Chapter 2
Electromechanical Energy Conversion

Electrical machines contain stationary and moving parts coupled by an electrical or magnetic field. The field acts on the machine parts and plays key role in the process of electromechanical conversion. For this reason, it is often referred to as the *coupling field*. This chapter presents the most significant principles of creating a force or torque on the machine moving parts. In all the cases considered, the force appears due to the action of the electrostatic or magnetic field on the moving parts of the machine. Depending on the nature of the coupling field, the machines can be magnetic or electrostatic.

2.1 Lorentz Force

Electrical machines perform conversion of electrical energy to mechanical work or conversion of mechanical work to electrical energy. The basic principles involved in the process of electromechanical conversion are presented in the considerations which follow.

One of the laws of physics which is basic for electromechanical conversion is Lorentz law which determines the force acting upon a charge Q moving with a speed v in the electrical and magnetic fields:

$$\vec{F} = Q\vec{E} + Q\left(\vec{v} \times \vec{B}\right). \tag{2.1}$$

In electrical machines, the operation is most often based on the magnetic coupling field. Conductors and ferromagnetic parts in a magnetic field are subjected to the action of electromagnetic forces. Magnetic induction $\boldsymbol{B}$ in (2.1) is also called *flux density*. Electrical current existing in the conductor is a directed motion of electrical charges. Therefore, (2.1) can be used to determine the force acting upon conductors carrying electrical currents.

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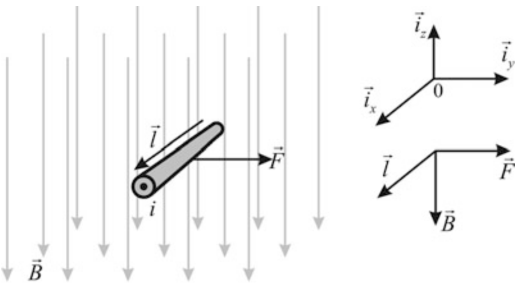

Fig. 2.1 Force acting on a straight conductor in homogeneous magnetic field

Figure 2.1 shows straight portion of a conductor of length l with electrical current i which is placed in a homogenous magnetic field with flux density $\boldsymbol{B}$. Electromagnetic force $\boldsymbol{F}$ acting on the conductor depends on current i, conductor length l, flux density $\boldsymbol{B}$, and angle between directions of the field and the conductor. In the example presented in Fig. 2.1, the conductor is perpendicular to the direction of the field. Applying the Cartesian coordinate system with axes x, y, and z, the vectors of magnetic induction $\boldsymbol{B}$, conductor length, and force can be expressed in terms of the corresponding unit vectors.

$$\begin{aligned}\vec{l} &= l\,\vec{i}_x,\\ \vec{B} &= -B\,\vec{i}_z,\\ \vec{F} &= i\cdot\left(\vec{l}\times\vec{B}\right).\end{aligned} \tag{2.2}$$

Since the vector $\boldsymbol{B}$ is orthogonal to the conductor, the module of the force vector is equal to $F = l{\cdot}B{\cdot}i$. Direction of the force is determined by the vector product. The right-hand rule[1] can be used to determine quickly the vector product direction. If the considered part of the conductor makes a displacement Δy along the axis y, corresponding mechanical work is $\Delta W = F\Delta y$. At a constant speed of motion v_y, the mechanical power assumes the value $p_m = Fv_y$. In the case when the force acts in the direction of motion, power p_m is positive, and the system operates as a motor, delivering mechanical work and power. Otherwise, the motion and force are opposed, power p_m is negative, and the mechanical work is converted to electrical energy, while the system operates as a generator.

[1] The right-hand rule requires thumb and forefinger to assume right angle. The middle finger should be perpendicular to both. Now, with forefinger alligned with vector l and middle finger alligned with $\boldsymbol{B}$, thumb determines the direction of force. Alternatively, direction of any vector product can be determined by an imaginary experiment, where the first vector of the product (l in (2.2)) is rotated toward the second vector ($\boldsymbol{B}$). Envisaging a screw that is turned by such rotation, the screw would advance along the axis perpendicular to $l - B$ plane. The direction of the vector product is determined by the advance of the (right) screw.

