Chapter 13
Characteristics of DC Machines

Working with DC machines requires the knowledge on their electrical and mechanical properties, parameters, and limitations. This chapter introduces and explains the concept of rated quantities and discusses the maximum permissible currents in continuous, steady-state service of DC machines. It also defines the safe operating area of DC machines in T_{em}–Ω_m plane, both in steady-state operation and during transients. For the sake of readers that meet electrical machines for the first time, the concepts of rated[1] current, rated voltage, mechanical characteristics, natural characteristics, rated speed, rated torque, and rated power are introduced and explained in this chapter. The need to use machines at higher speeds and with reduced flux is discussed and explained, introducing at the same time the constant flux operating region and the field-weakening operating region. The problems of removing the heat caused by the conversion losses are analyzed along with the performance restrictions imposed by temperature limits. Besides, an insight is given into possible short-term overload operation of DC machines. Principal conversion losses in DC machines are analyzed, discussed, and included in power balance. This chapter closes by discussing permissible operating areas in torque-speed plane. The steady-state safe operating area in T_{em}–Ω_m plane is also called exploitation characteristics. It is introduced and explained along with the transient safe operating area, also called the transient characteristic. Discussion and examples within this chapter are focused on separately excited DC machines.

[1] In electrical engineering, the concept of rated voltages, currents, and other similar quantities is widely used. Considered quantities are usually the ones that contribute to thermal, mechanical, dielectric, or other stress that may have potential of damaging electrical machine, transformer, or other electrical device or to increase its ware and reduce the expected lifetime. The rated value is most usually set by the manufacturer as a maximum value to be used with the considered device. Continued operation with voltages and currents that exceed the rated values causes permanent damage to machine windings, magnetic circuits, or other vital parts. For some quantities such as electrical current, the rated value can be surpassed during very short intervals of time without causing damages. The rated values are usually set somewhat below the level that damages the device. This is done to allow a certain safety margin.

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13.1 Rated Voltage

The winding rated voltage is the highest voltage that can be permanently applied to the winding terminals without causing breakdown or accelerated aging of electrical insulation. In some cases, the rated voltage can be briefly exceeded without causing any harm.

Electrical insulation separates the winding conductor from the walls of the slot where the conductor is placed. In addition, the electrical insulation separates this conductor from other conductors. A loss of insulation leads to a short circuit between individual conductors, between the winding terminals, as well as between the winding and the magnetic circuit or the machine housing. Each of the accidents mentioned results in permanent damage and interrupts the operation of the machine. In most cases, it has to be sent to the workshop or repair service.

The electrical field which exists in insulation is proportional to the supply voltage. The insulation is characterized by the rated voltage U_n as well as by the breakdown voltage U_{max}.

The breakdown voltage causes instantaneous damage of insulation. With breakdown voltage applied, the electrical field strength in critical parts of the insulation material exceeds the dielectric strength of the material and destroys insulation. Cumulative ionization of otherwise nonconductive dielectric provides a virtual short circuit between two conductors, or between the winding terminals, or a short circuit between the winding and earthed metal parts of the machine.

The rated voltage is lower than the breakdown voltage. Supplying the winding with a voltage which is higher than the rated but lower than the breakdown voltage does not necessarily lead to breakdown. Increased electrical field strength established with voltages above the rated lead to accelerated degradation and aging of the insulation material. This phenomenon reduces the expected lifetime of the insulation. Insulation aging is related to electrical, chemical, and thermal processes within dielectric materials. In most cases where the voltage is maintained within the limits of the rated voltage, expected lifetime of insulation materials and systems reaches 20 years. Continued operation with voltages exceeding the rated by 8% may halve the insulation lifetime.

13.2 Mechanical Characteristic

Mechanical characteristic is function $T(\Omega)$ or $\Omega(T)$ which gives relation between the angular speed of rotation and electromagnetic torque in steady-state operating conditions, with no variations of the speed, current, or flux of electrical machine. For separately excited DC machine, where the excitation winding and armature windings have separate supply, mechanical characteristic is given by expression

$$T_{em} = k_m \Phi_f \frac{U_a}{R_a} - \frac{k_m k_e \Phi_f^2}{R_a} \Omega_m$$

13.3 Natural Characteristic

Natural characteristic is mechanical characteristic obtained in the case when all the voltages fed to the machine are equal to the *rated voltages*.

13.4 Rated Current

Rated current I_n is the maximum permissible armature current in continuous operation. Namely, it is the largest current that can be maintained permanently, which at the same time does not cause any overheating, damages, faults, or accelerated aging. In AC machines, the rated current implies the rms value of the winding current.

Electrical currents in windings of electrical machine produce losses and develop heat, increasing the temperature of conductors and insulation. Current in the windings creates Joule losses which are proportional to the square of the current. The temperature of the machine is increased in proportion to generated heat. Increased *temperature difference* between the surface of an electrical machine and the environment gives a rise to heat transfer from the machine to the environment. The heat can be passed by conduction, convection, and radiation. Heat conduction takes place through the machine parts that are in touch with cold external solids such as the machine basis or flange. Heat convection relies on natural or forced streaming of air along the machine sides. Heat radiation is electromagnetic process caused by thermal motion of charged particles on the surface of electrical machine, and it depends on absolute temperature. The heat radiated from a warm machine to cold environment is larger than the heat absorbed by the machine due to radiation caused by the environment.

When the surface temperature of electrical machine exceeds the temperature of the environment, the heat is transferred to the environment, and the machine is cooled. The power P_T of the heat transfer defines how many joules of heat are transferred in each second. The processes of the heat transfer are nonlinear. Yet, for the range of temperatures encountered in operation of electrical machines, the power P_T can be considered proportional to the temperature difference, $P_T = \Delta\theta / R_T$, where R_T is *thermal resistance* of an electrical machine with respect to the environment, proportional to the surface of the machine and characteristics of this surface. Power P_T is expressed in W, temperature $\Delta\theta$ in $^{\circ}$C, while thermal resistance R_T is expressed in $^{\circ}$C/W. The equilibrium is established when the machine temperature reaches the value that results in heat transfer P_T which is equal to the total losses within the machine. Total losses of electrical machine are denoted by P_γ. With $P_T = P_\gamma$, machine temperature remains constant. The higher the losses in a machine, the higher the temperature reached in the steady state. Considering a DC machine, an increase of armature current I_a increases Joule effect losses. They are proportional to I^2, and they increase temperature of DC machine. Excessive temperature increase may damage some critical machine parts, such as the insulation of the windings.

Electrical insulation is made of paper, fiberglass, lacquer, or other materials. It separates copper conductors of machine windings from other conductive parts of machine, as well as from other conductors, thus preventing short circuits of the windings or their individual turns. The insulation can be damaged if temperature exceeds the maximum permissible limit for given insulating material. The insulation of thermal class A is damaged if the temperature exceeds 105 °C. For insulation of class F, the temperature limit is 155 °C. Therefore, DC machine with class F insulation would have higher permissible temperatures and higher rated current. When DC machine operates in steady state with the rated armature current, the losses are expected to heat the machine up to the temperature limit. With currents in excess to the rated, DC machine in continuous service would overheat. At the same time, temperature would exceed the limits and damage insulation or some other vital part of the machine. Other than insulation, there are other machine parts that are sensitive to elevated temperatures. Permanent magnets, ferromagnetic materials, and even the elements of steel construction of the machine could deteriorate and fail due to excessive temperatures. Ferromagnetic materials lose their magnetic properties when heated up to Curie temperature. Permanent magnets could be permanently damaged (demagnetized) by overheating. The elements of steel construction such as the shaft and bearings could be damaged due to thermal dilatation, changes of steel properties, and failure in lubrication of the bearing at high temperatures.

The rated current I_n is the highest armature current I_a in continuous service that does not cause any damage or fault and does not shorten the expected lifetime of the machine.

13.5 Thermal Model and Intermittent Operation

Conversion losses in an electrical machine lead to an increase of temperature of the magnetic circuit and windings. Machine is warmer than the environment, and it transfers heat to the environment. If the heat generated by the losses within the machine is equal to the heat transferred to the environment, the system is in steady-state conditions, and the temperature does not change. The temperature remains constant in cases where the heat generation remains in equilibrium with the heat emission. In other words, the *heating* has to be equal to *cooling* in order to achieve a constant temperature. The maximum permissible temperatures of vital parts of the machine are determined by endurance of the electrical insulation, magnetic circuit, windings, bearings, and housing. Variation of temperature in a machine is determined by thermal resistance R_T and thermal capacity C_T, the two machine parameters discussed further on. The former determines the heat emission from the machine into the environment, while the latter determines the heat accumulated within the machine. If thermal capacity is sufficiently large, the machine can endure

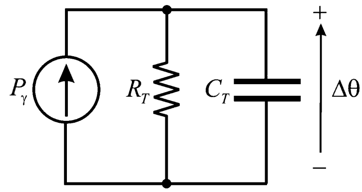

Fig. 13.1 Simplified thermal model of an electrical machine

a short-term overload. In overload conditions, the current exceeds the rated, losses in the machine are increased, and the heat is generated in excess to the heat removed by cooling. Excess heat is accumulated in thermal capacity of the machine, and the temperature rises. With sufficiently large thermal capacity, the temperature does not reach the limits for significant interval of time. During that time, the armature current exceeds the rated value, but it does not cause any damage. Hence, depending on thermal parameters of the machine and the current amplitude, an overload condition is permissible for certain interval of time. The impact of thermal resistance, thermal capacity, and power losses on temperature change is shown in Fig. 13.1, which represents a simplified thermal model of an electrical machine.

Electrical machines are made of copper, iron, aluminum, and insulating materials. Each material used in manufacturing electrical machines has its own specific heat. Specific heat represents the energy that raises by 1 °C the temperature of the unit mass. By multiplying specific heat and mass of the part, one obtains thermal capacity of considered part, expressed in terms of J/°C. Based on the assumption that all parts of the machine are at the same temperature, simplified thermal model is obtained and shown in Fig. 13.1. Parameter C_T of the thermal model represents the sum of thermal capacities of all machine parts. Total thermal capacity C_T determines the heat which causes the machine temperature to increase of one degree, under condition with no heat being released into environment. The part of the loss power liable to the temperature rise is

$$C_T \frac{\mathrm{d}(\Delta\theta)}{\mathrm{d}t}.$$

The remaining part of the loss power is transferred to the environment. Whenever the machine temperature exceeds the environment temperature by $\Delta\theta$, the power of heat emission to the environment, also called cooling power, assumes the value of

$$\frac{\Delta\theta}{R_T}.$$

Thermal resistance R_T is expressed in °C/W. It determines temperature rise $\Delta\theta$ required to obtain the cooling power $\Delta\theta/R_T$. The heat is transferred by convection, conduction, and radiation. Thermal resistance depends on the surface area exposed toward the environment, on the airspeed, on properties of the surface which radiates

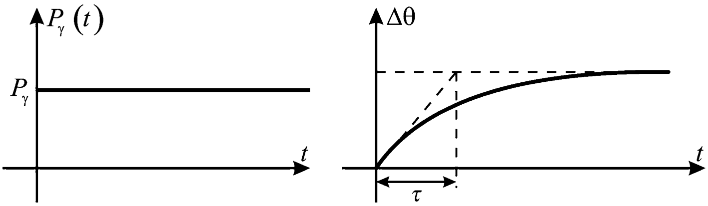

Fig. 13.2 Temperature change with constant power of losses

the heat, and also on other circumstances and parameters. The change of the machine temperature is determined by differential equation

$$P_\gamma = \frac{\Delta\theta}{R_T} + C_T\,\frac{\mathrm{d}(\Delta\theta)}{\mathrm{d}t}, \tag{13.1}$$

where the first factor describes the cooling power, while the second factor represents the rate of the heat accumulation in thermal capacity of the machine. The equation is derived under assumption that all the machine parts have the same temperature. It should be noted that the equation resembles the one describing the voltage change in parallel *RC* circuit with resistor R_T and capacitor C_T, supplied from current source of the current P_γ. Hence, thermal model of the machine of Fig. 13.1 is dual with the electrical *RC* circuit supplied from a current source. The voltage corresponds to the temperature increase $\Delta\theta$, while the current corresponds to the power of losses.

In cases where $\Delta\theta(0) = 0$, and where the power of losses is represented by $P_\gamma(t) = P_1 h(t)$, the temperature of the machine changes according to expression

$$\Delta\theta = R_T P_1\left(1 - e^{-\frac{t}{\tau}}\right), \tag{13.2}$$

where $\tau = R_T C_T$ is *thermal time constant*. The thermal time constant of small electrical machines reaches several tens of seconds. Large machines may have their thermal time constants of several tens of minutes. Hence, the thermal processes within the machine are relatively slow. In Fig. 13.2, temperature change is presented for the case when the machine starts from the standstill, with the initial temperature equal to the ambient temperature. The operation proceeds with constant losses, and the temperature rises exponentially, according to the law given in (13.2). The temperature increase $\Delta\theta$ approaches to the steady-state value $\Delta\theta = P_\gamma R_T$ after 3τ .. 5τ , where R_T is the thermal resistance and τ is the thermal time constant.

If power of machine losses changes due to variations of the current, torque, or the rotor speed, the machine temperature follows variations of power losses with certain delay, determined by the thermal time constant. Figure 13.3 shows temperature changes of an electrical machine having periodic changes of the armature current. The intervals with considerable armature current are followed by the intervals when the current is equal to zero. Former intervals are associated with losses, while the

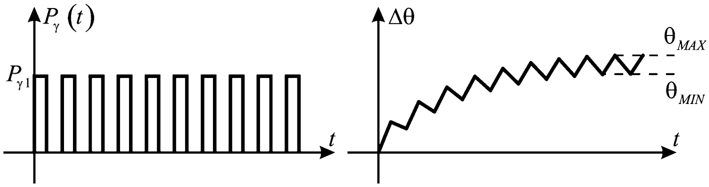

Fig. 13.3 Temperature change with intermittent load

latter interval passes with no Joule effect losses. Presented operating mode is called *intermittent mode*. At steady state, temperature oscillates between θ_{MAX} and θ_{MIN}. In hypothetical prolonged no load conditions, the machine would reach the ambient temperature θ_a, that is, the temperature of the environment. The value of θ_{MIN} is higher than the ambient temperature θ_a. Assuming that the interval with the armature current is prolonged indefinitely, the temperature will reach the value $\theta_\infty = \theta_a + \Delta\theta = \theta_a + P_{\gamma 1} \cdot R_T$, where $P_{\gamma 1} = R_a I_1^2$. The value of θ_{MAX} is lower than θ_∞. The power of losses $P_{\gamma 1}$ is shown in Fig. 13.3, and it corresponds to the operation with armature current I_1. In intermittent mode, this current can exceed the rated current and do no harm.

With $I_a > I_n$, power losses of $P\gamma 1$ in continuous service cause the overheating. Eventually, the machine temperature would reach θ_∞ and damage some vital parts of the machine. Nevertheless, the thermal capacity of the machine permits overload condition $I_a > I_n$ with higher loss power $P_{\gamma 1}$, but only for a short interval of time, determined by the thermal time constant $\tau = R_T C_T$. Hence, any electrical machine can withstand certain overload of limited duration. If such short time overloads are followed by intervals with no losses, the heat accumulated during overload pulses is released into ambient during prolonged intervals of time. With an adequate cooling, short time overloads can be repeated in the manner shown in Fig. 13.3. The overload intervals can be also followed by the intervals with loads that are sufficiently small to ensure a sufficient decrease in the machine temperature (cooling) before the next overload pulse.

When the load torque and the machine current exhibit periodic changes, it is possible to identify load cycles that comprise one overload interval followed by an interval with reduced losses. If the period of the load cycling is shorter than the thermal time constant τ, then the temperature oscillations $\theta_{MAX} - \theta_{MIN}$ are relatively small. With load cycle periods significantly shorter than τ, the difference $\theta_{MAX} - \theta_{MIN}$ becomes negligible. In such cases, the temperature increase $\Delta\theta$ depends on average losses within each load cycle. Notwithstanding periodic overloads, machine does not get damaged if the average power of losses does not exceed the rated loss power $P_{\gamma n}$, permissible in continuous operation. Neglecting all the losses except the ones in the armature winding, the rated power of losses can be estimated as $P_{\gamma n} = R_a I_n^2$. The losses during the overload interval of the load cycle may be considerably larger than the rated losses and yet maintain safe operation of electrical machine. The heat impulses generated during the overload intervals in

Fig. 13.3 are released into the ambient during the intervals with reduced losses. Intermittent temperature rise $\theta_{MAX} - \theta_{MIN}$ during overloads is inversely proportional to the thermal capacity C_T.

Based on the performed analysis, it is concluded that the current of an electrical machine may exceed the rated current during short time intervals, provided that the overload intervals are much shorter compared to the thermal time constant of the machine. The overload pulses can come as a train, provided that the light load intervals in between secure sufficient cooling and sufficient drop of the machine temperature prior to arrival of the next pulse. With load that comes as a train of pulses, machine operation is safe if the repetition period is shorter than the thermal time constant, and provided that the average power of losses does not exceed the losses incurred with nominal current.

Question (13.1): While operating with the rated load, the steady-state temperature of an electrical machine is only slightly increased above the ambient temperature. Is it designed properly?

Answer (13.1): No. Small steady-state operating temperatures show that the power of losses within the machine is rather small. This means that there is plenty of room for increasing the current density and increasing the magnetic induction. The flux increase accompanied by the increase in the armature current raises the output torque and power. Hence, the machine under consideration can produce significantly higher torque and power compared to the power declared as the rated. Consequently, too much copper and too much iron are used to make the machine. Other than being more expensive, the machine is also larger and heavier. The same load requirements can be met by electrical machine of much smaller size and weight.

Question (13.2): Electrical machine has not been used for a long time. After turning on, the armature current reaches the value of $2I_n$, twice the rated current. Prevailing losses are the winding losses. These are proportional to the square of the current. The iron losses can be neglected. The thermal time constant is 60s. Determine the longest permissible time the machine can be operated with $2I_n$ with no damages.

Answer (13.2): Permanent operation with twice the rated current produces the power of losses which is four times larger than the rated power of losses. Therefore, the machine would heat up rather quickly. Theoretical value of the steady-state temperature in this mode is four times higher than the permissible temperature. The limit temperature is reached after t_1, where $1 - \exp(-t_1/\tau_T) = ¼$. It follows that $t_1 = 17.26$ s.

Question (13.3): Electrical machine is loaded in such a way that every 10s a pulse of armature current appears having the amplitude of $2I_n$. This pulse is followed by an interval with no current. Prevailing losses in the machine are the winding losses, which are proportional to the square of the armature current, while the iron losses can be neglected. It is known that the thermal time constant is $\tau_T \gg 10$s. Determine the longest permissible width of the armature current pulse which does not cause damages to the machine.

