguide if the slot is made on the wide side.

By placing two waveguides on top of each other and making a common hole through the walls (on the broad side), some of the waves from one waveguide can be allowed to leak into the other. That way one can make wave dividers and wave combiners.

If a semiconductor diode is placed across the waveguide (and one end is directed as a separate wire), we get a detector that gives a signal proportional to the intensity of the waves passing (an example is given to the far right in Fig. 16.4).

If the wavelength is less than the wider dimension of the waveguide, the electric field may form several different patterns/distributions (multiple "modes") in rectangular (and circular) waveguides. Waves that have different motion patterns, or modes, go at slightly different speeds through the waveguide. For certain layouts, this is unfortunate. A "single-mode" solutions are preferable. We do not enter any mode other than TE10 in this round.

It is an interesting challenge to use Maxwell's equations to determine the direction of a TE10 wave when we have a drawing of the field distribution in a waveguide (see problems at the end).

16.4 Single-Mode Optical Fibre

A single-mode optical fibre consists of a very thin cylindrical core made of very pure silica or fused quartz and diameter only a few microns, surrounded by a layer of another type of glass with a slightly different refractive index than the core. It is usually surrounded with plastic sleeves of different types.

The core may also be made of plastic, but the attenuation is then larger than for glass. Plastic core fibres can only be used for communications over short distances.

It is common to hear that in an optical fibre the light stays in the fibre because of total reflection (based on Snel's refraction law). We have partly done the same earlier in the book.

For large-diameter optical fibres in relation to the wavelength, it is perfectly appropriate to use such an explanatory model. In that case, the interface between the core and the casing satisfies the preconditions we made when we derived reflection laws based on Maxwell's equations.

When the diameter of the core of the optical fibre is shrunk to about six times the wavelength, it will be different. Then we can no longer consider the light as plane waves, because plane waves will not survive in such a fibre.

Then there are other solutions of Maxwell's equations that force themselves forward. In Fig. 16.7, is shown the cross section of several possible solutions of the wave equation for this type of geometry and wavelength. We show different patterns that show where the electromagnetic field is greatest (red and blue only indicate that if the electric field across the fibre in a red area has a maximum value, the field in a

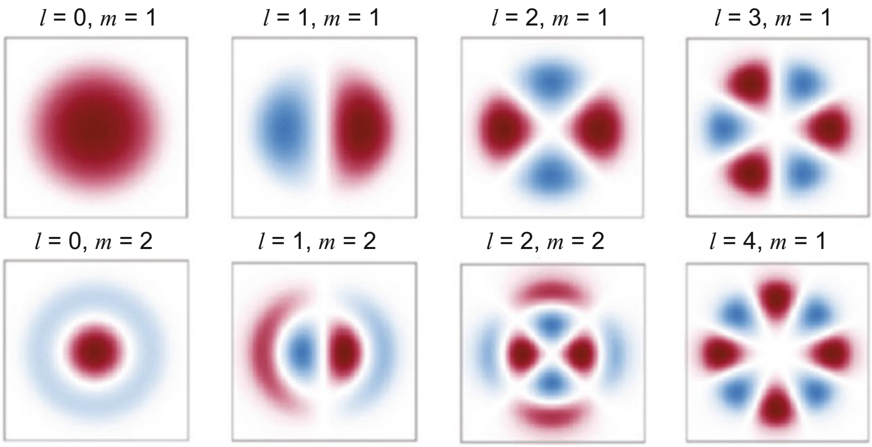

Fig. 16.7 Distribution of electric field across an optical fibre for eight different "modes". Only the simplest survives in a "single-mode fibre" ("single-mode fibre"). Red and blue signify different directions of the electric field in the two areas. The modes are classified using two numbers that give the symmetry properties of the mode. Try to find out what the two parameters really tell us. Figure generated with the software RP Fibre Power. R. Paschotta, Modified from original [2]. Reproduced with permission from the author and publisher

blue area is negative along the current direction). We say that the field has different modes to organize itself within an optical fibre. A complete description of the modes would require a three-dimensional sketch, but we do not go into detail here.