In a conductor moving through a homogeneous magnetic field, induced electromotive force e depends on flux density $\boldsymbol{B}$, speed of motion, and conductor length l. The product of the electromotive force e and current i is equal to the product of the force and speed ($p_m = Fv_y$), as shown later on. Assuming that energy losses are negligible, the motor operation can be perceived as a lossless conversion of electrical power p_e to mechanical power p_m. The relevant powers are represented by derivatives of the electrical energy and mechanical work.

2.2 Mutual Action of Parallel Conductors

In the previous example, the case is considered of a conductor in a homogenous *external* magnetic field which exists due to the action of external current circuits or permanent magnets. Force acting on conductors also exists in the case when there is no externally brought field, but there are two conductors both conducting electrical currents.

Magnetic field created by one of the conductors interacts with the current in the other conductor, according to the principle presented in Fig. 2.2. The result of this interaction is force acting on the conductor.

When currents in the conductors have the same direction, the force tends to bring the conductors closer. In the case when directions of the currents are mutually opposite, the force tends to separate the conductors.

In the case being considered, the force acting on parallel conductors is very small. If two very long and thin parallel conductors, each with current of 1A, placed at a distance $d = 0.1$ m are considered, one of the conductors will be found in the

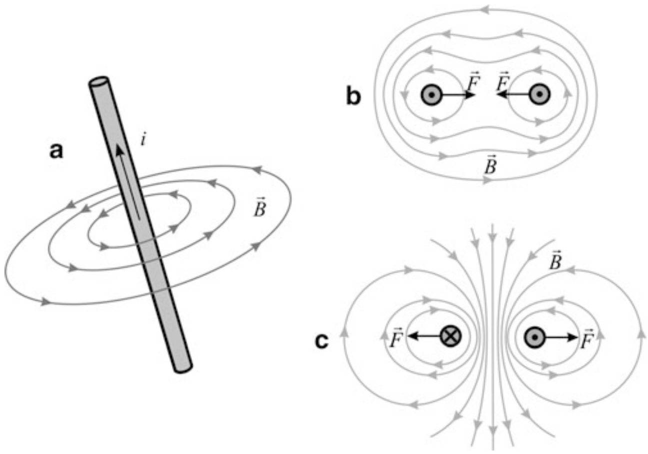

Fig. 2.2 (**a**) Magnetic field and magnetic induction of a straight conductor. (**b**) Force of attraction between two parallel conductors. (**c**) Force of repulsion between two parallel conductors

magnetic field created by the other conductor. The magnetic induction created by a very long conductor is given by (2.3):

$$B = \mu_0 \frac{I}{2\pi d} = 4\pi \cdot 10^{-7} \frac{1}{2\pi \cdot 0.1} T = 2 \cdot 10^{-6} T. \tag{2.3}$$

Since the magnetic field and magnetic induction are orthogonal to the conductor, the electromagnetic force acting on a part of the conductor 1 m long is given by (2.4):

$$F = l \cdot B \cdot I = 2 \cdot 10^{-6} \mathrm{N}. \tag{2.4}$$

Magnetic circuits are discussed later in the book. They are made of ferromagnetic materials with high permeability μ, and they direct the magnetic flux[2] to paths with low magnetic resistance. The lines of magnetic field are concentrated into magnetic circuits in a way that resembles electrical current being contained in conductors and windings. In turn, there is a considerable increase of the flux density $\boldsymbol{B}$, resulting in an increase in electromagnetic force and power of conversion, both proportional to $\boldsymbol{B}$.

2.3 Electromotive Force in a Moving Conductor

A considerable number of electrical machines convert mechanical work to electrical energy, like synchronous generators in electrical power plants. Figure 2.3 shows the principle where mechanical work and mechanical power are used to obtain electrical energy and electrical power. The figure shows straight part of conductor of length l, moving along y-axis at a speed v. The conductor placed in a homogeneous magnetic field is moved by action of an external force $\boldsymbol{F}_s$. Direction of vector $\boldsymbol{B}$ is opposite to direction of axis z.

Motion of a conductor in magnetic field causes induction of electrical field $\boldsymbol{E}_{ind}$. Induced field strength can be measured by an observer moving together with the conductor and cannot be sensed by an immobile observer. The field strength can be calculated on the basis of (2.1), expressing the force acting upon a moving

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[2] