

Chapter 7

Sound



Abstract The prime theme of this chapter is reflection of (sound) waves at interfaces between two media with different wave velocities (or different acoustical impedances). Such reflections are used in, apart from other situations, ultrasound imaging in medicine, e.g. of fetuses during pregnancy. If a wave moves in an extended medium with reflective boundaries at both ends, a wave of arbitrary shape will go back and forth repeatedly with a fixed time period determined by the wave velocity and the distance between the reflecting ends. We argue that this lies at the core of musical instruments, and not pure standing waves, used as the paradigm in most physics textbooks. We then present the tone scale and go on to define sound intensity, both physically and in relation to human hearing. The chapter ends with a discussion of beats, Doppler shifts and shock waves.

7.1 Reflection of Waves

Sound waves are reflected by a concrete wall, and light waves by a mirror, whereas the waves on a guitar string are reflected at the ends where the string is clamped. Reflection of waves under different circumstances is a topic that we would encounter time and again in this book. Mathematically, reflections are treated by the introduction of so-called boundary conditions. As mentioned earlier, the same differential equation for wave motion can arise in various contexts, yet the solutions differ markedly because the boundary conditions are not identical. The first and perhaps the simplest illustration of this is wave motion along a string of finite length, where physical conditions at the “boundaries” (the ends of the string) play a decisive role in the wave motion.

Suppose that we have a taut string, one end of which is attached to a large mass, and that we produce a transverse “pulse” by providing a sharp blow to the other end (see Fig. 7.1). The pulse will move along the string at the rate $\sqrt{S/\mu}$, where S is the tension and μ the mass per unit length. The shape of the pulse is preserved.

When the pulse reaches the clamped end of the string, the amplitude at this end must necessarily vanish. This means that the pulse close to this end will be compressed and the force across the string will increase significantly. Since the endpoint

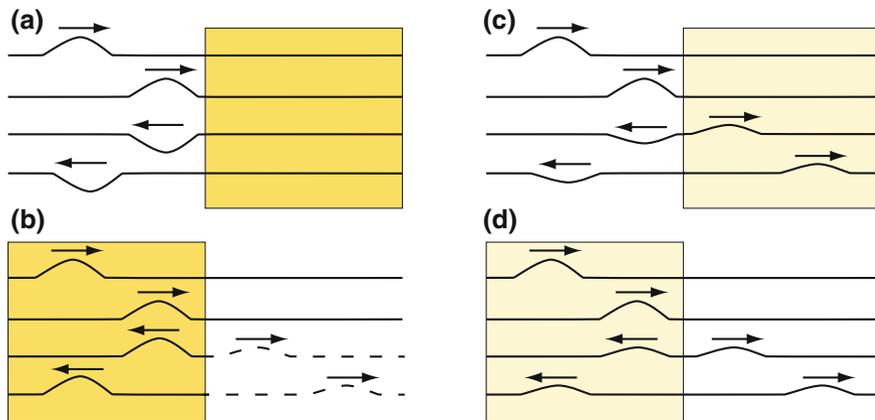


Fig. 7.1 A transverse wave reaches an interface between two media. The wave is drawn for four successive instants. In **a**, the wave goes from a region of low impedance (white) to one of much larger impedance (yellow). The wave is fully reflected and the result gets the opposite sign of the incoming. In **b**, the impedance at the interface decreases (the wave comes from a high impedance region and meets a low impedance region). Here, reflection of energy is near total, but if we consider only amplitudes, the effect is more modest (indicated by the dotted line). Panels **c** and **d** illustrate a case where the amplitudes of the reflected and transmitted wave are equal

cannot move, the compressed string experiences a force in the opposite direction, which creates an imbalance between the amplitude and the travel speed, compelling the pulse to turn back along the string. However, the wave that travels backwards will have an amplitude opposite to that of the original (incoming) pulse (case **a** in Fig. 7.1). No energy is lost (to a first approximation) since loss on account of friction requires the frictional force to work over a certain distance, while we have assumed that the end point is completely fixed.

Another extreme is that where the end is free to move. This can be achieved, for example by holding the string at one end and allowing it to fall freely downwards and let the end move freely in air (disregarding air resistance). However, this is not a good model, since the tension in the string is not defined. A much better model is a string of large linear mass density (mass per unit length) connected at the free end to a string of significantly smaller linear mass density, and subject the entire structure to a fairly well-defined tensile force. It will be convenient to call the former a thick and the latter a thin string.

A pulse transmitted along the thick string will move normally until it reaches the boundary between the two strings. The disturbance that reaches the thin string will give it a significantly greater impact than if the string were of a uniform density. There is again a mismatch between amplitude and velocity, resulting in reflection, but the result in this case is a reflected pulse with same amplitude as the original pulse. In this case, however, some of the wave (and energy) will also propagate along the thin string. If the thin part has a significantly smaller density, almost all energy will be reflected (case **b** in Fig. 7.1).

The terms “a massive structure” and “a thinner or thicker string” (signifying linear mass density) are not sufficiently precise words, and it is better, when one is discussing production and transmission of sound, to use the term “acoustic impedance”, defined below:

Acoustic impedance is defined as acoustic pressure (sound pressure) divided by acoustic volume flow rate (details in the next subsection).

Meanwhile, we will content ourselves with qualitative descriptions, but will continue to employ the term “impedance”, even though our understanding of acoustic impedance is still vague. Based on this understanding, the rules for reflection and transmission of waves at an interface can be enunciated as follows:

It can be shown both experimentally and theoretically that:

- Waves that strike an interface beyond which the impedance of the medium *increases*, split so that the reflected part is of the opposite sign to that of the incident wave. The transmitted wave has the same sign as that of the incoming wave.
- Waves that strike an interface beyond which the impedance of the medium *decreases*, split so that the reflected part is of the same sign as that of the incident wave. The amplitude of the transmitted wave also has the same sign as that of the incident wave.
- The fraction that is reflected or transmitted depends on the relative impedance change in relation to the impedance of the medium the wave originates from. If there is no impedance change, nothing is reflected; if the relative impedance change is infinitely large, all energy is reflected.

In Fig. 7.1, the waveform at the instant the wave strikes the interface is not shown on purpose, but a detailed profile can be constructed using the method outlined in Fig. 7.2. In the figure, total reflection is illustrated. Correct waveform before the interface is drawn, and we let an upward pulse approach the interface. A downward virtual pulse is also made to travel towards the interface with the same velocity. The virtual pulse has an equal and opposite amplitude to that of the incoming pulse if total reflection occurs against a medium with very high impedance (e.g. by attaching a string to a massive structure). The shape of the actual wave during and after reflection is found by adding the waveforms of the original and virtual pulses (the resultant is indicated by the thick dotted line in the figure). Eventually, only the virtual wave survives in the region to the left of the interface, and further wave evolution follows the motion of the virtual pulse alone.

This model can be easily modified to deal with the case in which the wave approaches a medium with much lower impedance, which leads to almost total

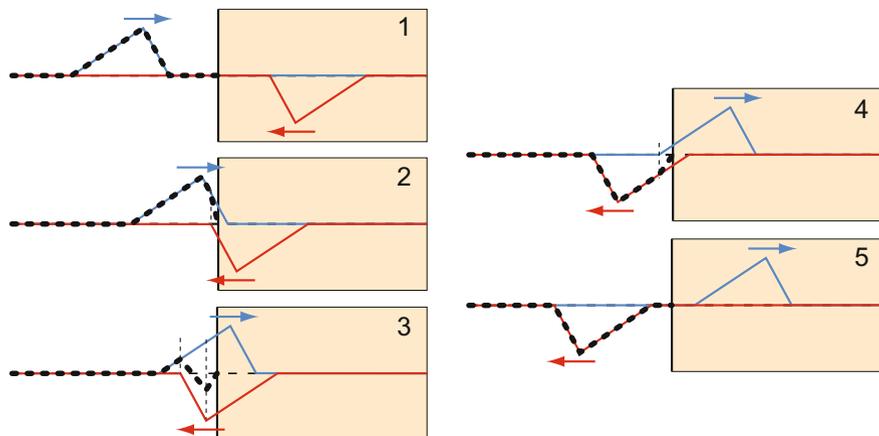


Fig. 7.2 A model for portraying the time development of the waveform of the reflected part of a transverse wave reflected at an interface between two media. See the text for details

reflection with no change in the sign; it can also be modified to handle cases where a part of the wave is reflected and a part transmitted.

We will return to a more detailed description of reflection and transmission of electromagnetic waves when they meet an interface between two media.

7.1.1 Acoustic Impedance *

We will in this chapter speak about “acoustic impedance” rather loosely. However, for the sake of those who wish to acquire greater familiarity with acoustic impedance, this subsection provides a slightly more detailed description. Go ahead to the next sub-chapter if you are not interested in spending more time on this topic at this point.

The notion of acoustic impedance arose when we discussed reflection of waves at the interface of two media. Let us delve a little deeper into this issue. There are several variants of acoustic impedance.

“**Characteristic acoustic impedance**” Z_0 is defined as:

$$Z_0 = \rho c \tag{7.1}$$

where ρ is the mass density of the medium (kg/m^3), and c is the speed (m/s) of *sound* in this medium. Z_0 depends on the material and its units are Ns/m^3 or Pa s/m .

The characteristic impedance of air at room temperature is about 413 Pa s/m . For water, it is about $1.45 \times 10^6 \text{ Pa s/m}$, i.e. about 3500 times larger than the characteristic impedance of air.

Differences in characteristic acoustic impedance determine what fraction of a wave is transmitted and what fraction is reflected when a “plane wave” reaches a plane interface between two media.

The big difference in characteristic acoustic impedance between air and water means that sound in the air will be transmitted into water only to a small extent, and sound in water will penetrate into air only to small extent. Most of the sound will be reflected at the interface between air and water.

In Chap. 6, we found that the sound speed in air or water was given (using c instead of v) as:

$$c = \sqrt{K/\rho}$$

where K is the modulus of compressibility and ρ is the mass density. Upon eliminating ρ by using the definition of characteristic impedance in Eq. (7.1), we get:

$$Z_0 = K/c . \quad (7.2)$$

This expression gives us another idea of what influences the *characteristic* acoustic impedance. For a particular system, e.g. a musical instrument, an different measure is often used:

“**Acoustic impedance**” Z is defined as:

$$Z = \frac{p}{vS} \quad (7.3)$$

where p is the sound pressure, v is the particle speed (over and above the contribution of thermal movements) and S is the pertinent cross-sectional area (e.g. the mouthpiece of a trumpet).

There is a close analogy between acoustic impedance and impedance in electromagnetism. For this reason, the definition of acoustic impedance is often compared with Ohm’s law, and Z is sometimes called “sound resistance” or “audio impedance”.

If you wish to learn more about acoustic impedance, the following article might be of interest: “What is acoustic impedance and why is it important?” available on: <http://www.phys.unsw.edu.au/jw/z.html> (accessed May 2018).

7.1.2 Ultrasonic Images

Characteristic acoustic impedance will change with mass density and the modulus of compressibility [see Eqs. (7.1) and (7.2)]. Precise correspondence is not so easy to obtain from these equations since the speed of sound also depends on the same quantities.

Nevertheless, there are differences in the characteristic acoustic impedance of, e.g. blood and heart muscle. The characteristic acoustic impedance of a foetus and foetal fluid are different. Therefore, if we send sound waves to the body, some of



Fig. 7.3 Ultrasound images of two foetuses. On the left is a 3D image of a foetus about 11 weeks old. On the right is a sectional image (2D) of a foetus about 18 weeks old. Reproduced with permission from the owners (private ownership)

the sounds will be reflected from the interfaces between blood and heart muscle, and between the placenta and the foetus.

However, there is a huge difference between the characteristic acoustic impedance of air and body. In order to get sound efficiently in and out of the body during an ultrasound examination, a gel is applied on the skin, which reduces friction and acts as a conductor of the ultrasonic waves from the ultrasound probe. This material should have approximately the same characteristic acoustic impedance as the tissue the sound is going to enter.

After reflection at interfaces between different impedances, the sound will be captured as an echo, provided that the original sound pulse has already ceased before the echo returns. By analyzing the echo as a function of time delay, we will be able to determine distances. And if we can send sound in well-defined directions, we will also be able to form images of what is inside the body. Figure 7.3 shows a pair of ultrasonic images of a foetus.

Much interesting physics goes into the design of the sound probe in ultrasound surveys. We can control the beam in two directions by causing interference between many independent transmitters on the surface of the sound probe. Control of the sound beam is achieved by systematically changing the phase of the sound for each single transducer on the ultrasound probe. Focusing for the sake of reducing diffraction can also be done by similar tricks. We will return to this in later chapters.

It should be added that there are major similarities between ultrasound surveys, for example, of foetuses and mapping of the seabed for oil exploration. In the latter case, a number of sounders (and microphones) are used along a long cable towed along the seabed. Echo from different geological layers in the ground with different acoustic impedances is the starting point for finding out where to expect oil and where there is no oil and how deep the oil lies.

Many physicists in this country, educated at NTNU, UofO or other institutions, have helped develop ultrasound and seismic equipment. The Norwegian company

Vingmed has been a world leader in developing ultrasound diagnostics equipment. Vingmed has now been purchased by General Electric, but Norwegian scientists trained in physics and/or informatics still play an important role in the development. Similarly, we have taken an active part in seismic surveys as well. A great deal of interesting physics lies behind these methods, and these principles are sure to find other applications in the years to come. Perhaps *you* will become one of the future inventors by exploiting these ideas?

7.2 Standing Waves, Musical Instruments, Tones

7.2.1 Standing Waves

When a persistent wave travels along a taut string that is firmly attached to a massive object at one end, the wave will be reflected from the endpoint and travel backwards along the string with an amplitude opposite to that of the incoming wave. If there is negligible loss, the incident and (an equally strong) reflected wave will add to each other (superposition principle). Let the incoming wave be a harmonic wave described in the following form:

$$y(x, t) = A \cos(\omega t + kx)$$

for $x \geq 0$. That is to say, the wave comes from “the right” (large x) and is *moving toward the origin*. The string is tied to a massive object at the origin, which gives rise to a reflected wave that can be described by the equation:

$$y_r(x, t) = -A \cos(\omega t - kx) .$$

We have chosen to describe the waves in a somewhat unusual way to ensure that the amplitude at the origin is exactly the same for incoming as reflected wave, but with the opposite sign. The two contributions will then cancel each other exactly at the origin.

The superposition principle allows us to express the resultant of the incoming and reflected waves as follows:

$$y_{\text{sum}} = A \cos(\omega t + kx) + [- A \cos(\omega t - kx)]$$

for $x \geq 0$.

We have the trigonometric identity

$$\cos a - \cos b = -2 \sin \left(\frac{a+b}{2} \right) \sin \left(\frac{a-b}{2} \right) .$$

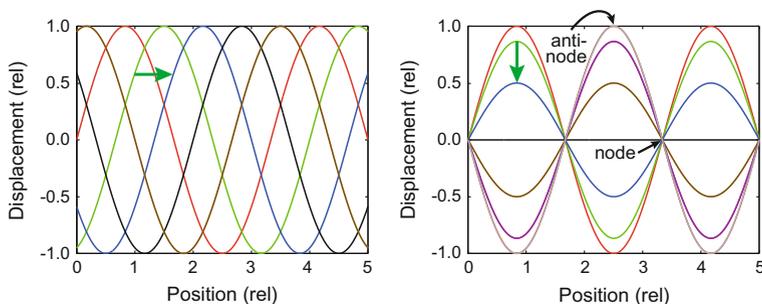


Fig. 7.4 A travelling (*left*) and a standing wave (*right*) as a function of position at various times. The green arrow shows how the wave changes from one point of time (*green curve*) to a subsequent time (*blue curve*). Pay particular attention to how we use the words “anti-node” and “node” in the standing wave

By using the above identity for our sum of an incoming and a totally reflected wave on a string, we find:

$$y_{\text{sum}} = -2A \sin(kx) \sin(\omega t) . \quad (7.4)$$

In this expression, we have taken account of the fact that the reflection occurs against a medium with greater impedance, so that the wave is reflected with the opposite sign.

The important point about Eq. (7.4) is that *the coupling between position and time is broken*. Maximum amplitude in a given position is achieved at *times* for which $\sin(\omega t) = \pm 1$, and these times have nothing to do with position. Similarly, the *positions* where the maximum amplitude occurs is determined solely by the term $\sin(kx)$, which does not change with time. These characteristic features are displayed in Fig. 7.4.

Remark: In the foregoing account, we have assumed that the incoming wave is harmonic, but beyond this we have *not* imposed any requirements on the three main parameters needed to describe a wave: amplitude, phase and frequency. Regardless of the values chosen for the three parameters, standing waves will result after a total reflection as described above, but *this holds only for a pure harmonic wave!*

Standing waves are an important phenomenon when one is dealing with harmonic waves; they can arise with sound waves, water waves, radio waves, microwaves and light—indeed, for all approximately harmonic waves!

The addition of several harmonic waves which do not all have the same phase cannot give rise to standing waves with fixed nodes and anti-nodes like those shown in the right part of Fig. 7.4.

7.2.2 Quantized Waves

Identical endpoints

Suppose that a string is clamped at *both ends* and that we manage to create a wave pulse similar to the one in Fig. 7.1. The wave pulse will be reflected each time it reaches an end, and the wave will travel to and fro indefinitely, provided that there is no energy loss. The same sequence repeats over and over again with a time period T , which is the time taken by the wave to go back and forth once.

The time period equals the total distance back and forth divided by the velocity of the wave; that is:

$$T = \frac{2L}{v}$$

where L is the distance between the two identical ends of the string. The frequency of the periodic movement comes out to be $f = 1/T$, or

$$f = \frac{v}{2L} . \quad (7.5)$$

If we use the general relationship $\lambda f = v$ for a wave, we can assign a kind of wavelength λ to the wave along the string:

$$\lambda = 2L .$$

This will in general not be a harmonic wave.

It is somewhat artificial to speak of the wavelength *inside* the instrument, but it becomes meaningful as soon as we consider the sound created by the instrument in the surrounding air.

Note that these relationships are generally applicable and are *not* limited only to harmonic waves!

A wave moving along a string clamped at *both ends* will have a “quantized” time course with a period given by the time the wave needs to travel back and forth along the string once.

The same regularity will hold also, to take another example, for an air column inside a flute (where there is low impedance at both ends).

Figure 7.5 attempts to highlight reflection of waves for two equal endpoints. The figure shows the wave pulse as a function of position at different instants of time (*left*). After an interval equal to the time period, we are back to the same situation as

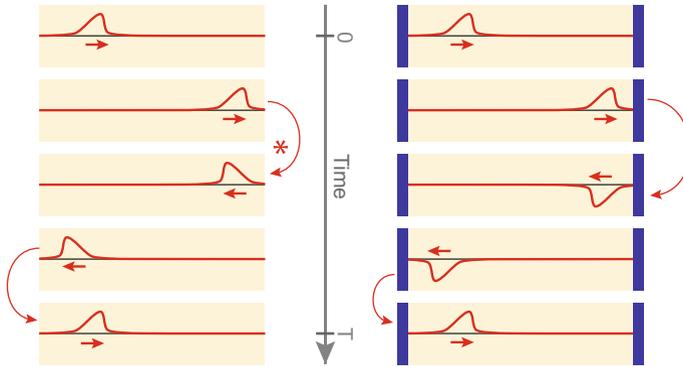


Fig. 7.5 A wave pulse with an arbitrary amplitude travels without loss between two reflecting ends. The wave pulse may be a pressure pulse in air (to the *left*) or a mechanical transverse wave pulse on a string (to the *right*). There are no change in sign of the wave pulse in the first case, but the sign is changed for the latter kind of reflection. See explanations in the text

we had at the beginning of the interval. In the case a flute, a (weak) sound wave will emerge from the flute every time the internal wave reaches the open end (marked with an asterisk in the figure). The player then has to add to the wave pulse at correct time once every period to compensate for the loss to the surroundings. The time period of emitted sound will be the same as the time taken by the wave peak to make one round trip inside the flute.

One should note that it is completely possible to admit more wave peaks within the fundamental period of time we have considered so far. Figure 7.6 attempts to specify a hypothetical case where there are three identical wave peaks evenly distributed over the fundamental time period. The frequency of the sound emerging from a flute sustaining such a wave will be three times the fundamental frequency. It is important to remember that the to and fro movement of a wave does not affect the movement of other waves, even though the total amplitude is a sum of the several independent contributions (assuming that a linear wave equation describes the movement).

Nonidentical endpoints

For a wave moving in a musical instrument where one end of an air column has a high impedance and the other a low impedance, the conditions are different than when the impedance is the same at both ends. An example is an organ pipe sealed at one end, the other end being open to ambient air. In such a case, a wave reflected from the low impedance end will continue in the opposite direction with unchanged amplitude, while the wave amplitude will change sign when reflection occurs at the high impedance end.

In such a situation, the wave as a whole will experience a sign reversal by travelling up and down the pipe once. If the wave makes a second round trip, its sign will change again. This means that a wave must make two journeys back and forth twice (cover a distance $4L$) for it to repeat itself. Figure 7.7 provides an illustration of this principle.

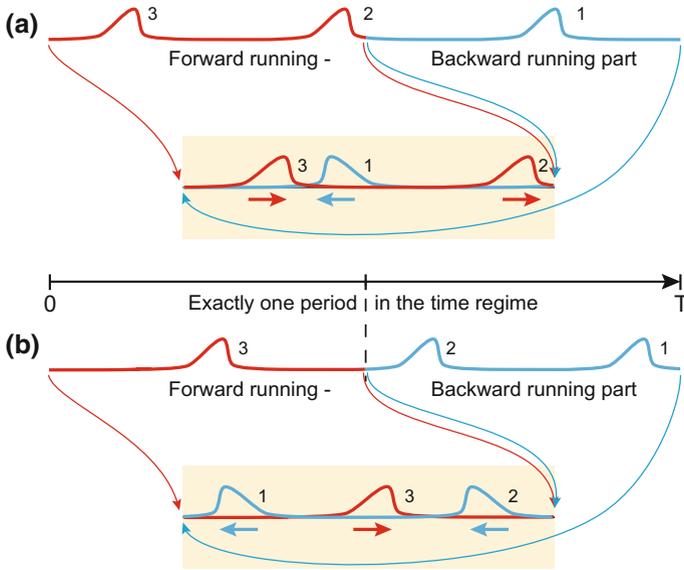


Fig. 7.6 A wave that travels back and forth between two identical ends a distance L apart will have a fundamental frequency $\nu/2L$. However, it is possible to add more than one wave peak to the fundamental wave. In this figure, three equivalent peaks equally spaced within the $2L$ distance are depicted at one instant of time (a) and at a slightly later time (b). Wave peaks travelling to the right are indicated by red, and peaks travelling to the left by blue. The resulting wave is always the sum of these two contributions. In this case, the sound will have a frequency three times the fundamental

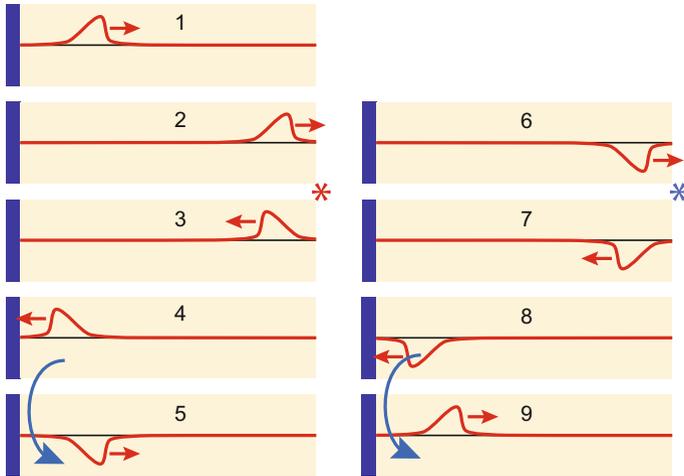


Fig. 7.7 A wave peak must travel twice back and forth in order to regain its initial amplitude when one end of an air string has a high acoustic impedance and the other end low. The massive blocks drawn to the left indicate high acoustic impedance. At this end, the wave undergoes a sign reversal upon reflection

The period for a wave that is reflected at two unlike ends is

$$T = \frac{4L}{v} .$$

The corresponding frequency is

$$f = \frac{v}{4L} . \quad (7.6)$$

The wave can also have any shape whatever; it is only the duration of the period that counts in this context.

As in the case when both ends have the same impedance, we can have an integral multiple of the fundamental frequency, but with one notable exception. We cannot have an even number of identical sequences during the fundamental period of time (you can verify this yourself by making a drawing similar to Fig. 7.7). We can therefore have only an odd multiple of the fundamental frequency given in Eq. (7.6). We usually write this in the form:

$$f = \frac{(2n - 1)v}{4L} \quad (7.7)$$

where $n = 1, 2, 3, \dots$

7.2.3 Musical Instruments and Frequency Spectra

Some musical instruments, such as a drum, provide transient sounds, while other instruments emit more or less persistent “tones”. A tone can be characterized as deep/dark or high/light. The pitch height depends on the frequency of the fundamental tone. The sound of an instrument can be “sampled” and displayed as a time series (a plot of the signal strength in the time domain). The frequency content can be determined experimentally, for example, by Fourier transformation of the time series.

Pure sinusoidal form occurs rarely in the time series of sounds from real instruments. Why is it so difficult to generate harmonic waves from a musical instrument?

It becomes easy to understand that the waveform is not harmonious when we look into the mechanism for the production of sound in a musical instrument. When we pluck a guitar string, it becomes obvious that we are unable to produce a perfect sinusoidal wave. The deviation from a sinusoidal shape will depend on where the string was plucked. This can be easily seen from a Fourier analysis of the sound, since the intensity distribution among the different harmonics depends on where the string is plucked.

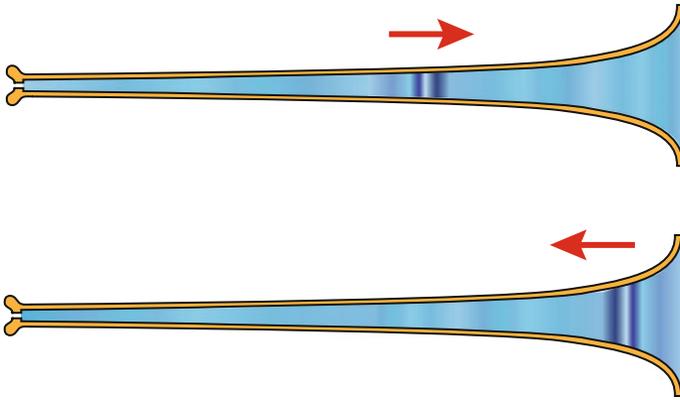


Fig. 7.8 In a musical instrument, a wave goes back and forth with the speed of sound in air and is reflected at each end of the instrument. If we analyse the sound signal with Fourier transformation, we can get many harmonics in addition to the fundamental tone. The harmonics are *not* independent of each other, and their existence only means that the pressure wave is not harmonious

We know that when someone plays the trumpet, the air passes through the tightened lips of the player in small puffs, and it is obvious that these puffs will not lead to sinusoidal variations for the resultant pressure waves (illustrated in Fig. 7.8). In a clarinet or oboe or a transverse flute, we create air currents and vibrations where turbulence plays an important role. The air eddies are nonlinear phenomena and will not lead to sinusoidal timescales for the pressure waves. It is therefore quite natural that the pressure waves in the instrument do not become harmonic. Nonharmonic waves inevitably lead to more harmonics in the frequency spectrum, something already pointed out in the chapter on Fourier transformation. There is no mystery about it.

Nonlinear effects are present in virtually all musical instruments. For string instruments, the vibration and rotation of the string affect in return the contact between the (violin) bow and the string. This results in continuous small changes in the vibration pattern, even though the salient features last long. It is the nonlinearity that gives life to the sound of the instrument and makes it difficult to generate synthetic sound that is as lively as that which emanates from musical instruments.

When the sound waves in the instrument are almost periodic but do not have sinusoidal time periods, the frequency spectrum will consist of several discrete peaks separated by the fundamental tone frequency. How should we determine the tone of the sound? It is the fundamental tone that determines the pitch we perceive with our hearing.

Curiously enough, it is possible that a frequency spectrum may lack the peak corresponding to the fundamental tone and still our ear will perceive the pitch of the fundamental tone. Figure 7.9 shows a small segment of a time signal from a tuba, an instrument that plays low-frequency tones. The time display shows a periodic signal, but a waveform that is far from a pure sine. The frequency spectrum shows a number of peaks, and it is clear that the peaks have frequencies that are integer multiples of

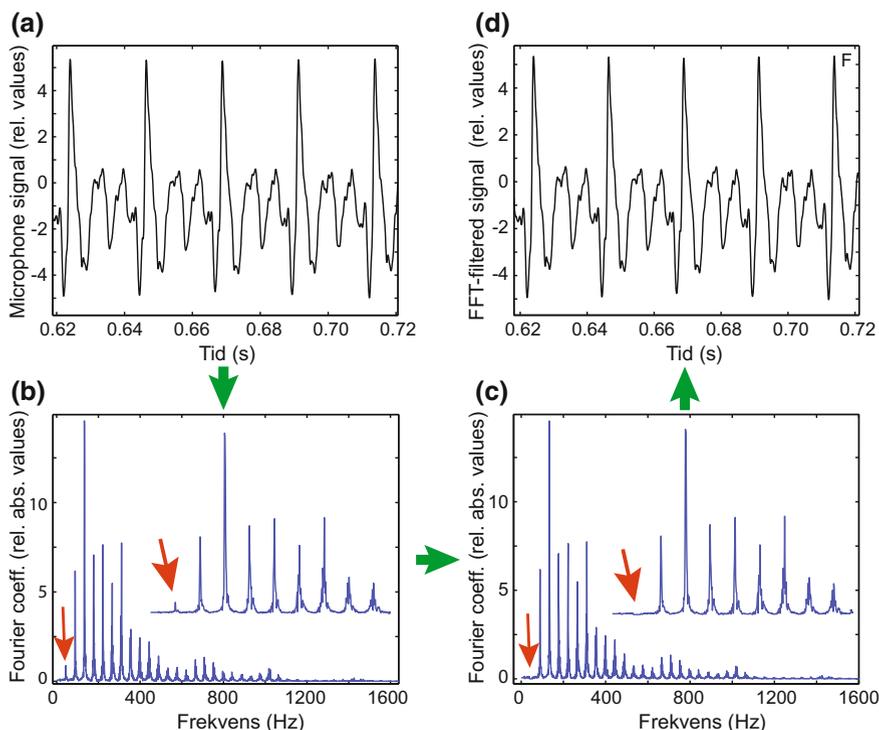


Fig. 7.9 It may happen that the intensity of the fundamental tone is much less than any of the harmonics. In such cases, we can remove the fundamental tone completely without the time signal changing noticeably. Also when we listen to the sound, the pitch will be determined by the periodicity of the time signal rather than the frequency of the harmonics

the fundamental frequency. However, the intensity of the fundamental frequency is quite small.

For the sake of a little amusement, the fundamental tone was completely removed from the frequency spectrum¹ and an inverse Fourier transform was calculated over the entire frequency domain. The result was a time profile visually indistinguishable from the original time signal of the tuba sound (see Fig. 7.9). If we listen to the filtered signal, we do not hear any difference either (at least not easily).

Let us use Fig. 7.9 to point out an important message regarding our perception of the pitch of a tone. *The fundamental tone is found by requiring all peaks in the frequency spectrum to have frequencies equal to an integral multiple of the fundamental frequency. The fundamental frequency does not have to be present.*

¹Both from the positive and the negative half of the frequency domain, due to folding in Fourier transform.

There can also be “holes” in the series of harmonics (e.g. only third, fifth and seventh harmonics exist). Nevertheless, only one frequency satisfies the requirement that the frequency spectrum consists of components that are harmonics of the fundamental.

7.2.4 *Wind Instruments*

When we pluck on a guitar string, the resulting wave will travel back and forth along the string, and the total movement (at each time and each instant) will be the sum of the forward and reverse wave.² However, the energy imparted to the string by the act of plucking eventually changes to sound that disappears in the surroundings and it also heats the string, since it bends a bit here and there and is not fully elastic. The oscillations of the string will die out in a matter of seconds, which is several hundred times longer than the time the wave needs to make one round trip along the string.

A wind instrument (such as a flute, trumpet, clarinet, oboe) is a little different from a guitar string. With such an instrument, a musician can keep a steady volume of the sound for a long time—until he/she has to pause for breath. For wind instruments, therefore, we have a (quasi)-steady-state excitation of the instrument as long as we keep blowing air into it.

In a trumpet, 40–50% of the energy in a wave disappears when the wave reaches the funnel-like opening of the instrument. This means that only 50–60% of the sound energy of the wave is reflected, and the musician must supplement the reflected wave to uphold a steady state situation.

The pace at which the musician blows air must have proper timing relative to the reflected waves, in order to get good sound intensity. This may seem like a difficult task, but sound waves reflected from the end of the instrument back to the mouthpiece of a brass wind instrument impress on the musician’s lips, making it easy to provide new puffs at the right time. Finesse is achieved by tightening and shaping the lips and how forcefully the musician squeezes air through the lips.

For a flute, the reflected wave will affect the formation of new air eddies, which ensures proper timing also for such instruments.

There is some leeway with respect to timing (a slightly higher or lower frequency of air blows than that corresponding to the wave speed and the length of the instrument), but too great a departure will not lead to a sonorous success, because new air blows will not work in unison with the reflected waves.

²There are details to the movement of a wave on a guitar string not mentioned here. These are easier to understand when we use numerical methods to calculate wave movements in Chap. 8.

7.2.5 Breach with Tradition

In this subsection, we will discuss a traditional way of presenting the physics behind different instruments. The purpose is to show that such an approach can easily lead to misunderstandings.

Figure 7.10 shows a traditional representation of what characterizes the physics behind an organ pipe. The organ pipe is chosen as a concrete example. The same conditions apply to all instruments with corresponding impedance termination at the end of an air column. The illustration focuses on the notion of standing waves, as described in Eq. (7.4). The wavelength is determined by the requirement that there be either an anti-node or a node in the standing waves at the end of the air column inside the instrument (“anti-node” corresponds to maximum and “node” to zero amplitude). We must distinguish between the pressure and the displacement, since we know that in a harmonic sound wave there is a phase difference of 90° between the two.

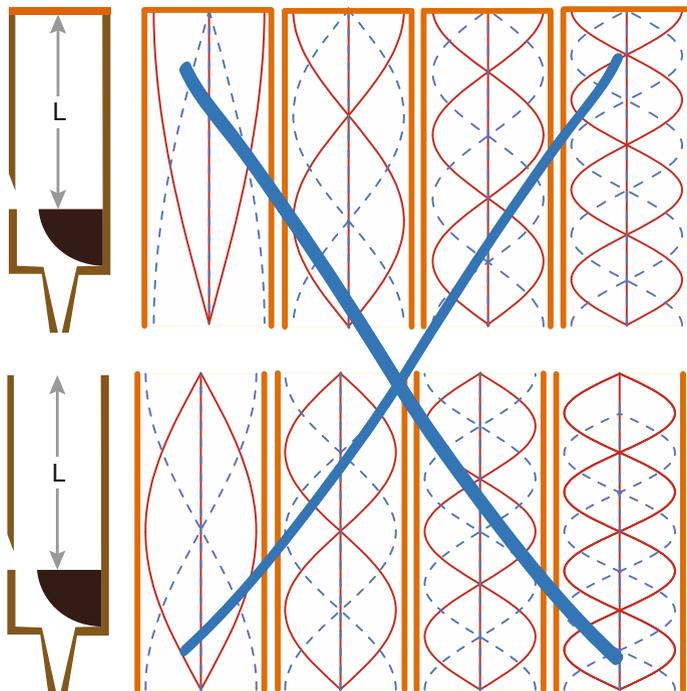


Fig. 7.10 Ostensible displacement amplitude (dashed blue curve) and pressure amplitude (solid red curve) for sound waves in an open and closed organ pipe (or instrument with corresponding acoustic endpoints). This is a standard illustration found in most textbooks in this field. However, the figure is liable to misunderstanding, which is why it has been marked with a large cross. See the text for details

For an instrument that is closed at one end and opens in the other, there will be an anti-node for pressure and a node for displacement at the closed end. The converse holds for open ends, that is, a node for the pressure and anti-node the displacement.

The conventional account is correct, provided that there is a pure harmonic wave in the instrument (only one frequency component), which provides a perfect standing wave. The problem is that the proviso is seldom met in practice!

This is compensated by drawing standing waves also for the higher harmonics, and the figure shows how these waves appear in addition to the wave of the fundamental frequency. One is given the impression that one need only add the separate contributions to get the correct result.

However, the recipe cannot work. There are phase differences between the harmonic frequency components in a Fourier analysis of the sound. These phase differences are vital for reproducing the original time profile of the sound. The phase differences are conspicuously *absent* in Fig. 7.10.

The phase difference means that there will be no standing wave inside the instrument! It becomes meaningless to talk about anti-nodes and nodes inside the instrument. Application of these terms at the endpoints does have a certain justification. However, in our explanatory model, it is more natural to associate this with the rules for reflection of waves.

At the open ends, the air molecules move more easily than inside the pipe. Impedance outside the pipe being lower than that inside, we demand that waves reflected at such an interface do not change sign upon reflection. This means that there is maximum movement of the air molecules at ends that are open.

Similarly, air molecules will find it difficult to move against a massive wall, for example, at the close ends of a closed organ pipe. Accordingly, waves reflected at the closed end will have a sign opposite to that of the incoming wave, with the result that the displacement of the molecules at the boundary becomes zero.

For the pressure wave, the argument is reversed.

We are led to the same conclusion, but for the end faces only, whether we base our argument on reflection of waves or on standing waves for the fundamental frequency, but there is disagreement everywhere else.

Animation

It may be instructive to see how a wave evolves inside a wind instrument. We can make an animation in a simple way and the procedure is shown in Fig. 7.11. We have chosen an animation based on a wave travelling back and forth inside the instrument with negligible loss (but still sufficient to permit detection of the sound emitted by the instrument). We have also chosen a situation where both ends have the same impedance, and the impedance is lower at the ends than inside the instrument, so that a wave is reflected without a sign reversal.

We have chosen a real audio signal from an instrument (French Horn) and picked exactly one period of the signal. The starting point and endpoint are arbitrary, and the signal has been divided so that one half indicates the wave amplitude at different positions for the part of the wave moving toward the opening (where some of the

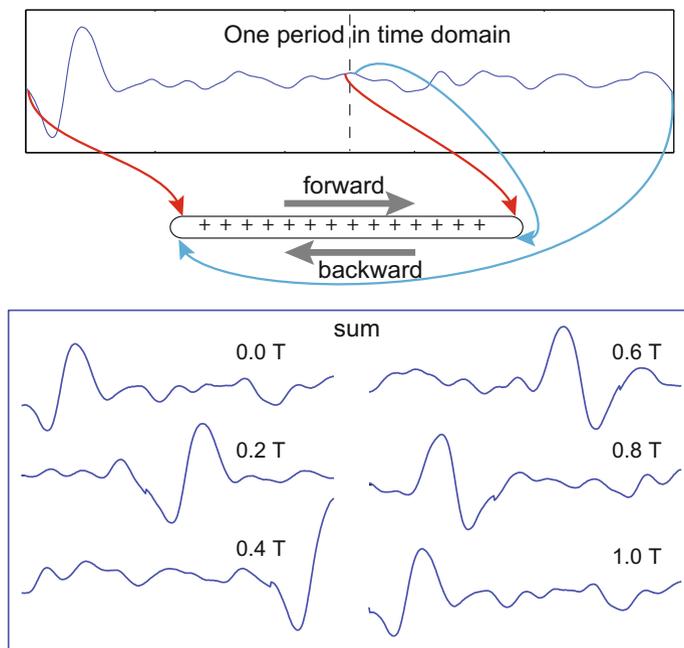


Fig. 7.11 Total sound pressure at different positions within an instrument can be found by summing the sound pressure of the forward and backward waves. One whole period of the sound output from the instrument must be divided equally between the forward and backward waves, as shown at the bottom. By moving forward the waveform (cyclically), keeping pace with the wave, we create an animation of total sound pressure vs position with the passage of time. We see that it is meaningless to talk of standing waves in a case where the frequency spectrum has many harmonics (the time profile is very different from a that of pure sinusoid)

sounds are released). The other half represents the reflected wave. The part that has just been reflected is very close to the opening of the instrument. The part that was reflected half a period earlier has travelled all the way back to the mouthpiece of the instrument.

The wave inside the instrument can be found by adding the forward wave and the backward wave at each point.

Animation is achieved by cyclic stepwise movement of the waveform, each step or frame representing a later instant. The last point of the forward wave becomes, as we move to the next instant, the first point in the backward, while the last point in the backward wave becomes the first point in the forward wave.

Figure 7.11 shows some examples of how the wave looks at six instants (separated by a fifth of the period). We can follow the dominant wave peak and see that it first travels towards the open end of the instrument, but is reflected and moves away from this end during the next half of a period, at the end of which a new reflection occurs.

The animation is meant to show that there is really nothing here that would evoke standing waves, as described by Eq. (7.4) and in the left part of Fig. 7.4. The model based on a wave that travels back and forth provides by far the more faithful description of the state of affairs.

The upshot of the foregoing discussion is that it is more appropriate, when one is describing the process underlying the production of sound in a musical instrument, to speak of a “trapped moving wave” than of a “standing wave”. Joe Wolfe at The University of New South Wales, Australia, focuses on trapped moving waves in his outstanding Web pages about audio and musical instruments (see references at the end of the chapter).

Concrete examples of quantization

It may be useful to look at some concrete examples of quantization (or lack of quantization) of frequencies from different “musical instruments”.

For a 94-cm-long brass tube (internal diameter about 15 mm) two series of measurements were made. In the first, one end of the tube was placed just next to a speaker where a pure tone was played with a tunable frequency. At the other end of the tube, a small microphone was placed for monitoring the signal strength. When the frequency was varied from about 150 to about 1400 Hz, resonances (sound intensity at the location of the microphone) were observed at frequencies of approximately 181, 361, 538, 722, 903, 1085 and 1270 Hz. This corresponds to nf_1 ($n = 1, 2, \dots, 7$), with f_1 calculated from a tube open at both ends [Eq. (7.5)].

When we used the tube as a makeshift trumpet, we could turn (by tightening our lips more from one variant to the next) generating sound with frequencies (ground tone) of about 269, 452, 622, 830 and 932 Hz, that is, to say completely different frequencies than the resonant frequencies at both ends open! The frequencies here correspond approximately to $\frac{1}{2}nf_1$ ($n = 3, 5, 7$), with f_1 pertaining to a tube open at both ends [Eq. (7.5)]. This is in perfect accord with the frequencies predicted by Eq. (7.7) for an instrument closed at one end and opened at the other.

For a trumpet, the situation is a little different. The trunk of the tube in a trumpet results in nonlinear effects because the effective length of the tube is slightly different for different frequency sounds. The tract also causes the sound to come into the surroundings in a more efficient manner than with instruments made hundreds of years ago. The mouthpiece also has complicated acoustic features, but we will not go into these details here.

In Fig. 7.12 is shown an example of a continuous sound from a trumpet, both considered in the time domain and the frequency domain. In this case, the fundamental tone and higher harmonics are present at the same time, and the amplitude ratio between them appears in the frequency domain (often called “frequency spectrum” or “Fourier spectrum”).

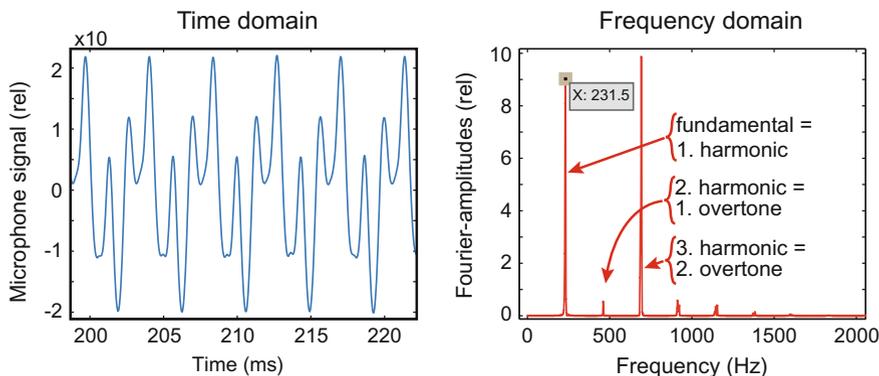


Fig. 7.12 Example of time frame and frequency picture of the sound of a B trumpet playing a “C” note (which is really a B, see next sub-chapter). It is obvious that the time signal is not a pure sinusoid, but a mixture of several frequencies, as revealed by the frequency spectrum. Note that the fundamental frequency is a part of the harmonic range, while the fundamental frequency is not counted in the numbering of so-called overharmonics

Note the asymmetry in the time frame of the sound from the trumpet. The maximum peak is found once again to be as large as the negative peak half a period later. Similar to the second largest peak. This corresponds well with the picture that a wave peak undergoes a sign change after one round trip, but the wave peak returns to the original after two round trips.

It is this asymmetry of the signal itself that causes integer harmonics to almost disappear in the Fourier analysis of the sound, as we see in the right part of Fig. 7.12.

Examples of nonquantization

The importance of reflection of waves and wave velocity within the instrument for obtaining a given (quantized) frequency can be grasped by referring to Fig. 7.13. Here, we have sampled the sound of a mouthpiece from a trumpet (removed from the trumpet itself) while the musician has changed the tightening of the lips slightly up and down. The time signal is analysed by a form of time-resolved Fourier transformation (wavelet analysis, which we will return to later in the book). In such a diagram, peaks in the frequency spectrum are shown as a function of time. We can see that the pitch of the fundamental tone here can be varied continuously. There is no quantization, because there is no reflection of the sound waves of some importance. Frequency is determined exclusively by the rate of air blows through the lips, and here there is no physical process that could impose quantization on frequency.

The harmonics also appear in the wavelet diagram, but since we use a logarithmic frequency scale (y-axis), it does not become the same distance between the different harmonics.

It is interesting to note that we get many harmonics even when the mouthpiece is used alone. This means that it is the slightly chaotic opening/closing of the lips that

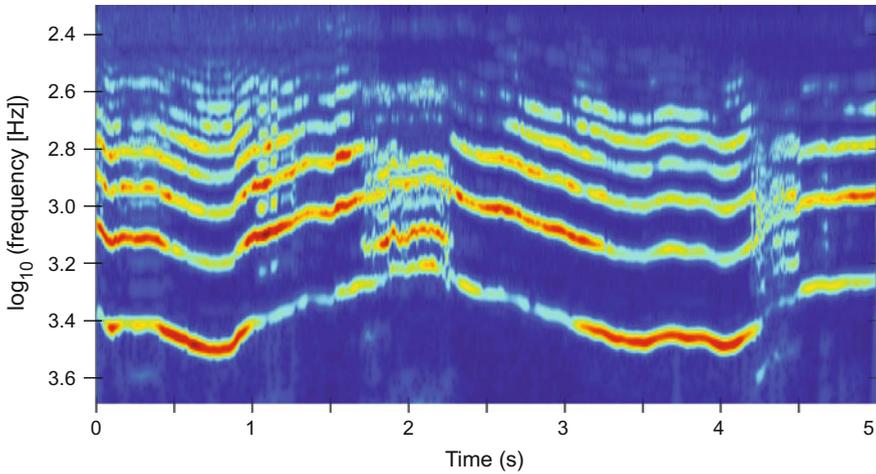
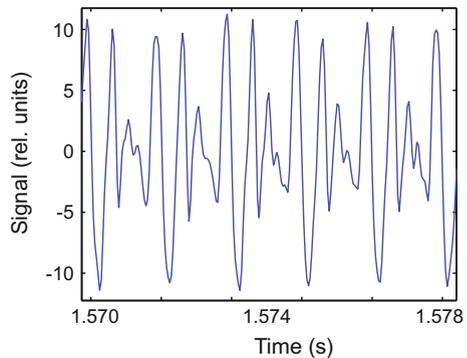


Fig. 7.13 An advanced form of time-resolved Fourier transformation (wavelet transformation) of the sound from the mouthpiece of a trumpet. Time is measured along the x -axis, the logarithm of the frequency along the y -axis. The intensity of the frequency spectrum is highlighted with colours. See also the text for comments

Fig. 7.14 A slice of the time picture of the sound from a mouthpiece shows that the sound pressure does not vary harmoniously with time. Note that the asymmetry seen in the trumpet signal in the left part of Fig. 7.12 is no longer present, even though the same mouthpiece was used in both cases



make sure the airflow does not acquire a sinusoidal time course. This is confirmed by Fig. 7.14, which shows a small section of the time signal from the mouthpiece sound. In other words, it is not the trumpet itself that creates the harmonics. What matters more is the action of the tight lips whereby small air puffs are ejected in a rather erratic manner. On the other hand, back and forth passage of the wave in the trumpet leads to the quantization of whatever tones are emitted by the instrument.

Later in the book, we will use wavelet transformation for further analysis of sound. It will then be seen that Fourier transformation often furnishes a picture that lacks life and nuance. In reality, the harmonics do not exist at the same intensity all the

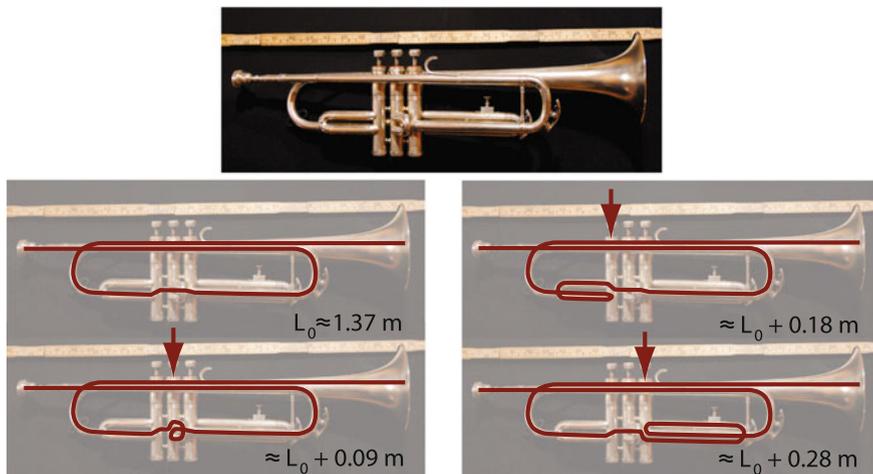


Fig. 7.15 The air column in a trumpet is slightly funnel shaped from the mouthpiece to the outer opening. Valves permit changing the length of the air column. For a B-trumpet (the fundamental tone is a B when no valves are pressed) the length of the air column is about as long as specified

time. The intensity distribution of the harmonics varies as shown in Fig. 7.13. This is one reason why sound from real musical instruments has often more life in it than synthetically produced sound.

7.2.6 How to Vary the Pitch

It is interesting to see how we may change the pitch in different instruments. For a guitar, it is obvious that we should change the length of the vibrating part of the string. Since the *tension* is largely unchanged when we press a string against a fret in the neck of the guitar, the velocity of the waves remains unchanged. When we choose to reduce the *length* of the string, the time taken by a wave to go back and forth decreases proportionally, and the frequency rises according to the relationship $f = v/2L$.

In a brass wind instrument, such as a trumpet, the length of the air column in the instrument changes when the valves are pressed. For a trumpet, when the middle valve is pressed, the air is diverted to a small extra loop. If only the first valve is pressed, the extra loop is about twice as long if only the middle valve is pressed, and if only the third valve is pressed, the extra loop is about three times as long. In Fig. 7.15, the data for the effective air column length for different single valves are given in bold letters. Several valves can be pressed simultaneously, and then the total air extension will equal the sum of all the additional loops that are inserted.

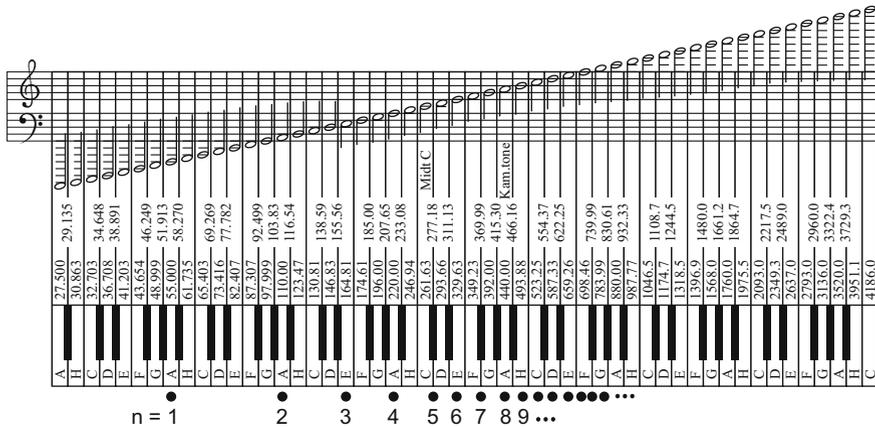


Fig. 7.16 Tones on a piano along with calculated frequency on a tempered scale. The figure is inspired from [1] but completely redrawn

7.2.7 Musical Intervals

In the Western musical tradition, tones are graded on a scale of 12 semitones, which together span a frequency range over which the frequency increases by a factor of 2.0. This means that for a tone C there is a new tone C with a base tone of a frequency twice as large as that of the first one. The tone range is called *an octave*.

The semitones (or half-tones) in-between are chosen so that there is a constant frequency ratio between a tone and the lower semitone. Since there are 12 such steps to achieve an octave, it follows that the ratio of the frequency of one tone and the lower semitone must be

$$2^{1/12} \approx 1.0595$$

provided that all steps are equal. A scale defined in this way is called *tempered*. Figure 7.16 shows the frequencies on a tempered scale if we assume that one-stroke A should have a frequency of 440.00 Hz.

Two tones from, for example, a violin can together sound particularly pleasant if their frequency ratio equals an integer fraction (where only numbers up to 5 are included). The ratio between the frequency of an E relative to the C below on a tempered scale is about 1.260. This is close to 5:4, and the leap is called a (major) *third*. Similarly, the frequency of an F relative to the C is equal to 1.335, which is close to 4:3, and the jump is called *fourth*. Finally, we may note that the relationship between a G and the C below is 1.4987 which is very close to 3:2, a leap called *fifth*.

It is feasible to create a scale where the tones are *exactly* equal to the integer fractions mentioned above for selected tones. Such a scale is called “just”. Certain combinations of tones sound more melodious than on a tempered scale, but the drawback is that we cannot transpose a melody (displace all the tones by a certain number of semitones) and maintain the melodious character.

In Fig. 7.16, some interesting inscriptions can be seen at the bottom. If we start with a low A with frequency 55 Hz ($n = 1$), the frequency of the first overtone ($n = 2$) will be twice as large (110 Hz). The difference between the fundamental tone and the first overtone is a whole octave.

The frequency of the second overtone ($n = 3$) will have $3 \times 55 \text{ Hz} = 165 \text{ Hz}$, which almost corresponds to an E, and the third overtone ($n = 4$) will have the frequency $4 \times 55 \text{ Hz} = 220 \text{ Hz}$, which is the next A. This amounts to two overtones within one and the same octave.

Continuing in the same vein, one sees that there are four overtones within the next octave and eight within the following octave. In other words, the higher harmonics will eventually stay closer than the semitones. That is why we can almost play a full scale without the use of valves in a lute, by forcing the instrument to emit sound mostly at the higher harmonics.

On a trumpet, the fundamental tone (which corresponds to $n = 1$) is achieved if the lips are pressed together only moderately. The frequency of the fundamental tone can be increased in leaps (n increases) by tightening/pressing the lips more and more. The air that escapes through the lips will then come in a tighter bunches than when the lips are more relaxed.

In Fig. 7.12, we saw that the frequency of the fundamental tone for a B trumpet was about 231.5 Hz. This should be a B, and those familiar with the tone scale will know that a B is the semitone that lies between A and H. From Fig. 7.16, we see that this is as it should be. By slightly varying lip tension, the tone from the trumpet can be changed quite a bit (even I can vary the frequency between about 225 and 237 Hz for the current B). Good musicians take advantage of this fine-tuning of the pitch when they play.

7.3 Sound Intensity

Sound may be so weak that we do not hear it, or so powerful as to become painful. The difference is in the *intensity* of the sound, and the sound intensity is defined as:

Sound intensity is the time-averaged energy transported per unit time and area in the direction of the sound.

Alternatively, the sound intensity can be defined as the time-averaged energy per unit area and time flowing normally across a surface in the direction of propagation of the wave.

Sound intensity is measured in units of watt per square metre: W/m^2 .

It is also possible to work with an “instantaneous” intensity (as opposed to the time average), but this will depend on both position and time. For sound waves, the local instantaneous intensity I_{ins} will be given by:

$$I_{\text{ins}}(\vec{r}, t) = \vec{p}(\vec{r}, t) \cdot \vec{v}(\vec{r}, t) \quad (7.8)$$

where \vec{p} is the local pressure (strictly, the pressure deviation relative to the mean) and \vec{v} is here *the local velocity of air molecules* at the same place and time (*not* the speed of sound!).

Remark: A useful rule of thumb will now be derived. Instead of looking at the amount of energy, we may consider what work the sound wave is able to perform. Work is force times distance, and the force that can work on a cross-sectional area A is the local pressure in the sound wave multiplied by the area (actually the excess or deficit pressure in the sound wave multiplied by the area).

Work is “force times distance”, and if the wave moves a distance Δx in a time Δt , then it follows that:

$$\begin{aligned} \text{Instantaneous intensity} &= \frac{\text{work that can be performed}}{\text{area and time}} \\ I_{\text{ins}} &= \frac{p A \Delta x}{A \Delta t} \\ I_{\text{ins}} &= p \frac{\Delta x}{\Delta t} \approx p v \end{aligned}$$

which is the desired rule of thumb.

In the previous chapter, a harmonic sound wave was described in terms of η , the local displacement of the molecules, by the following equation:

$$\eta(x, t) = \eta_0 \cos(kx - \omega t)$$

where η_0 is the maximum displacement relative the equilibrium position (in addition to the thermal movements!).

The speed of the molecules executing in the motion is the time derivative of the displacement η :

$$\frac{\partial \eta}{\partial t} = \omega \eta_0 \sin(kx - \omega t) .$$

It was also shown that the same wave can also be described as a pressure wave by using the equation:

$$p(x, t) = kK \eta_0 \sin(kx - \omega t) .$$

where K is the compressibility module for the medium in which the sound wave is moving.

The instantaneous intensity will now be the product of the local velocity of the molecules and the local pressure as described in Eq. (7.8). The wave is assumed to be longitudinal and moving in the x -direction, so that velocity and pressure have the same direction. Accordingly:

$$I_m = p \frac{\partial \eta}{\partial t} = k\omega K \eta_0^2 \sin^2(kx - \omega t). \quad (7.9)$$

The wavenumber k and the angular velocity ω must satisfy the relation

$$v = \frac{\omega}{k} = \sqrt{\frac{K}{\rho}}$$

where v now stands for the speed of sound, K is the modulus for bulk elasticity and ρ the mass density.

Whence follows the expression for the time-averaged intensity:

$$I = \frac{1}{2} k\omega K \eta_0^2 = k\omega K \eta_{\text{rms}}^2 = 4\pi^2 \frac{K}{v} (f \eta_{\text{rms}})^2$$

since the time-averaged value of \sin^2 equals $1/2$. Here, η_{rms} is the root mean square displacement of the air molecules, or $\eta_{\text{rms}} = \eta/\sqrt{2}$. [The reader is reminded that we are speaking here of the collective displacement of the molecules *over and above* the thermal motion of “individual” molecules.]

It will be useful to eliminate K , the bulk modulus for compressibility, and use the amplitudes of displacement and pressure, together with mass density, sound speed, wavelength and frequency. After some trivial manipulation of the above expression, one can show that:

$$I = \frac{(p_{\text{rms}})^2}{\rho v} \quad (7.10)$$

where p_{rms} is the root mean square deviation of the pressure fluctuation, ρ is the mass density of air and v is now the speed of sound in air.

Further, it can be shown that:

$$I = 4\pi^2 \rho v (f \eta_{\text{rms}})^2 \quad (7.11)$$

where λ is the wavelength of the sound in air, that is, $\lambda = v/f$ where f is the frequency of the sound.

Equation (7.10) shows that sound with different frequencies will have the same intensity if the pressure amplitude is the same.

Equation (7.11) shows that sounds of the same intensity, but different frequencies, have displacement amplitudes η_{rms} which are inversely proportional to the frequency, hence proportional to the wavelength.

It is much easier to measure pressure fluctuations than displacements of molecules. Therefore, Eq. (7.10) is the version that finds practical applications when sound intensities are to be measured and reported.

Before looking at some examples of intensity values, let us return briefly to Eq. (7.9). The equation shows the instantaneous value of energy transport as a function of position and time. The expression is always positive (since $\sin^2 > 0$). It is an important characteristic of waves! The molecules that propagate the wave swing back and forth, but their mean position remains fixed, and does not move with the wave (apart from thermal movement). Yet, energy is transported onward from the source of the wave, and it normally never returns to the source.

It is of some interest therefore to integrate over time all energy transmitted from the source to the wave. We can do that by looking, for example, at total energy per time going through a spherical shell around the source of the waves. The unit for such integrated intensity is watts.

A human voice during normal conversation produces a total power of about 10^{-5} W. If one shouts, the power may amount to about 3×10^{-2} W. In other words, the production of a usable sound wave does not require an unusual expenditure of power.

The figures for the human voice may seem strange when we know that a stereo system can produce powers at 6–100 W. Of course, a stereo system used at 100 W produces a sound far more powerful than human voice can provide. Nevertheless, the difference in intensities of sound from a human voice and a stereo system is striking.

The reason for the big difference is that only a small part—a few per cent for ordinary speakers—of the power supplied to the speakers is converted into sound energy. For special horn speakers, the efficiency can reach up to about 25%. The rest of the energy is converted to heat.

7.3.1 *Multiple Simultaneous Frequencies*

In the derivation of Eq. (7.10), we assumed a single harmonic wave. We will now consider waves with many different frequencies occurring simultaneously?

We must distinguish between correlated and uncorrelated waves. If we send one and the same harmonic signal simultaneously to two stereo speakers, the sound waves from the two sources will be correlated. At some places in the room, the waves will be added constructively. The amplitude can be twice as large as that from a single speaker, in which case the intensity would increase by a factor of four. Elsewhere in the room, the waves will be added destructively and, in the extreme case, will nullify each other. The intensity at such a place would be zero.

For uncorrelated waves (no temporal coherence, see Chap. 15), there will be no fixed pattern of intensifying or nullifying waves at various positions in the room. It will change all the time. For those cases, the following applies:

When we measure sound intensities, the contributions are usually uncorrelated. The sound intensity is then equal to the sum of the intensities of the separate contributions.

7.3.2 Audio Measurement: The Decibel Scale dB(SPL)

Sound intensity can be specified in watts per square metre, as described above. However, it is not a convenient scale. One reason for this is that human hearing has a more logarithmic than linear response. This means that the ear perceives changes in volume based on percentage change compared to the existing sound level. Increases the sound intensity from 10^{-5} to 10^{-4} W/m², the change is perceived to be approximately as large as when the sound intensity increases from 10^{-3} to 10^{-2} W/m².

Therefore, a logarithmic scale for sound intensity, the so-called decibel scale, has been introduced. The sound intensity I relative to a reference intensity I_0 is given in the number of decibels as follows:

$$\beta = L_I = (10 \text{ dB}) \log \frac{I}{I_0} . \quad (7.12)$$

The unit “bel” is named after Alexander Graham Bell, the inventor of the telephone. The prefix “deci” comes from the factor of 10 that is introduced to get simple working values. The decibel scale is used in many parts of physics, not just when we deal with sound intensity.

In principle, we can choose any reference value and can say, for example, that the sound intensity 10 m away from the speakers in the example above is 26 dB higher than the sound intensity 200 m away (check that you understand how the number 26 arises).

In some contexts, it becomes necessary to specify sound intensity on an absolute scale. This can be achieved by using a well-defined reference value specified on an absolute scale. For sound, the following absolute scale is often used:

$$L_{I_{abs}} = 10 \text{ dB(SPL)} \log \frac{I}{I_{abs.ref}} = 10 \text{ dB(SPL)} \log \frac{p^2}{p_{abs.ref}^2} . \quad (7.13)$$

SPL stands for *sound pressure level* and the reference value is 1000 Hz audio with sound pressure $p_{rms} = 20 \mu\text{Pa}$ (rms). This sound pressure corresponds approximately to an intensity of 10^{-12} W/m² and represents about the lowest intensity a 1000 Hz sound may have for a human being to perceive it. This corresponds to approximately the sound intensity 3 m away from a flying mosquito.

It is amazing that the displacements $\eta(x, t)$ of tiny volumes of air molecules for such a weak sound wave is only of the order an atomic diameter. Our ear is a very sensitive instrument!

Although dB(SPL) has been chosen with reference to human hearing, this is nevertheless a purely physical measure of intensity, based solely on W/m^2 . dB(SPL) can be used for virtually all frequencies, regardless of whether a human being can hear the sound or not.

The conversion from intensity to the square of sound pressure is given by Eq. (7.10).

In practice, the term SPL is frequently omitted when sound intensity is specified. This is unfortunate, for when one says that the sound intensity is 55 dB, the statement is, in principle, incomplete because the reference has not been specified. If it had been stated instead that the sound intensity is 55 dB(SPL), it would have implied that the reference level is as indicated above, and that the sound level has been specified on an absolute scale.

7.3.3 *Sound Intensity Perceived by the Human Ear, dB(A)*

Several factors must be taken into account when sound intensities are specified. The definition in Eq. (7.13) is based on a reference sound with a frequency of 1000 Hz. However, we hear sound in a wide range of frequencies, and the ear does not perceive sound with different frequencies as equally intense, even if the number of watts per square metre remains unchanged. We find it harder to hear sounds of frequencies which are lower and higher than sound with average frequencies. The dB(SPL) decibel scale refers to intensity values of sound, irrespective frequencies. In order to get a measure of perceived loudness of a sound, we need to take into the consideration the properties of the human ear.

Figure 7.17 shows equal-loudness contours for different frequencies, that is, the physical intensity in dB(SPL) required to give the same perceived loudness as the frequency varies. Several curves are recorded, since the relative change in frequency varies somewhat with how loud the sound is initially.

The unit phon device indicates the intensity of pure tones. 1 phon corresponds to 1 dB(SPL) at the frequency 1000 Hz. The sound intensity corresponding to a given number of phon varies greatly with the frequency of the pure tones. For example, we see from Fig. 7.17 that a pure 20 Hz sound of 100 dB(SPL) volume is perceived to be equally intense as a pure 1000 Hz sound of 40 dB(SPL). We further see that the sound intensity at 100 Hz must be about 25 dB(SPL) to be audible. Furthermore, an audio intensity of 40 dB(SPL) at 1000 Hz corresponds to the intensity of 55 dB(SPL) for sound of 10,000 Hz.

The curves, issued by the International Organization for Standardization (ISO), were updated in 2003. The year indicates that it is not easy to determine such curves as long as there are significant individual variations. People with obvious hearing deficiencies are probably not used when the data for such curves are collected!

Fig. 7.17 Sound levels at different frequencies giving approximately the same sensation of loudness (see text). Lindosland, Public Domain, Modified from original [2]

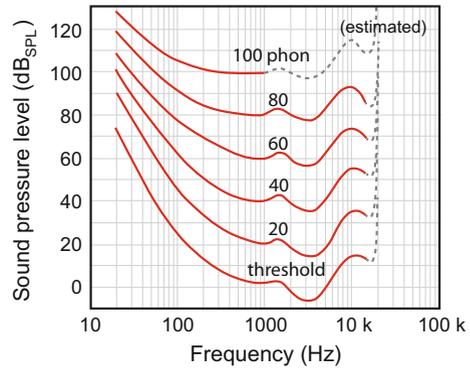
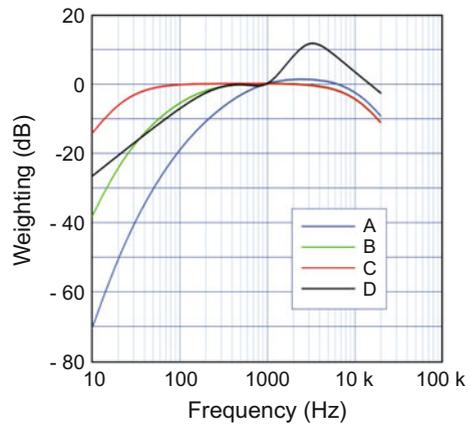


Fig. 7.18 Weighting curves used to indicate the perceived strength of a signal that has many different frequencies at the same time. The graphs give rise to dB(A)-scale, dB(B)-scale, etc. Lindosland, Public Domain, Modified from original [3]



It goes without saying that the decibels scale as presented in Eq. (7.13) cannot be used to indicate *perceived* sound intensity in humans, which becomes particularly demanding when the sound is composed of multiple frequencies. For this reason, an intensity measure is introduced so that different frequencies are weighted according to how intense the sound appears to the *ear*. There are various weight functions, giving rise to dB(A)-scale, dB(B)-scale, etc. Figure 7.18 shows examples of the most common weighting curves.

The curves show that low frequencies count much less than average frequencies when dB(A)-scales are to be determined, as compared to a pure dB-scale as defined in Eqs. (7.12) or (7.13).

The reason for employing different weight functions is based on the phon curves in Fig. 7.18. If the intensity is high, the ear weights various frequencies a little differently than if the intensity is low. dB(A) is best suited for mean and low-intensity

levels, whereas, for example, dB(C) or dB(D) is best suited for measurements at high intensities.

Concrete example of calculation

Let us go through an example of the slightly more appropriate procedure that needs to be used when sound contains multiple frequencies.

Suppose that a sound consists of a pure 100 Hz signal and a pure 1000 Hz signal and that the signals are uncorrelated. Assume that, taken individually, the two components are of equal strength on the dB(SPL) scale, for example, 80 dB(SPL) each. The sound intensity of the composite signal on a dB(SPL) scale would then be:

$$\begin{aligned}
 L &= 10 \text{ dB(SPL)} \log \frac{p_{tot}^2}{p_{abs.ref}^2} = 10 \text{ dB(SPL)} \log \frac{p_{100 \text{ Hz}}^2 + p_{1000 \text{ Hz}}^2}{p_{abs.ref}^2} \\
 &= 10 \text{ dB(SPL)} \log 2 \frac{p_{1000 \text{ Hz}}^2}{p_{abs.ref}^2} = 3 + 80 \text{ dB(SPL)} = 83 \text{ dB(SPL)} .
 \end{aligned}$$

However, in a dB(A) scale, the calculation would go like this: The contribution from the 1000 Hz signal should be weighted with a weight factor 1.0, that is, effectively as 80 dB(SPL). However, the contribution from the 100 Hz signal is to be weighted by a factor of -20 dB, that is, we must subtract 20 dB from the 80 dB the sound would have on a dB(SPL) scale, because it is placed on a dB(A)-scale. 80 dB(SPL) corresponds to

$$\frac{p^2}{p_{abs.ref}^2} = 10^8$$

and 60 dB(weighted) corresponds to

$$\frac{p^2}{p_{abs.ref}^2} = 10^6 .$$

The sum comes out to be:

$$\begin{aligned}
 L &= 10 \text{ dB(A)} \log \frac{p_{tot, weighted}^2}{p_{abs.ref}^2} = 10 \text{ dB(A)} \log \left(\frac{p_{100 \text{ Hz, weighted}}^2}{p_{abs.ref}^2} + \frac{p_{1000 \text{ Hz, weighted}}^2}{p_{abs.ref}^2} \right) \\
 &= 10 \text{ dB(A)} \log(10^6 + 10^8) = 80.04 \text{ dB(A)} .
 \end{aligned}$$

In other words, sound at 100 Hz contributes hardly anything to the perceived intensity as compared with sound at 1000 Hz.

We often see tables with sound intensities in different circumstances, and a typical example is shown below:

| | |
|---------------------------------------|-----------|
| Audibility threshold at 1000 Hz . . . | 0 dB(A) |
| Whispering | 20 dB(A) |
| Quiet radio at home | 40 dB(A) |
| Conversation | 60 dB(A) |
| General city traffic | 70 dB(A) |
| Loud music | 100 dB(A) |

It is most common in such overviews to use the dB(A) scale, but presented just as “dB”. In principle, we should state the intensities in dB(A), dB(B), etc., instead of just dB, to point out, first, that the values refer to an absolute scale, and second, that the contributions from different frequencies have been weighted, in order to show the *perceived sound intensity* and not a measurement of sheer physical intensity.

For our ear to experience that the sound level has doubled, the sound intensity must increased by 8–10 dB(A).

As for large sound intensities, we know that:

| | |
|-----------|---|
| 85 dB(A) | prolonged exposure can lead to hearing loss |
| 120 dB(A) | acute exposure can cause hearing loss |
| 130 dB(A) | causes pain (“Pain threshold”) |
| 185 dB(A) | causes tissue damage. |

Data like these vary from source to source and must be taken with a pinch of salt. It is clear, however, that loud noise can destroy the hairs in contact with the basilar membrane in the inner ear (see Chap. 3). Too many persons regret that they were tempted to listen to such powerful music that hearing impairment became permanent. Also note that with very powerful sound, ordinary tissue is torn apart and shredded, so that the body as such degenerates completely. Powerful sound is not something to play with!

7.3.4 Audiogram

We can test our hearing by visiting an audiologist, or by using available computer programs and the computer’s sound card (but the accuracy is often dubious). In fact, there even are smartphone apps for this type of test. The result of a hearing test is often displayed as a so-called audiogram, and an example is given in Fig. 7.19. An audiogram is constructed such that if a person has normal hearing, her/his audiogram should be a horizontal straight line at the 0 dB level (or at least within the blue band the figure between -20 and $+10$ dB). If the person has impaired hearing for some frequencies, the curve will be below the 0 level. The distance from the null line indicates the difference in the sensitivity of the test person at a particular frequency compared with the norm.

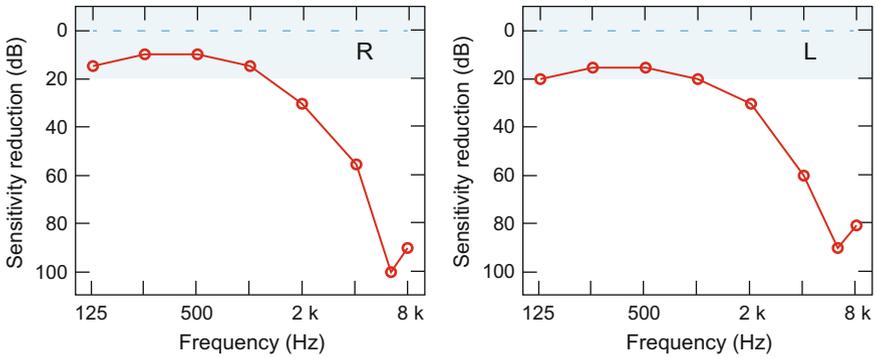


Fig. 7.19 Example of an audiogram recorded by an audiologist. The curves show age-related hearing loss in a 68-year-old man. **R** and **L** stand for the right and left ear, respectively. Normal hearing is within the blue area between -10 and $+20$ dB

Figure 7.19 shows that the person tested has normal hearing for 500 and 1000 Hz in the left ear but has impaired hearing loss for all other frequencies. The hearing loss is 80–90 dB in both ears at 8 kHz. This means that the person is practically deaf at high frequencies. This is an example of age-related hearing impairment. It is small wonder that older people have trouble understanding conversations between people because the most important frequency range in this context is between 500 and 4000 Hz.

Remarks: You have previously worked with Fourier transformation of sound. If the Fourier transform with appropriate calibration provides a measure of the sound intensity at different frequencies, you should be able to calculate dB(A) values, dB(B) values, etc. using the curves in Fig. 7.18. As you can see, you can create your own sound-measuring instrument! (But calibration must be done!)

dBm

Finally, another dB-scale will be defined that is widely used in physics, namely the dBm scale. This is an absolute scale where I_0 is selected equal to 1 mW. The dBm scale is used in many parts of physics, often associated with electronics, but rarely when the sound level is reported. The scale is generally used to specify radiated power from, for example, an antenna. If a source yields 6 dBm, it means that the radiated power is

$$10^{6/10} \text{ mW} = 4 \text{ mW} .$$

7.4 Other Sound Phenomena You Should Know

7.4.1 Beats

When we listen to two simultaneous sounds with approximately the same frequency, it may sometimes appear that the *strength* of the sound varies in a regular manner. Such a phenomenon is called “beating” or “producing beats”. The word “beat” is used because the resulting sound appears to the listener as a regular beat.

Mathematically, this can be displayed in approximately the same way as in the expression of a standing wave. However, for our new phenomenon, it is not interesting to follow the wave’s propagation in space. The interesting thing is to consider how the sound is heard at one spot in the room.

The first step is to add two sinusoidal oscillations:

$$y_{\text{sum}} = A \cos(\omega_1 t) + A \cos(\omega_2 t) .$$

This sum is mathematically equivalent to a formula similar to that found earlier:

$$y_{\text{sum}} = 2A \cos \left[\frac{1}{2}(\omega_1 + \omega_2)t \right] \cos \left[\frac{1}{2}(\omega_1 - \omega_2)t \right] .$$

If the two (angular) frequencies are nearly equal, a mean and differential value can be inserted as $\bar{\omega}$ and $\Delta\omega$ in the formula, which yields the following result:

$$y_{\text{sum}} = 2A \cos(\bar{\omega}t) \cos \left(\frac{\Delta\omega}{2} \right) t . \quad (7.14)$$

This expression is mathematically speaking a product of a “mean frequency oscillation” factor and a “difference frequency oscillation” factor, which is nearly independent on each other.

If the frequency differences are too small to be distinguished by the ear, the mean frequency oscillation factor $\cos \bar{\omega}t$ in Eq. (7.14) will correspond to approximately the same auditory experience as if only one of the two sounds was present. The difference frequency oscillation factor $\cos(\frac{1}{2}\Delta\omega t)$, however, oscillates with a much lower frequency than the original. For example, if we listen to two equally sounds simultaneously, with frequencies of 400 and 401 Hz, the difference frequency oscillation factor will be a $\cos(\pi t)$. Once a second, this factor will vanish, and the total sound will disappear. The listener will experience a sound of almost the same frequency as that of a single original sound, but with a volume fluctuating at a frequency of 1 Hz. This pulsation in the volume is known as “beating”.

Fig. 7.20 When two sound signals with nearly the same frequency are added, the intensity of the resulting signal will vary in time in a characteristic manner

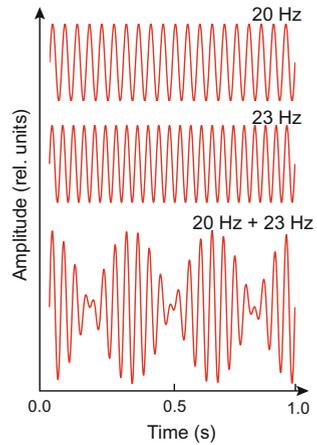


Figure 7.20 shows an example of beating. There are two signals with 20 and 23 Hz respectively, and we follow each of the signals and their sum over a period of one second. We see that in the sum signal there are three “periods” with strong and weak sound within the interval we consider. Note the factor $\cos(\frac{1}{2}\Delta\omega t)$ in Eq. (7.14), and that *half* of the difference of the two frequencies (that are added) corresponds, in our case, to 1.5 Hz. Why does one see three “periods” in the intensity of the beat plot in Fig. 7.20? This is a detail you should notice and understand, because it creeps into several different contexts (Hint: How many times is a sine curve equal to zero during one period?).

There are more puzzles to the beat sound phenomenon. A Fourier analysis of the signal described by Eq. (7.14) gives two peaks corresponding to ω_1 and ω_2 only. There are *no* peak corresponding to the difference frequency. Why do we then experience beating and not two simultaneous sounds with slightly different pitch?

If the difference in the two frequencies is increased, we will eventually hear two separate tones and no beat. Thus, the beat phenomenon is a result of our ear and further processing in the brain. Detailed explanations are found in textbooks in auditory physiology and perception.

However, we suggest a numerical experiment: Make a sum of two sine signals with identical amplitudes and the frequencies 100 and 110Hz. Let the signal last for at least hundred 100Hz periods. Calculate the Fourier transform. The result is as expected.

Calculate then the signal squared (each element is the square of the same element in the previous signal) and perform the Fourier transformation. Notice the frequencies of the peaks now!

This numeric experiment is of interest since many detectors for oscillatory phenomena in physics do not respond to the momentary amplitude of the signal, but to the *square* of the amplitude (to the intensity instead of the amplitude). Whether or not this is applicable to the beat sensation is just a speculation. Our ears are mainly “square law detectors” since phase information is lost for frequencies higher than ≈ 1 kHz.

7.4.2 *Sound Intensity Versus Distance and Time*

When sound propagates in air, little energy is lost along the way. This means that the amount of energy crossing a spherical shell of radius of r_1 will be nearly the same as crosses a spherical shell of radius r_2 ($> r_1$). The local sound intensity is the amount of energy per unit area and time. Since the area of a spherical shell of radius r is $4\pi r^2$, the intensity will decrease as $1/r^2$ where r is the distance from the source.

Now, sound rarely spreads out in totally spherical wavefronts. The distance to the ground is usually significantly shorter than the extent of propagation in the horizontal plane. However, the relationship

$$\frac{I(r_2)}{I(r_1)} = \left(\frac{r_1}{r_2}\right)^2$$

applies reasonably well also to limited solid angles (as long as interference phenomena do not play a significant role).

This implies that if, at a concert, we are 10 m from the speakers, the intensity will be 400 times greater than for the audience 200 m away.

However, inside a room an audio pulse will be damped with the passage of time. The pressure waves lead to oscillations in objects, and many objects have a built-in friction where the sound energy is converted to heat. Various materials dampen sound more or less efficiently. A smooth concrete wall is not set into oscillation by sound waves, and sound is reflected from such a surface without much loss of energy. Walls covered with mineral wool or other materials that are more easily set into vibratory motion in response to a sound wave can dampen the sound much more effectively.

Walls and interior in a room can lead to major differences in damping. They affect the so-called reverberation time. In the Trinity Church (in Norwegian, Trefoldighet-skirken) in Oslo, with bare stone walls and few textiles, the reverberation time is so long that music with fast passages becomes fuzzy to listen to, especially when the audience is thin. In a room with a lot of textiles and furniture and people in relation to the overall space, the sound will die out appreciably faster. In an echo-free room, the floor, walls and ceiling are covered with damping materials, and the reverberation time is extremely short. For concert venues and theatre venues, it matters a great deal for a good overall sound experience that the reverberation time is adapted to the sound images that occur. Building acoustics are a separate part of physics, where good professionals are hard to find and therefore much sought after.

7.4.3 Doppler Effect

Most of us know that the sound of an ambulance siren changes pitch when the vehicle passes us. The phenomenon is called *Doppler effect*. We will now derive a mathematical expression for the observed frequency change.

Sound waves travel at a certain speed *in relation to the transporting medium*. No matter what speed the source has, and no matter what speed an observer has, the sound wave passes through, for example, air at rate $v = \sqrt{K/\rho}$ (symbols defined earlier).

To the left in Fig. 7.21, the wavefront is shown to be the maximum in the air pressure waves from a source that is at rest. The sound spreads smoothly in all directions, and as long as the source does not move, all wavefronts will have the same centre. To the right of the same figure, the wavefront is shown when the source of the sound has moved between each time a pressure wave started. Thereafter each pressure wave progresses unabated with the sound speed (e.g. in air).

This means that an observer positioned so that the source of sound approaches her/him will find that the wave peaks are more frequent (more wave peaks per second) than if the source were at rest. For an observer from whom the source of sound is receding, the opposite will be true. This means that the frequency experienced by an observe will differ in the two situations.

When the observer is at rest with respect to the air, the sound waves will approach her/him with the speed v . When the effective wavelength is as shown in the right part of the figure, it follows that the frequency as heard by the observer is f_o :

$$f_o = \frac{v}{\lambda_{\text{eff}}} .$$

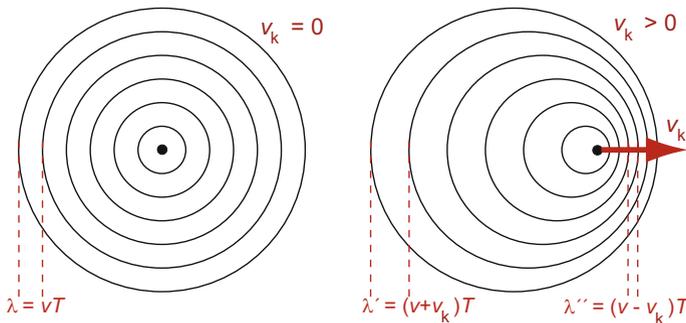


Fig. 7.21 Sound waves spread with the same speed in all directions in the medium through which the sound waves pass. The wavy peaks are equally far apart if the source is at rest in relation to the air. If the source moves relative to the air at the speed v_s , the wave peaks are closer together one side than on the other. The sound speed is set as v

When a source of sound with period T and frequency $f_s = 1/T$ approaches the observer with a speed v_s , one has:

$$f_o = \frac{v}{(v - v_s)T}$$

$$f_o = \frac{1}{1 - v_s/v} f_s \quad (7.15)$$

where v is the speed of sound in air. For an observer from whom the source is receding, the minus sign is to be changed into plus.

This version of Doppler effect can be described by saying that the wave speed relative to the observer (who is at rest) equals the speed of sound in air, while the effective wavelength is different from a situation where both source and observer are at rest.

A variant of Doppler effect is that when the source is at rest, but the observer is in motion. Then the velocity of the wave peaks relative to the observer is different from the sound velocity in air in general. However, the wavelength is unchanged.

The frequency experienced by the observer will then be proportional to the effective velocity of the wave peaks relative to the observer, compared with the speed with which the waves would have reached the observer if he/she and the source were at rest. For a stationary source, and an observer in motion with the speed v_o towards the source, we have the relation:

$$f_o = (1 + v_o/v) f_s \quad (7.16)$$

where f_s is again the frequency of the source.

It is perfectly possible to combine the two variants of Doppler effect discussed above, so that we get a more general expression that applies to situations where both the observer and the source are moving in relation to the air where the sound is spreading.

In Eq. (7.16), the frequency f_s can be replaced by the frequency an observer (suffix o) would have experienced if the source (index s) were in motion, i.e. with f_o given by Eq. (7.15). The result will then be:

$$f_o = \frac{v + v_o}{v - v_s} f_s \quad (7.17)$$

Here v is the speed of sound in air (e.g. 344 m/s), and v_s and v_o are, respectively, the speeds of the source and the observer relative to the air through which the

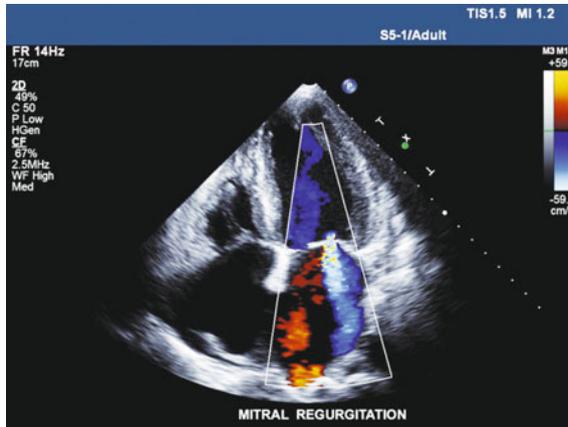


Fig. 7.22 Ultrasound picture of a human heart, superimposed with an ultrasound Doppler image of the blood flow, at a particular point in the heart rhythm. The picture reveals that a heart valve does not close properly during a ventricular compression. The picture is reproduced with a permission from Vingmed. It is difficult to understand a single picture like this. It is recommended to watch a video (search at YouTube with the search words: cardiac ultrasound Doppler heart)

sound is transmitted. In the equation, the following sign convention is observed: If the source moves towards the observer at a rate of v_s relative to air, v_s is positive. If the observer moves toward the source at the rate v_o relative to the air, v_o is positive.

Note that the sign is based on the relative motion between the source and observer as noted above, while the actual magnitude of the velocity is specified relative to air (or the medium through which the sound waves propagate).

Note that it is *not* irrelevant which is moving, the source or the observer. If the source approaches the observer at a speed close to the speed of sound in air, the denominator will tend to zero and the frequency perceived by the observer will tend to infinity. On the other hand, if the observer approaches the source at a speed equal to the speed of sound in air, he/she will perceive a frequency that is only twice the frequency of the source of sound.

Doppler shift is utilized today in ultrasound diagnostics. In Fig. 7.22, a combined ultrasound and ultrasound Doppler image of a heart is shown. The black and white picture shows the ultrasound picture, while the sector with colours indicates blood flow towards or away from us. The subject has a heart valve that does not close properly when the ventricle compresses.

7.4.4 Doppler Effect for Electromagnetic Waves

Applications of the Doppler effect for sound waves are based on a constant sound speed relative to the medium which the sound passes through. For electromagnetic waves, the situation is completely different. The velocity of light is linked in a not easily comprehensible way to our entire space/time concept, and the velocity of light in vacuum is the same regardless of the speed of the source and how an observer moves. When wavelengths are measured, length contractions are observed due to relativistic effects, and time dilation/contraction take place due to relativistic effects. Therefore, the derivation of Doppler effect for electromagnetic waves becomes a little more complicated than for sound; we will content ourselves by merely reproducing the final expression.

Doppler shift for electromagnetic waves in vacuum is given by the relation:

$$f_o = \sqrt{\frac{c+v}{c-v}} f_s . \quad (7.18)$$

Here c is the velocity of light, and v the velocity of the *source relative to observer*, $v > 0$ if the two approach each other. As before, f_s is the frequency of the wave emanating from the source.

This relation shows that light from distant galaxies will be observed to have a lower frequency if the galaxies are moving away from us. The effect is well known and is termed “red shift” in the observed spectra.

Redshift is more pronounced in the light from distant galaxies, as these (in accord with the Big Bang model for the universe) are moving away from us at high speed. The effect is so strong that parts of the visible spectrum are shifted into the infrared region.

This is one reason why the space telescope James Webb is equipped with infrared detectors.

7.4.5 Shock Waves *

From the right part of Fig. 7.21, it appears that the pressure waves lie closer to an sound source moving relative to air than if the source had been at rest. However, the figure was based on an implicit assumption, namely that the source of sound does not catch up with the sound waves generated by it. In other words, the sound source moves at a speed less than the speed of sound in air (or the medium under consideration).

What happens if the audio source moves *faster* than the speed of sound? This state of affairs is depicted in Fig. 7.23. To go from the case at the right of Figs. 7.21, 7.22 and 7.23, we must, however, consider the situation where the source moves at precisely the speed of the sound. In this situation, the pressure waves at the front of the source pile up on each other, and we can have enormous pressure variations within relatively short distances. Such a situation is called a shock wave, a shock front or even a “sound barrier”.

Considerable energy is needed to penetrate the sound barrier. The intensity of the shock front can reach 160–170 MW/m². And, perhaps more importantly, the object that is “going through the sound barrier” must be robust enough to withstand the stresses when the pressure variations over the object become very large. The sound intensity of the shock wave is about 200 dB, so that persons aboard a plane passing through the sound wall must be shielded significantly to avoid permanent damage.

Remark: It is not the noise of the engine on the plane that gives rise to the shock wave. It is simply the pressure wave due to the airplane getting through the air. Engine noise comes as an addition to this main component of the pressure wave.

The speed of sound in air is usually taken as 340 or 344 m/s, which comes out to be around 1230 km/h. Fighter planes can fly faster than this, breaking the sound barrier on their way to the highest speeds.

The speed of a supersonic aircraft is given in terms of the Mach number, where:

$$v \text{ measured in Mach} = \frac{v_{\text{plane}}}{v_{\text{sound}}} .$$

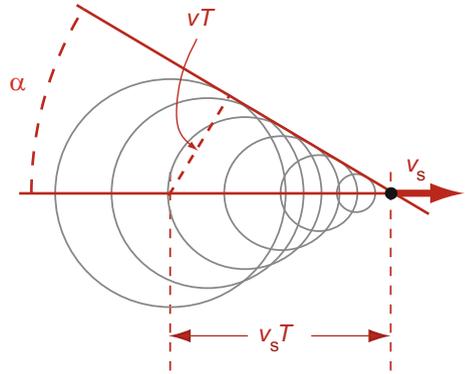
The Concorde aircraft had a normal transatlantic cruising speed of about 1.75 Mach, but a peak speed of approximately 2.02 Mach. The space shuttle had a speed of 27 Mach. Remember, in this connection, that the speed of sound in the rarefied air at high altitudes differs from the speed of sound at the ground level.

From Fig. 7.23, one sees that the shock wave forms the surface of a cone after the plane that is the source of the waves. The opening angle of the conical surface is given by:

$$\sin \alpha = \frac{v_{\text{sound}} \times t}{v_{\text{airplane}} \times t} = \frac{v_{\text{sound}}}{v_{\text{airplane}}} .$$

When a supersonic plane is flying at high altitudes, the aircraft will have gone past an observer on the ground several seconds before the observer hears the sound from the plane. Only when the shock wave reaches the earth-based, will he/she hear the plane, and that as a bang, which indicates that the pressure wave on the expanding

Fig. 7.23 Behind a supersonic plane, a shock waveforms (also called a shock front or a “sound barrier”) with a conical surface with the plane at the vertex. The angle of the cone depends on how much faster the plane moves with respect to the speed of sound



cone surface has reached the ground. The instant at which the bang is heard is not the moment when the plane crosses the sound barrier, but when the shock wave cone hits the observer.

In the case of the Concorde aircraft, the shock wave had a pressure of about 50 Pa at the ground when the plane flew at an altitude of 12,000 m. It was easy to hear the noise from the shock wave shortly after the plane had flown past. Similarly, in the Los Angeles district, we could hear a bang when the spaceship came in for landing on the desert strip a little northeast of the city.

Historically, the American Bell X-1 rocket-powered aircraft was the first vehicle to break the sound barrier. This happened on 14 October 1947; the aircraft then achieved a speed of 1.06 Mach.

7.4.6 An Example: Helicopters *

Few would think of helicopters in the context of supersonic speed, but we must. A Black Hawk helicopter has blades that rotate about 258 times per minute, which corresponds to about 4.3 rotations per second.

The rotor blades have a length of 27 feet, which corresponds to about 9 m.

The speed at the tip of the blade for a stationary helicopter (with the rotor running) is then:

$$\frac{2\pi r}{1/4.3} \text{ m/s} = 243 \text{ m/s.}$$

If the helicopter is flying at a speed of 100 km/h relative to the air, the speed of the blades relative to the air will be 360 m/s on one side of the helicopter. This is about equal to the sound speed!

Manufacturers of helicopters must find a balance between blade speed, the rotation rate and the flight speed in order to avoid problems with the sound barrier. The fact that the speed of the outer edge of the blade does not have the same speed relative

to the air through a full rotation makes the task a little easier than with a supersonic plane.

Anyway, it is interesting to calculate the radial acceleration for a point at the extreme end of a helicopter rotor blade. On the basis of the figures above, it follows that:

$$a_r = \frac{v^2}{r} = \frac{243^2}{9} \text{ m/s}^2$$

$$a_r = 6561 \text{ m/s}^2 \approx 670 \text{ g.}$$

In other words, enormous forces work on the rotor, and the material must be flawless to avoid accidents. It is not uncommon for a rotor blade to cost more than 100,000€ per piece.

7.4.7 Sources of Nice Details About Music and Musical Instruments

There is much fun associated with musical instruments. Physicists have contributed to better understanding of many details and continue to do so. Here are some interesting sources you can look at:

Joe Wolfe, Music Acoustics: Basics, The University New South Wales, Australia. <http://newt.phys.unsw.edu.au/jw/basics.html> (accessed May 2018). Highly recommended!

Alexander Mayer, RIAM (Reed Instrument Artificial Mouth). Institute of Music Acoustics, University of Music and Performing Arts Vienna. http://iwk.mdw.ac.at/?page_id=104&sprache=2 (accessed May 2018).

Seona Bromage, Visualisation of the Lip Motion of Brass Instrument Players, and Investigations of an Artificial Mouth as a Tool for Comparative Studies of Instruments. Ph.D. thesis, University of Edinburgh, 2007.

H. Lloyd Leno, Larry Fulkerson, George Roberts, Stewart Dempster and Bill Watrous: Lip Vibration of Trombone Embouchures. YouTube video showing lip vibrations when playing trombone: *Lip Vibration of Trombone Embouchures, Leno*, (accessed May 2018).

Barry Parker, Good Vibrations. The Physics of Music. The John Hopkins University Press, Baltimore, 2009.

7.5 Learning Objectives

After working through this chapter you should be able to:

- Explain general features of reflection and transmission of waves at an interface between two different impedance media.
- Explain conditions for the formation of standing waves, and how such waves are characterized, including the terms nodes and anti-nodes.
- Explain what determines the pitch of some different musical instruments, and how we can achieve different pitches with one and the same instrument.
- Calculate the frequency (approximate) for a vibrating string and for a wind instrument.
- Explain the concept “trapped moving wave” (as opposed to the traditional “standing wave pattern”) and explain advantages by this concept.
- Explain what we mean by frequency spectrum, fundamental frequency and harmonics when sound is analysed using, for example, Fourier transformation.
- Explain a tempered scale and calculate the frequency of any tone on a piano.
- Explain what is meant by beats, and derive a mathematical expression that shows that beating has something to do with the sound intensity.
- Calculate (when formulas are given) the amplitude of motion of air molecules and the amplitude of the pressure wave created by a harmonic sound wave with a specified dB value.
- Explain dB, dB(SPL), dB(A) and dBm scales.
- Explain the causes of Doppler shift in different contexts, derive formulas that apply to Doppler shift in air, and perform calculations based on these formulas.
- Explain shock waves, especially the “sound barrier” of supersonic aircraft and the like.

7.6 Exercises

Suggested concepts for student active learning activities: Acoustic impedance, reflective boundaries/interfaces, standing wave, node and anti-node, quantized wave, trapped moving wave, pitch, musical interval, tone scale, octave, sound intensity, difference between physical and phonetic intensity units, sound pressure limit, frequency dependency, decibel scale, dB(SPL), dB(A), audiogram, ultrasound, beating, Doppler effect, shock waves.

Comprehension/Discussion questions

1. For ultrasound examinations of, for example, a foetus, there must be at least as much sound reflected from the interface between the uterine wall and the amniotic fluid as from the interface between the amniotic fluid and the foetus. Why will not reflected sound from the first interface blur the image of the foetus?
2. Some piano tuners base their tuning on a frequency counter alone. Many believe that this is not a good way to tune. Can you give a reasonable explanation for such scepticism?
3. Try to give a verbal description of what is going on physically as we *begin* to blow air into an organ pipe and until the sound becomes stable.
4. We can create a tone by blowing air through a straight tube. By changing the tightening of the lips, we can produce different pitches. How is it related? What is the wave pattern inside the tube made by some of the sounds that can be generated? How do you suppose the spectrum would look like?
5. Can we get a standing wave by adding two waves moving in the opposite direction to each other, one having greater amplitude than the other, but the same frequency? Can we get a standing wave if we add two waves that move in the opposite direction to each other, where one has greater frequency than the other, but the same amplitude?
6. Are standing waves always quantized? Explain.
7. In music, an octave is characterized such that the frequency, for example, of a high C being twice the frequency of a C that is an octave lower. Suppose we have a properly tuned guitar, and we will amuse ourselves by tightening a string so that it will give an octave higher than it normally should be. How much more tightening do you need? [Is this a party game that can be recommended?]
8. A violinist sometimes touches the midpoint of a string while stroking the bow over the string. What does she accomplish with this trick?
9. When sound goes from air to water, which one of the following quantities stays constant: Wavelength, wave speed, frequency, amplitude of displacement of the molecules that propagate sound?
10. On a trumpet we can play different tones by pushing valves that cause air to pass through tubular loops (of different lengths) that extend the effective length of the air string within the instrument. How can we play different tones on a “post horn” or similar instruments where we cannot change the effective length? Can we play the same type of tunes on such an instrument as, for example, on a trumpet?
11. If we inhale helium and talk, we get a “Donald Duck voice” that is light and shrill. What is the reason for that? [Remember that inhaling too much helium can damage health and cause death, so be careful if you want to try this yourself!]
12. When we play an acoustic guitar (see Fig. 7.24), the sound becomes different depending on whether we strum the strings all the way down to near the saddle where the strings end or near the sound hole (or even closer to the middle of the string). What is the reason for the difference in tonal quality? And how would you characterize the difference?

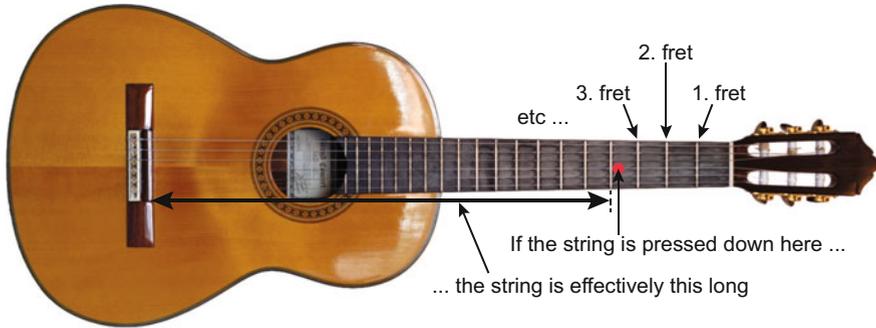


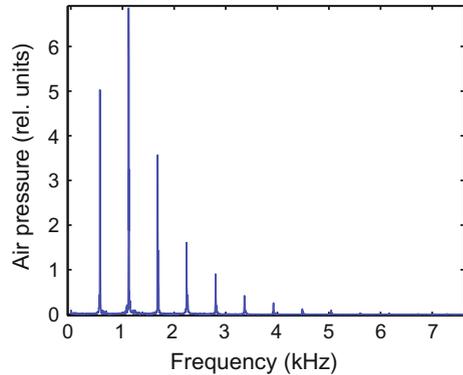
Fig. 7.24 On a classic guitar, a string is shortened if pressed against the first fret. The tone will then be a half-tone higher than with an open string. If the string is clamped at the second fret, the tone becomes two semitones higher, etc.

13. Does it make sense to say: Adding X dB to the sound corresponds to multiplying the intensity of the original sound wave with a definite factor?
14. Explain briefly the difference between dB, dB(SPL), dB(A) and dBm.
15. At an organ concert a listener noticed that after the organist had finished playing, it took a few seconds for the sound to subside totally. What is the reason that the sound dies out slowly? And what happened to the energy that was in the original sound?

Problems

16. An organ pipe is 3.9 m long, and open at the end. What tone do you suppose it emits (compare with Fig. 7.16).
17. The length of the free part of the strings on an acoustic guitar is 65 cm (that is, the part that can vibrate). If we clamp down the G-string on the fifth fret, we get a C (see Fig. 7.24). Where must the fifth fret be located on the guitar neck? The G has a frequency of about 196.1 Hz and the C about 261.7 Hz.
18. Use the information and answers from the previous assignment. For every semitone we go up from where we are, the frequency must increase by a factor of 1.0595. Calculate the position of the first fret, and to the sixth fret. Is the distance between the frets (measured in millimetres) identical along the guitar neck? Show that the distance between the frets is 0.0561 times the length of the string when it was clamped at the previous fret.
19. Check the frequencies indicated in Fig. 7.16. Supposed that we determined the frequency content of the sound data using Fourier transformation. For how long did we have to sample the sound to reach such precision? Is this a realistic way to determine the frequency accurately? Would it be more realistic to report the frequency with five significant digits for the highest frequencies than for the lowest? (Hint: Use the time-bandwidth product from the Fourier transform chapter.)

Fig. 7.25 Example of the frequency spectrum of a trumpet



20. Assume (for the time being) that the intensity of the sound that comes from a choir is proportional to the number of singers. How much more powerful, on a decibel scale, will a choir of 100 persons sound compared to a four-person choir (a quartet)?
21. Figure 7.25 shows the frequency spectrum of a trumpet sound.
 - (a) Estimate the frequency and relative pressure amplitude of the first five harmonics.
 - (b) What is the frequency of the fifth overtone?
 - (c) Assume that the intensity of the fundamental tone is 50 dB(SPL). Calculate the sound intensity in dB(SPL) for the entire trumpet sound (enough to include the first four (or five) harmonics).
 - (d) Calculate the sound intensity in dB(A) for the entire trumpet sound (enough to include the first four (or five) harmonics).
22. Suppose a person is lying on a beach and listening to a CD player placed 1 m from the head, and that the music has an intensity of 90 dB. How powerful will the music sound to someone who is 4 m away from the speaker? If the neighbour complains about the noise level, what can the first person do to resolve the conflict? Feel free to present a calculation to support your proposal.
23. Two strings on an instrument are both tuned to vibrate at 440 Hz. After a few hours, we notice that they no longer have the same frequency, because we hear a 2 Hz beat when we let both strings vibrate at the same time. Suppose one of the strings still vibrates at 440 Hz. Which frequency or frequencies can the other string have? How much has the tension changed on the string that has lost its tuning?
24. In this assignment, we will compare sound intensities, displacement amplitudes and pressure amplitudes. Remember to comment on the results you get in every part!
 - (a) What is the amplitude of air molecules when the sound intensity is 0 dB(SPL) at 1000 Hz? Repeat the same calculation for sound with intensity 100 dB(SPL).

- (b) What is the sound pressure amplitude (both in Pascal and in atmospheric pressure) when the sound intensity is 0 dB(SPL) at 1000 Hz? Repeat the calculation for sound with intensity 100 dB(SPL).
- (c) What is the displacement amplitude and the pressure amplitude for sound with the frequency 100 Hz and the intensity 100 dB(A)?
- (d) There is an upper limit for how large the sound pressure amplitude may be if the sound wave is to be approximately harmonic (sinusoidal). What is this limit? How powerful would the sound be at this limit (specified in dB(SPL))?
25. Suppose you drive a car at 60 km/h and hear that a police car with sirens approaches from behind and drives past. You notice the usual change in sound as the police car passes. Assume that the speed of the police car is 110 km/h and that the upper limit for the frequency of the siren (when heard inside the police car) is 600 Hz. What frequencies do we hear before and after the police car has passed us?
26. Suppose a fighter plane takes off from Bodø airport and reaches 1.75 Mach already at 950 m altitude. What angle does the shockwave have? How long does it take from the moment the plane passes directly above a person on the ground till the moment the person notices the shock wave? Disregard changes in the speed of sound with the height.
27. In an ultrasound examination of a foetus, the Doppler effect is used for measuring the rate of cardiac movement in the foetus. The sound has a frequency of 2.000000 MHz (2 MHz sharp), but the sound back has a frequency of 2.000170 MHz. How much speed had that part of the foster heart where the sound was reflected from, in the short period in which this measurement was made. Sound travels in the foetus with a speed of about 1500 m/s. [Optional additional question: How much time resolution is it possible to achieve for mapping cardiac movement in cases like this?]
28. The Crab Nebula is a gas cloud that can be observed even with small telescopes. It is the remnant of a supernova explosion that was seen on Earth July 4, 1054. Gas in the outermost layers of the cloud has a red colour that comes from hot hydrogen gas. On earth, the hydrogen alpha line $H-\alpha$ has a wavelength of 6562.82 Å. When studying the light from the Crab Nebula, the $H-\alpha$ line has a *width* of 56,942 Å.
- (a) Calculate the rate at which the gas in the outer part of the Crab Nebula moves. [Assume that the velocity of light is 3.0×10^8 m/s and that the relativistic Doppler shift for electromagnetic waves can be given approximately as $f_{\text{observ}} = (1 - v/c)f_{\text{source}}$ if the source moves away from the observer with speed v .]
- (b) Assume that the gas in the outer part of the nebula has moved at the same speed ever since the supernova explosion. Estimate the size of the Crab Nebula as it appears now. State the answer both in metres and in light years.
- (c) The angular diameter of the Crab Nebula when we see it from Earth is about 5 arc minute. An arc minute is 1/60 of a degree. Estimate the distance (in light years) to the Crab Nebula.
- (d) When did the explosion of the star actually take place (approximately).
- (e) In reality, the Crab Nebula is not spherical. Viewed from the Earth, it looks more elliptical with the largest and smallest angular diameters of 420 and 290 arc

- seconds, respectively. Even today, we do not know the distance to the Crab Nebula very accurately. Can you give a good reason for the inaccuracy based on the calculation you have made?
29. Perform a Fourier transform frequency analysis of the sound of two different musical instruments (record sound yourself via microphone and sound card on a PC, on a mobile phone, or use wav-files made available from our Web pages). Determine the frequency of the sound (fundamental tone) and find which tone on the scale it corresponds to. State approximately how many harmonics you find.
 30. The left part of Fig. 7.26 shows a time plot of the sound from a tuba. One student used Fourier transform to convert this signal to the frequency spectrum including the harmonics. The student then conducted an inverse Fourier transformation of the frequency spectrum and expected to recover the original time signal. He did not. The result is shown in the right part of the figure. What went wrong?
 31. A piano tuner first selects all three C-strings (all of which are activated by one key) to produce the 261.63 Hz frequency. [She actually starts with another frequency, but let's take this starting point here.] She now wishes to tune the F-strings by starting from C and using "re-tuning" where the frequency of F is exactly $4/3$ of the frequency of C. This she does for all three F-strings that are struck when we press the key. She then intercepts one of the three F-strings by listening to the beat frequency she gets when she presses the key. By adjusting the beat frequency correctly, she ensures that the string gets the correct frequency on a tempered scale (and can adjust the frequency of the other two F strings after the first). What beat frequency should she choose?
 32. Use the numbers for the length of the air column in a trumpet given in Fig. 7.15 to check that:
 - (a) the fundamental tone is about a B (indicate the frequency).
 - (b) that the elongation of the air column resulting from the depression of valve 1 corresponds approximately to a complete compared with that when no valves are pressed. Does the frequency go up or down when we press a valve?

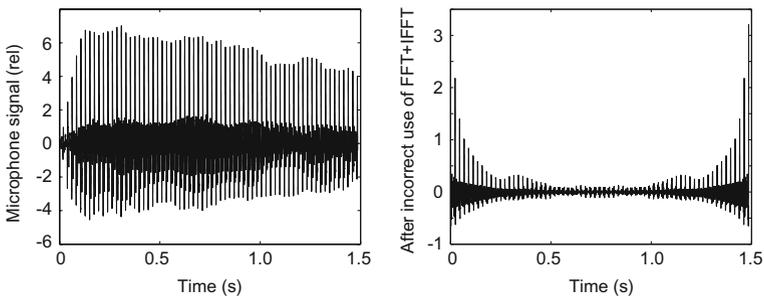


Fig. 7.26 See problem text

33. **Suggested Project:** We invite you to make your own “sound meter”. The procedure could be as follows:

- Sample sound from your laptop using, for example the program snippet 2 at the end of Chap. 5.
- Perform a Fourier transformation and get the frequency spectrum of the sound. The intensity for the different frequency components are then proportional to the square of the Fourier coefficients.
- Reduce the relative intensities for various frequency components according to for example the weight function for dB(A).
- Add the weighted intensities for the frequency components.
- Calculate the dB(A) value for your sound, using an arbitrary reference intensity.
- Borrow a sound meter from someone who has one and adjusts the reference intensity in the calculations until you get a similar reading on your own sound meter as for the commercial instrument.

In fact, it is reasonably easy to make your own sound meter in this manner. However, remember that the microphone on the computer as well as the digitizing circuit have their limitations. Especially, it is difficult to get a good determination of weak signals.

For strong signals, it is another serious problem: The sound may produce signals larger than the digitizing circuit can manage. In those cases, “clipping” will occur. It can be discovered if you plot the sampled signal in time domain. Sinus signals will then have a flat top, and no signal can exceed this limit.

For such signals, the sound meter will give wrong readings! However, it is reasonably easy to let your program display a warning in those cases.

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