The point is that when the diameter of the fibre is made smaller and smaller, the higher modes will not be able to propagate along the fibre. For an appropriate diameter, only the simplest mode will survive. If we shrink the diameter still further, even this mode will not survive over long distances.

An optical "single-mode fibre" is therefore characterized by a "cut-off wavelength" and can be used to "clean up" laser light that does not have a perfect Gaussian intensity profile.

When light in the infrared region is sent through a single-mode fibre that has the right dimensions and has ultrapure glass in the core, the loss is incredibly low! Furthermore, as seen from Fig. 8.6 in Chap. 8 the index of refraction n is very close to constant over a wide wavelength range in the infrared region (e.g. FK51A glass). Thus, dispersion is very low indeed.

Since both the loss and dispersion are incredibly low, the IR light can move in a very carefully defined manner, and short pulses can be sent many, many kilometres

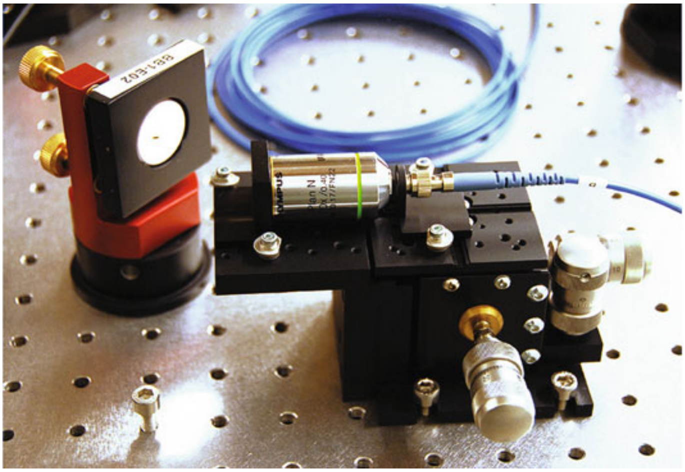

Fig. 16.8 To connect laser light from a laboratory laser into a single-mode optical fibre, one uses a microscope objective mounted on a three-dimensional stage with precision screws. The Airy disc at the waist of the laser beam after passing the microscope objective should be of the same size as the effective fibre diameter. Thus, diffraction comes into play even in this part of physics

before the pulse shape needs to be cleaned before the signal is forwarded. The result is a very high pulse rate and flow of information.

These are the optical fibres that ensure our impressive Internet. In other words: a special solution of Maxwell's equations, where initial conditions and boundary conditions are alpha and omega, in concert with development of materials with low dispersion, is what keeps the Internet going! Plane waves are not involved!

One disadvantage of using single-mode fibre is that the diameter of the core is so small that it is a challenge to get sufficient light from an laser beam in open air into the fibre. In our laboratory, we often use single-mode fibres, for example, to clean up laser light with a wavelength of 405 nm. The inner part of the fibre (where the light is going to go) is then only 2.7 μm in diameter. Around this core, a "cladding" zone with a lower refractive index extends to 125 μm and a "coating" zone is added to a diameter of 245 μm. Outside this comes a protective layer made of plastic.

If we start with a laboratory laser that normally sends the beam with a diameter of at least 1 mm into the open air, the beam must be focused strongly. It is done with a microscope objective (see Fig. 16.8). The end of the fibre must then be placed just in the focal plane of the focused beam, and the fibre must have a direction that completely coincides with the optical axis of the beam. It is a great patience test to get as much of the light into the fibre as possible! Also when the light is released by a single mode fibre, we often need to use a microscope objective to prevent the laser beam from diverging too much (freshly from Chap. 13 how is light going through round holes with very small diameter!).

For telecommunications, special adapters have been developed that make the connection far easier. In such systems, laser beams in air are not used at all.