Answer (13.3): Thermal time constant is significantly higher than the pulse period ($\tau_T \gg 10$ s). Therefore, temperature variations are slow, and the temperature changes within the pulse period are insignificant. For that reason, it is possible to consider the average power of losses. With any periodic current $i(t)$, the average power of Joule losses is calculated as

$$P_{\gamma(av)} = \frac{1}{T}\int_0^T R_a i_a^2 \mathrm{d}t = R_a\left(\frac{1}{T}\int_0^T i_a^2 \mathrm{d}t\right) = R_a I_{a(rms)},$$

where $I_{a(rms)}$ is the rms (*root mean square*) current. The rms current is also called *thermal equivalent* of periodic current $i(t)$. A resistor with constant $I_{a(rms)}$ will heat up very much the same as it does with periodic current $i(t)$. If the rms value does not exceed the rated value I_n, the machine does not overheat. A pulse of amplitude $2I_n$ that repeats every 10 s has the rms value of I_n provided that the pulse width is 2.5 s.

13.6 Rated Flux

Excitation flux Φ_f determines the current which has to be established in the armature winding so as to develop the desired torque, $I_a = T_{em}/(k_m\Phi_f)$. With higher values of flux, one and the same torque is obtained with lower armature current, and this reduces the winding losses. Therefore, it is of interest to increase the flux in order to reduce the required armature current. The excitation flux is limited by magnetic saturation of the ferromagnetic material. If the region of saturation is reached, any further increase of the excitation current results in a very small change of the flux. While operating in saturation region, the only effect of increasing the excitation current is the increase in power of excitation winding losses $R_fI_f^2$. Magnetic saturation becomes pronounced at the knee of the magnetizing curve $\Phi_f(I_f)$. The knee of the curve is the point where the initial, linear slope ends and the curve bends toward the abscissa, resulting in very small $\Delta\Phi/\Delta I_f$ ratio. The flux at the knee point is denoted by Φ_{fmax}. Since any further increase of the excitation current is of little practical effect, the value Φ_{fmax} represents the maximum flux. This flux can be used in order to reduce the armature current $I_a = T_{em}/(k_m\Phi_f)$ required to achieve desired torque T_{em}. Analysis performed in subsequent sections shows that, in certain operating conditions, the value of Φ_{fmax} cannot be used. An example is the operation at very high speeds, where the electromotive force $k_e\Phi_f\Omega_m$ must not exceed the rated voltage. In other cases, the flux Φ_{fmax} can be advantageously used, providing reduction in armature current. The knee point flux Φ_{fmax} is also called *rated flux*, and it is denoted by Φ_n.

13.7 Rated Speed

Rotor speed of electrical machine influences electromotive force induced in the windings. Rated speed is defined as the rotor speed where electrical machine with rated flux has electromotive forces equal to the rated voltage. Hence, when the machine with rated flux accelerates from zero to the rated speed, the electromotive forces increase from zero up to the rated voltage.

Electromotive forces should not exceed the rated voltage in order to avoid excessive voltage that could damage the insulation. Therefore, definition of the rated speed implies that the machine cannot exceed the rated speed yet maintaining the rated flux. In order to contain the electromotive forces between the rated limits, the flux has to be reduced as the speed goes beyond the rated speed.

Considering a DC machine with rated excitation flux and with a negligible resistance R_a, the electromotive force $E_a = k_e \Phi_n \Omega_m$ is induced, equal to the armature voltage U_a. At the rated speed, the electromotive force is equal to the rated voltage. Therefore, rated speed of DC machine is determined by relation

$$\Omega_n = \frac{U_n}{k_e \Phi_n} \tag{13.3}$$

Since the armature voltage should not exceed the rated value, the operation at speeds larger than the rated is possible only with a reduced flux. Otherwise, the electromotive force $k_e \Phi_n \Omega_m$ would exceed the nominal voltage $U_n = k_e \Phi_n \Omega_n$.

For small electrical machines, the voltage drop due to winding resistances cannot be neglected. Since $U_a = R_a I_a + E_a$, the difference between the armature voltage and the electromotive force cannot be neglected. In such cases, definition of rated speed is made more precise, and it includes the voltage drop across the armature resistance. The rated speed can be defined as the one that results in rated voltage across the armature winding of electrical machine which operates with the rated flux and the rated current. In such conditions, the electromotive force is equal to $E_a = k_e \Phi_n \Omega_n = U_n - R_a I_n$, while the rated speed is defined by expression

$$\Omega_n = \frac{U_n - R_a I_n}{k_e \Phi_n}. \tag{13.4}$$

13.8 Field Weakening

Operation of DC machines may require changes in the excitation flux. One example is the operation at speeds above the rated speed. In order to keep the electromotive force within the limits of the rated voltage, the flux should be reduced so as to maintain the relation $k_e \Phi_n \Omega_m < U_n$. Flux reduction at high speeds is called *field weakening*.

In addition, there are cases when the flux reduction is beneficial even at speeds below the rated. When electrical machine runs with relatively low electromagnetic torque, current is also low, as well as the power of losses in the winding. If at the same time the flux is kept at the rated value, the iron losses in the magnetic circuit become the principal conversion losses. In order to reduce the losses, it is necessary to reduce the flux. Lowering the flux increases the armature current required to develop desired torque, due to $I_a = T_{em}/(k_m \Phi_f)$. Operation with low T_{em} allows for relatively large flux reduction without any significant increase in armature current. For this reason, flux reduction at light load condition reduces the conversion losses.

13.8.1 High-Speed Operation

The electromotive force induced in the winding determines the voltage across the winding terminals. Therefore, the electromotive force must not exceed the rated voltage. The rated flux, voltage and speed are related by $U_n \approx k_e \Phi_n \Omega_n$. Therefore, electrical machine running with the rated flux Φ_n has the electromotive force that reaches the rated voltage as the speed approaches the rated speed. Any increase of the speed above the rated would result in excessive voltages. Therefore, the excitation flux has to be decreased as the rotor speed goes beyond Ω_n. Hence, electrical machines can maintain the operating speed $\Omega_m > \Omega_n$, provided that the flux is reduced so as to prevent the electromotive force from exceeding the rated voltage. For that to achieve, the flux should be varied according to the rotor speed. This change can be described by the function $\Phi(\Omega_m)$. Below the rated speed, $\Phi(\Omega_m) = \Phi_n$.

At speeds $\Omega_m > \Omega_n$, the electromotive force $E_a = k_e \cdot \Phi(\Omega_m) \cdot \Omega_m$ is induced in the machine. Neglecting the armature resistance and assuming that $E_a = U_n$, the flux to be used beyond the rated speed is

$$\Phi(\Omega_m)|_{\Phi > \Phi_n} = \frac{U_n}{k_e \Omega_m}. \tag{13.5}$$

From expression (13.3), which defines the rated speed, one obtains

$$\Phi(\Omega_m) = \Phi_n \frac{\Omega_n}{\Omega_m}, \tag{13.6}$$

which defines desired change of the flux at speeds above the rated speed. The flux is inversely proportional to the speed and varies according to $1/\Omega_m$. Variation of the flux which is necessary at speeds beyond the rated speed is defined by (13.6). The operating region where the speed is higher than the rated speed is called *field-weakening region*. If machine operates at speeds below the rated speed, it is possible to have the rated excitation flux Φ_n. For this reason, the operating region where $\Omega_m < \Omega_n$ is called *constant flux region*.

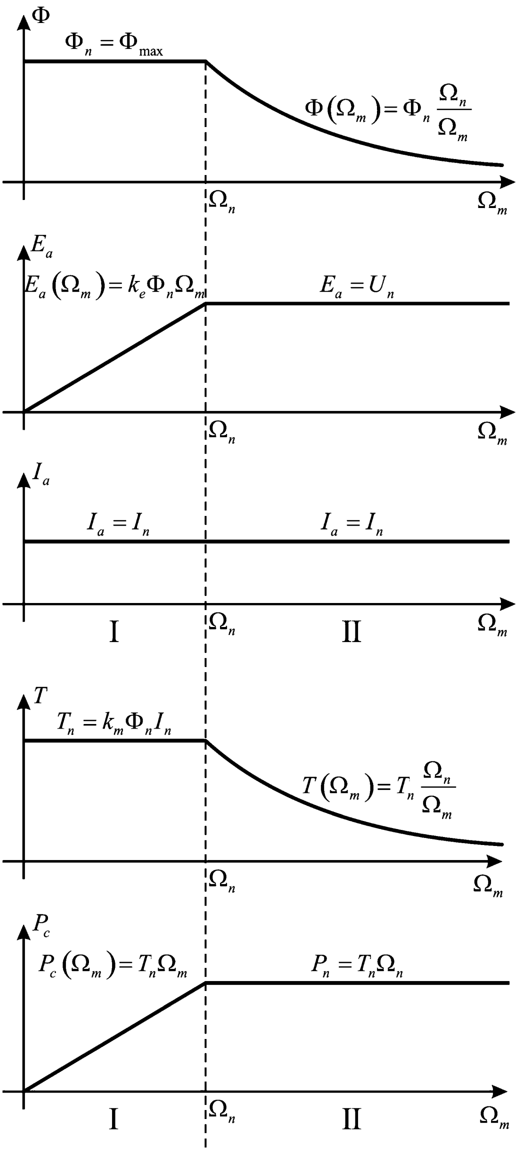

Fig. 13.4 Permissible current, torque, and power in continuous service in constant flux mode (*I*) and field-weakening mode (*II*)

13.8.2 Torque and Power in Field Weakening

The five diagrams in Fig. 13.4 illustrate the change of flux, electromotive force, power, torque, and current that can be maintained in field-weakening operation of DC machines. The rotor speed is on the abscissa of all diagrams.

It should be noted that diagrams in Fig. 13.4. represent the change in *available* values or *limit* values, that is, the values that can be used at steady state without causing any damage to the machine. Hence, any value below these limits can be used as well. Due to thermal capacity of the machine, the instantaneous values of the current, torque, and power may exceed the limit values for a brief interval of time. At the same time, it is of interest to notice that presented diagram represents the absolute values. The torque, current, and power may also take negative values. Therefore, conclusions derived here are applicable to both motors and generators. Diagrams of Fig. 13.4 and the corresponding conclusions are applicable to any machine studied in this book, with exception of symbols which is slightly different for AC machine.

13.8.3 Flux Change

Abscissa of all the diagrams in Fig. 13.4 represents the rotor speed. At speeds below the rated speed, the excitation flux is maintained at the maximum value, which is also the rated value. As the speed exceeds the rated speed, the flux decreases according to hyperbola $\Phi(\Omega_m) = \Phi_n\Omega_n/\Omega_m$. Machine can also use lower flux values, but they cannot exceed the values shown by the curve $\Phi(\Omega_m)$.

13.8.4 Electromotive Force Change

The electromotive force varies according to the law $E_a = k_e \cdot \Phi(\Omega_m) \cdot \Omega_m$. Below the rated speed, the flux is equal to the rated flux, and the electromotive force increases in proportion to the speed of rotation. In the region of the field weakening where $\Omega_m > \Omega_n$, the flux is inversely proportional to the rotor speed. If the speed increases, the electromotive force remains constant and equal to the rated voltage. If the flux is below the value determined by (13.5), the electromotive force will be smaller than the rated voltage. In the region of field weakening, the flux $\Phi(\Omega_m) = \Phi_n\Omega_n/\Omega_m$ has to be applied. If, at the same time, machine operates with very small torque and current, it is beneficial to reduce the flux even below the limit $\Phi_n\Omega_n/\Omega_m$, so as to reduce the iron losses.

13.8.5 Current Change

At steady state, the armature current must not exceed its rated value, $I_a^2 \leq I_n^2$. The current can assume any value below the rated, $|I_a| < I_n$. Exceeding the rated value in continuous operation results in overheating and may damage magnetic and/or current circuits of the machine. For this reason, diagrams in Fig. 13.4 indicate that the armature current applicable over long time intervals is limited by $|I_a| < I_n$ at all speeds.

13.8.6 Torque Change

Electromagnetic torque is determined by the product of the flux and current. During long-term operation at speeds below the rated speed, the flux and current can hold their rated values. Therefore, in these conditions, available electromagnetic torque is $T_n = k_m\Omega_n I_n$, also called *rated torque*. At speeds above the rated speed, the flux decreases according to expression $\Phi_n\Omega_n/\Omega_m$. Therefore, the available torque in the region of flux weakening is equal to $T(\Omega_m) = T_n\Omega_n/\Omega_m$. The product of torque which decreases with the speed and the speed results in a constant power. For this reason, the change of the available torque in the field-weakening mode is called *hyperbola of constant power*.

13.8.7 Power Change

Electrical power converted to mechanical power is equal to the product of the torque and the rotor speed. In constant flux region, at speeds below the rated, the available torque is constant, thus the available power increases proportionally with the speed. This region is also called *the region of constant torque*. In the region of field weakening, the power is equal to the rated power $P_n = T_n\Omega_n$, since $P_c = T_{em}\Omega_m = \Omega_m(T_n\Omega_n/\Omega_m) = T_n\Omega_n = P_n$. Therefore, the available power in field-weakening region is constant and equal to the rated power P_n. Another name for the field-weakening region is *the region of constant power*. Power P_n is called *rated* power.

13.8.8 The Need for Field-Weakening Operation

Applications of electric motors often require high values of electromagnetic torque at small speeds and small torques at high speeds. A DC motor used for propulsion of electrical vehicles can serve as an example. While setting in motion a heavily loaded vehicle which has to manage a very steep slope, the motor has to deliver a very large torque. In such case, it is important to develop the torque required to get over the hill, while it is acceptable to operate the vehicle and the motor at a low speed. The same vehicle may have to move unloaded over prolonged, flat path, where the motion resistances are low, and where the motor delivers relatively small torque. At the same time, it could be required to complete such motion quickly, and this calls for high vehicle speeds and high rotor speeds.

The first example requires high-torque, low-speed operation, while the second example calls for low-torque, high-speed operation. These requirements correspond to a hyperbola $T(\Omega)$ in T–Ω plane. The curve $T(\Omega_m) = T_n\Omega_n/\Omega_m$ is called hyperbola of constant power. DC machines with permanent magnets cannot operate in the field-weakening mode. There are no practical ways to reduce the flux of the magnets. Therefore, these machines cannot go beyond the rated speed. Hence, the constant power mode is inaccessible for this kind of DC machines. Whenever the need exists for such machines to provide the constant power at high speed, the problem cannot be solved on electrical side of the system. Instead, it is necessary to use mechanical coupling with variable transmission ratio. In cases when the electrical motor cannot increase the speed, as it has reached the rated rotor speed, transmission ratio of mechanical coupling can be changed so as to obtain higher load speeds with the same motor speed. Variable transmission ratio is used in road vehicles as well. Automobiles with internal combustion engines (ICE) are usually equipped with variable transmission gears. Torque-speed characteristics of internal combustion engines do not include constant power range hyperbola in $T(\Omega)$ plane. In order to provide for both the low-speed, high-torque operation and the high-speed, low-torque operation, it is necessary to change the transmission ratio of the gears that pass the ICE torque to the wheels. The use of electrical machines capable of providing constant power operation in field-weakening region removes the need for additional gears.

13.9 Transient Characteristic

The transient characteristic is the area in $T(\Omega)$ plane which comprises all T–Ω points attainable in short time intervals. That is, it is a collection of all the operating regimes the machine can support for a short while. Peak values of the torque which can be developed at a given speed depend on the excitation flux $\Phi_f(\Omega_m)$ and on the peak value of the armature current. In DC machines, instantaneous value of the current is limited by characteristics of mechanical commutator and on the maximum current of the semiconductor power switches used to build the switching power converter that supplies the motor. An example of transient characteristic is shown in Fig. 13.5.

13.10 Steady-State Operating Area

The steady-state operating area includes all the T-Ω points in $T(\Omega)$ plane where the machine can provide continuous service for a very long time. In the field-weakening region, the area is limited by the hyperbola of constant power, $T_{em}(\Omega_m) = T_n\Omega_n/\Omega_m$, while in the constant flux region the limit is $T_{em}(\Omega_m) = T_n$. An example of steady-state operating area is shown in Fig. 13.5. Since operation of electrical machines includes all four quadrants, the steady-state operating area exists in all four quadrants as well.

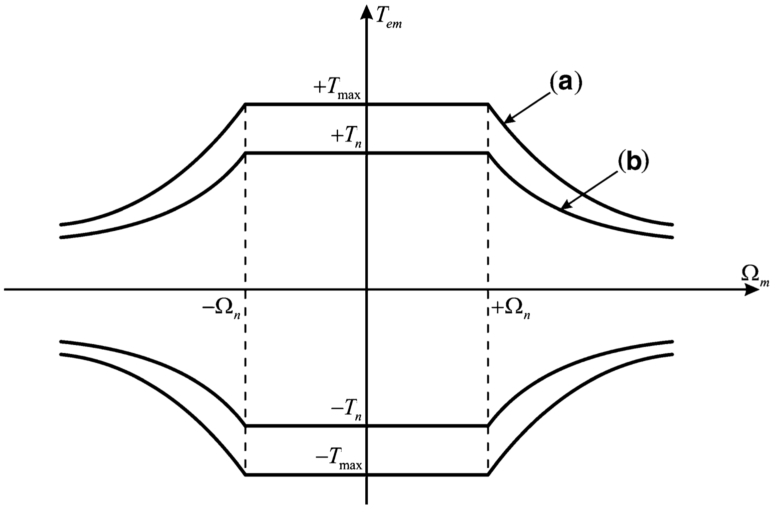

Fig. 13.5 (**a**) Transient characteristic. (**b**) Steady-state operating area

13.11 Power Losses and Power Balance

For the purpose of getting a better insight into the process of electromechanical energy conversion in DC electrical machines, it is required to study the power flow and the power of losses in the machine. Power balance equation includes the factors such as the iron losses in magnetic circuits, Joule losses in windings (also called *copper losses*), and mechanical losses due to rotation, also called losses in mechanical subsystem.

Losses in the compensation winding and the auxiliary poles windings are neglected, as these parts are not represented in each DC machine. While deriving the power balance, it is considered that the machine operates at steady state, with no variation of the armature current, excitation flux, torque, or the rotor speed. The power balance is shown in Fig. 13.6. The individual power components and losses are explained hereafter.

13.11.1 Power of Supply

The electrical sources feed the excitation and armature windings and supply the electrical power to the machine. The electrical power is $P_f + P_a = U_f I_f + U_a I_a = R_f I_f^2 + (R_a I_a^2 + E_a I_a)$.

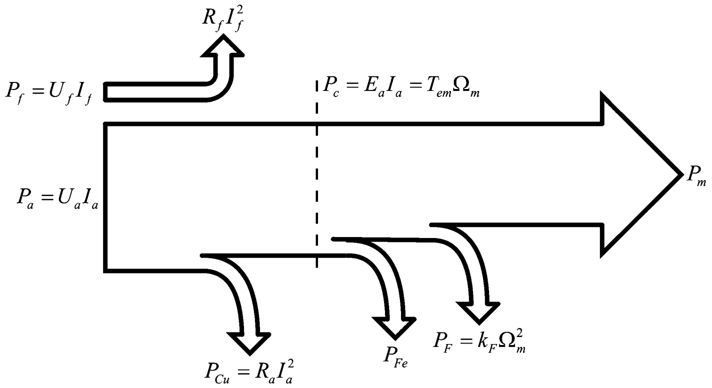

Fig. 13.6 Power balance

13.11.2 Losses in Excitation Winding

Due to Joule effect, the power of losses in the excitation winding is converted to heat. It is equal to $R_f I_f^2$, and it is also referred to as *copper losses* in excitation winding.