16.5 Learning Objectives

After working through this chapter, you should be able to:

- Explain the term skin depth when an alternating current passes through a metal wire.
- Explain the concept of skin depth when electromagnetic waves meet a metal surface.
- Know what parameters affect the size of the skin depth and know about skin depths for a few frequencies and metals.
- Explain that a simple analysis of skin depth may have significant weaknesses.
- Explain the distribution of electrical and magnetic fields and electrical currents in the walls inside a TE10 rectangular waveguide if you are given a figure like Fig. 16.6.
- Explain why Snel's refraction law is not relevant for explaining how a single-mode optical fibre works.
- Indicate why single-mode fibres are attractive in research and technology.
- Explain why it is a challenge to connect light from an open laboratory laser into a single-mode optical fibre, as well as coupling from such fibre back to a free laser beam in air.

16.6 Exercises

Suggested concepts for student active learning activities: Skin depth, waveguides, single mode, multimode, field distribution, wave pattern, boundary conditions, rectangular waveguide, circular waveguide, cut-off wavelength/frequency, optical fibre.

Comprehension/discussion questions

1. Why does an old-fashioned aluminium casserole not work on an induction cooker?
2. What is the big difference between the physics involved when sending electromagnetic waves against a piece of glass and a corresponding piece of metal?
3. Why do we need to change the dimensions of a rectangular waveguide when we switch the frequency of microwaves to be transmitted through the waveguide?
4. The cross section of a conductor used in power lines can sometimes look as shown in Fig. 16.9. Try to explain why the conductor is built in this special way.

Problems

5. (a) Can you tell from the field distribution shown in Fig. 16.10 the direction in which the microwaves propagate in the rectangular waveguide?
   (b) Point out where there must be charges on the inner surface of a waveguide, and how these charges must move as the microwaves move through the waveguide.

Fig. 16.9 Cross section of a type of conductor used in power lines hanging between large masts across large parts of the country. Clark Mills, Wikimedia Commons, CC BY-SA 3.0, [3]

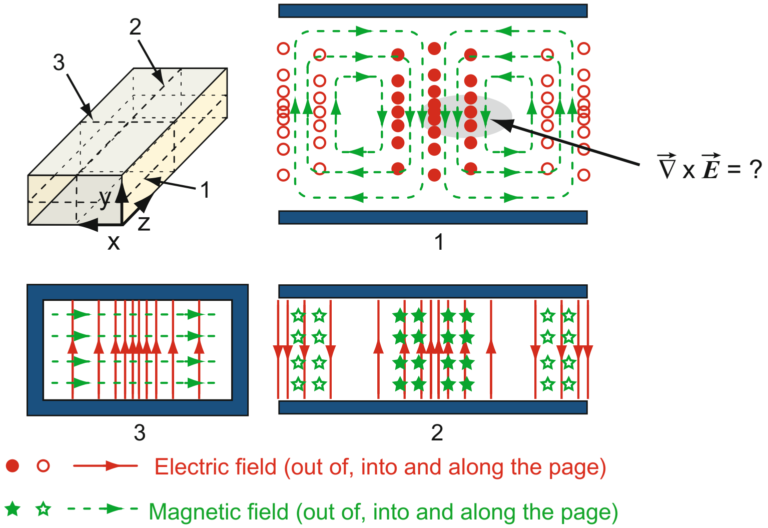

Fig. 16.10 Consider the field distribution in the shaded area and calculate the specified size. With the help of Maxwell's equations, you should be able to predict the time development

(c) Occasionally, for various reasons, we want to make narrow slits across a wall of a waveguide. In which direction should the gap be made to disturb as little as possible the propagation of the wave ? Justify as always the answer.

6. (a) A single-mode fibre designed for light of wavelengths between 450 and 600 nm has a core diameter of about 3.5 μm. Calculate about how many per cent of the energy flux in a laser beam we had received into the fibre if such a fibre was directly inserted into the beam from a regular laboratory laser without using a microscope objective to focus the beam onto the fibre. The beam diameter for many laboratory lasers is about 1.5 mm.
(b) When the light returns from the fibre to air at the other end, we get diffraction. Calculate the beam diameter 1 m after the light went out of the fibre.
(c) Approximately how much focal length should be on a microscope objective that we can place just after the light emanates from the fibre to create a laser beam about 1.5 mm in diameter?
7. Go to the web pages of, for example, ThorLabs (www.thorlabs.de) and search for "single-mode optical fibre" to find the mode field diameter (the diameter of the core) of three different single-mode fibres (calculated for different wavelengths). (Do not get confused by the cladding and coating diameters. The cladding is glass

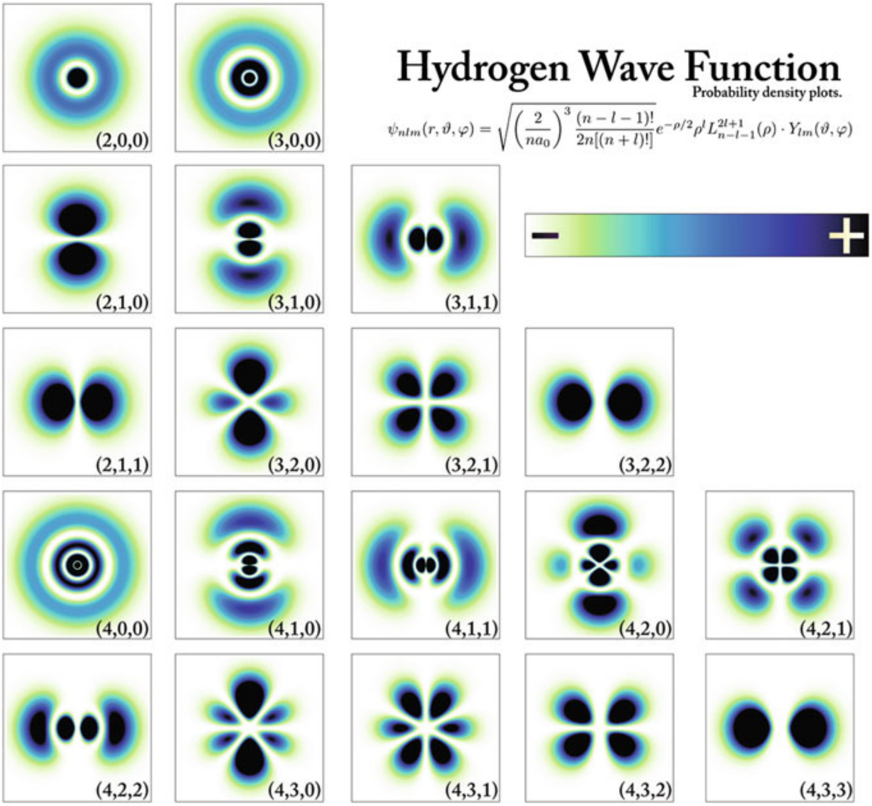

Fig. 16.11 Electron orbitals for the hydrogen atom in a quantum mechanical description. Read the assignment text for details. PoorLeno, Public Domain, Modified from original [4]

with a different index of refraction than the core, and the coating is often plastic to protect the tiny glass string). You may want to check data for "Single-Mode FC/PC Fibre Optic Patch Cables" to find fibres for widely different wavelengths. Do you find some regularity as to how the diameter of the core varies according to the wavelength?

8. Fig. 16.7 shows the patterns in the different modes of how electromagnetic waves (light) can organize themselves as they pass through an optical fibre. In Fig. 16.11, there is an overview of some of the common electron orbital of the hydrogen atom. The orbits show an average of how the quantum mechanical wave function differs when we cross the atom. The distribution in Fig. 16.7 is based on classical physics, yet there are certain similarities between the two figures. We need to be somewhat careful about the comparison since the fibre is a two-dimensional problem while the electrons are linked to a three-dimensional problem. In spite of this: Can you understand that there is a kind of "quantization" both in the classical system and in the quantum mechanics? What is the underlying reason that we get "quantization" in these cases.

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