13.11.3 Losses Armature Winding

Due to Joule effect, the power of losses in the armature winding is converted to heat. It is equal to $R_a I_a^2$, and it is also called *copper losses* in armature winding.

13.11.4 Power of Electromechanical Conversion

Power $P_c = E_a I_a = (k_e \Phi_f \Omega_m) I_a = (k_m \Phi_f I_a) \Omega_m = T_{em} \Omega_m$ is power of the electromechanical conversion. Electrical power $E_a I_a$ is converted to mechanical power $T_{em} \Omega_m$, and both of them are equal to P_c.

13.11.5 Iron Losses (P_{Fe})

The iron losses depend on magnetic induction B and the frequency of its changes. Hence, the iron losses take place in those parts of the magnetic circuit where the magnetic field pulsates or revolves. On the other hand, in magnetic circuits where the magnetic field does not change neither its strength nor its orientation, the iron losses are equal to zero.

In DC machines, the stator has the excitation winding with DC currents. Alternatively, permanent magnets are used instead of the excitation winding. In both cases, consequential magnetic field within the stator magnetic circuit does not change and, hence, does not produce the iron losses.

The rotor of a DC machine revolves in magnetic field of the excitation winding. Therefore, magnetic induction revolves relative to the rotor magnetic circuit. The frequency of changes of magnetic induction depends on the rotor speed. Due to magnetic induction changes, there are iron losses due to hysteresis and losses due to eddy currents. The iron losses have not been taken into account while developing the mathematical model. Yet, it is of interest to include them in the power balance equation. They are dependent on the square of the excitation flux, which determines magnetic induction. In addition, the losses are dependent upon the speed of rotation, because the field pulsates with respect to the iron sheets with a frequency which is equal to the angular speed of the rotor. The iron losses within the rotor are denoted by P_{Fe}. They exist even in cases when the armature winding is disconnected and does not have any current.

The energy that accounts for these losses comes through the shaft, from the mechanical subsystem. When DC machine operates as a motor, braking torque $T_{Fe} = P_{Fe}/\Omega_m$ is subtracted from the electromagnetic torque T_{em}, reducing the torque passed to the work machine. Creation of the torque T_{Fe} can be explained in terms of joint action between the eddy currents in rotor iron and the excitation field that passes through the rotor. The torque T_{Fe} resists the motion in both directions of rotation.

13.11.6 Mechanical Losses (P_F)

In the process of rotation, one part of the energy is spent on overcoming the friction in bearings and the air resistance.[2] The rotor ends are supported by ball bearings which carry the rotor weight and provide support. The bearings are made in such way that the rotor can revolve freely. The friction between bearings and the rotor is very low, and it has a minor contribution to the machine losses. In the course of rotation, the rotor surface slides with respect to the air at certain peripheral speed, creating in such way the air resistance. Electrical machines could have their own cooling, provided by fixing a fan to one end of the rotor shaft. In the course of rotation, the fan creates an axial component of airstream which helps in removing the heat and provides better cooling of the machine. In this case, the air resistance is significantly higher. Besides the air resistance of the rotor, fan-cooled machines also have the losses due to the braking torque of the fan.

[2] Resistance of the air can be modeled by expression $T_{air} = k_{air}\Omega_m{}^2$. If the air resistance prevails among internal motion resistances, corresponding power is proportional to the third degree of the rotor speed, that is, $P_{air} = k_{air}\Omega_m{}^3$.

13.11.7 *Losses Due to Rotation* (P_{Fe} + P_F)

The sum of all the losses caused by rotation is called *rotation losses*, notwithstanding whether specific loss is of electrical or mechanical nature. Considering DC machines, losses due to rotation are equal to $P_{Fe} + P_F$. Friction in the ball bearings, the air resistance, and the braking torque T_{Fe} due to the iron losses[3] belong to the internal motion resistances, namely, to phenomena that resist the rotation and originate from the electrical machine alone. The sum of internal motion resistances is often modeled by an approximate expression $T_F = k_F \Omega_m$.

The iron losses are due to hysteresis and eddy currents. The angular frequency ω_m of the pulsation of magnetic induction in the rotor magnetic circuit is determined by the rotor speed Ω_m. For DC machines considered since,[4] $\omega_m = \Omega_m$. Since the power of iron losses in the rotor is $P_{Fe} = k_V {\omega_m}^2 + k_H \omega_m$, the corresponding braking torque is $T_{Fe} = k_V \omega_m + k_H$. The losses due to hysteresis are usually much lower than the eddy current losses, thus $T_{Fe} \approx k_V\ \omega_m$. Braking torque T_{Fe} due to eddy currents in rotor magnetic circuit of DC machines corresponds entirely to the model $T_F = k_F \Omega_m$, but this is not the case with the air resistance torque which depends on $(\Omega_m)^2$. If torque T_{Fe} prevails, it is then justified to consider that the sum of motion resistances gets proportional to the speed. In this case, corresponding power of losses due to rotation is modeled by expression $P_F = k_F {\Omega_m}^2$.

13.11.8 *Mechanical Power*

Electrical machine delivers to work machine mechanical power P_m. Mechanical power is obtained by subtracting the losses due to rotation from the power of electromechanical conversion. Mechanical power is equal to $P_m = T_m \Omega_m = P_c - P_{Fe} - P_F = T_{em} \Omega_m - T_{Fe} \Omega_m - k_F {\Omega_m}^2$. This power is delivered to the work machine via shaft. The work machine resists the motion by the torque of the same magnitude ($T_m = P_m / \Omega_m$), acting in the opposite direction.

Question (13.4): Consider a DC machine having rated flux and the armature current $I_a = 0$. The machine rotates at a constant, rated speed Ω_n. The torque required for maintaining the rotation is provided by a driving machine coupled via shaft. Power of conversion $P_c = E_a I_a$ is equal to zero. Are there any losses in the

[3] Losses in the rotor magnetic circuit and their place in the power balance are different in DC, asynchronous, and synchronous machines.

[4] Namely, DC machines analyzed in this chapter have two magnetic poles of the stator (and two magnetic poles of the rotor). Hence, they have one pair of magnetic poles. Electrical machines with multiple pole pairs are described in the subsequent chapters. DC machines with $p > 1$ pairs of magnetic poles and with the rotor speed of Ω_m have the angular frequency of the magnetic induction pulsations of $\omega_m = p\Omega_m$.

rotor? Is there any torque acting on the rotor? Describe behavior of the rotor in the case that the coupling with the driving machine is broken.

Answer (13.4): The rotor revolves in the magnetic field of the stator excitation. Relative motion of the field with respect to the rotor magnetic circuit gives a rise to losses due to hysteresis and eddy current within the rotor iron sheets. Eddy currents that exist in the rotor interact with the magnetic field created by excitation winding. Therefore, minute forces are generated, resulting in torque T_{Fe} that acts against the motion. The torque which resists the motion is equal to $T_{Fe} = P_{Fe}/\Omega_n$, where P_{Fe} are the iron losses in the rotor. In order to keep the rotor spinning, this torque has to be fed from the driving machine. If the shaft is not coupled to the driving machine, the rotor would, due to the braking torque, gradually slow down. The losses in iron of the rotor would heat the rotor on account of its kinetic energy ½ $J\Omega_n^2$.

13.12 Rated and Declared Values

The rated parameters of DC machines have been defined in the preceding section. Rated current is the highest permissible current in continuous service that does not cause any damage or failure of DC machine. Rated voltage is the highest voltage which can be maintained permanently without causing breakdown or accelerated aging of electrical insulation. In similar way, the rated levels have been defined for the remaining variables. Rated value of each quantity should be understood as the highest acceptable value to be used in continuous service. Rated quantities are characteristics of the considered electrical machine or its vital parts.

In addition to the rated quantities, the concept of *declared* quantities is frequently encountered as well. Declared values are equal to or lower than the rated values. They are usually written on the plate affixed to the machine and/or presented in catalogue data concerning the machine. Declared quantities are specified by the manufacturer. By specifying a declared quantity, the manufacturer gives a warranty that the machine can bear it during permanent operation without damage. For that reason, they cannot be higher than the rated quantities. A declared quantity can be lower than the rated. Manufacturer can intentionally give declared quantity which is lower than the rated. There could be commercial reasons for such derating. An example to that is the case when manufacturer has large-scale production of 100-kW machines and receives request to deliver only one 90-kW machine. Manufacturing of a single machine is very expensive. Therefore, he would find no economic interest to make and deliver a single machine of 90 kW. Instead, manufacturer takes one machine of 100 kW and affixes a plate declaring it as 90-kW machine. In doing so, the manufacturer gives a warranty that the machine can develop 90 kW during permanent operation and disregards the fact that the actual power can be higher.

13.13 Nameplate Data

Basic data concerning an electrical machine are written on its nameplate. In addition to declared speed and power, declared values are also given for current, voltage, and torque. A plate may contain the following data:

- Declared current of armature winding (I_n)
- Declared current of excitation winding (I_{fn}) (optional)
- Declared voltage of armature winding (U_n)
- Declared voltage of excitation winding (U_{fn}) (optional)
- Declared speed of rotation
- Declared torque
- Declared power
- Declared power factor (for AC machines)
- Method of connecting three-phase stator winding (for AC machines)

Declared speed is usually expressed in revolutions per minute [rpm]. Thus n_n[rpm] = Ω_n[rad/s]·(30/π). Declared speed is related to the declared operating conditions. When the declared operating conditions correspond to the rated, declared speed is equal to the rated speed.

The rated speed of DC generators corresponds to the speed when the generator with rated flux operates with rated current and provides rated voltage to electrical loads. Generator current produces voltage drop $R_a I_G$ which is subtracted from the electromotive force. Since $U_n = k_e \Phi_n \Omega_n + R_a I_a = k_e \Phi_n \Omega_n - R_a I_G$, the rated speed of the generator results in electromotive force $E_a = U_n + R_a I_n$. Therefore, $\Omega_n = (U_n + R_a I_n)/k_e \Phi_n$.

The rated speed of DC motors corresponds to the speed when the motor with rated flux ($\Phi = \Phi_n$) operates with rated current. The motor current is directed from the source toward the motor, and therefore $U_n = E_a + R_a I_n$. Therefore, $U_n = k_e \Phi_n \Omega_n + R_a I_n$. At the rated speed, the electromotive force is $E_a = U_n - R_a I_n$. Therefore, $\Omega_n = (U_n - R_a I_n)/k_e \Phi_n$.

Question (13.5): For generator of known parameters $U_n = 220$ V, $I_n = 20$ A, $R_a = 1\ \Omega$, and $k_e \Phi_n = 1$ Wb, determine the rated speed.

Answer (13.5): Rated speed of the generator is $\Omega_n = (U_n + R_a I_n)/k_e \Phi_n = 240$ rad/s, corresponding to $n_n = 2{,}292$ rpm.

Question (13.6): For motor of known parameters $U_n = 110$ V, $I_n = 10$ A, $R_a = 1\ \Omega$, and $k_e \Phi_n = 1$ Wb, determine the rated speed.

Answer (13.6): Rated speed of the motor is $\Omega_n = (U_n - R_a I_n)/k_e \Phi_n = 100$ rad/s, corresponding to $n_n = 955$ rpm.

Definition of the rated rotor speed for DC machines may include the voltage drop across the armature resistance. In this case, the rated speed calculated for DC generator is different than the rated speed calculated for DC motor.

Analysis of electrical machines mostly assumes that the rated speed is the ratio of the rated voltage and flux, $\Omega_n \approx U_n/(k_e\Phi_n)$. This definition neglects the voltage drop R_aI_a. The ratio $U_n/(k_e\Phi_n)$ gives the speed that results in electromotive force equal to the rated voltage, provided that DC machine has rated excitation. Disregarding the voltage drop, one obtains the rated speed as $U_n/(k_e\Phi_n)$, while the speed of rotation with rated voltage, rated current, and rated flux will be slightly different. The approximation made is $U_n = R_aI_n + k_e\Phi_n\Omega_n \approx k_e\Phi_n\Omega_n$, and it results in $\Omega_n = U_n/(k_e\Phi_n)$, slightly higher than the speed measured on DC motor running in rated conditions and slightly lower than the speed measured on DC generator running in rated conditions.

In all analyses and calculations where resistance R_a is neglected or it is unknown, it is justifiable to assume that $U_a \approx k_e\Phi\Omega_m$, and that the rated speed is $\Omega_n = U_n/(k_e\Phi_n)$.

In solving the problems where the value of R_a is given, the voltage drop R_aI_a should be taken into account. Then, it is not justified to consider that $\Omega_n \approx U_n/(k_e\Phi_n)$. Expression $\Omega_n = (U_n - R_aI_n)/k_e\Phi_n$ determines the rated speed for motors, while expression $\Omega_n = (U_n + R_aI_n)/k_e\Phi_n$ determines the rated speed for generators.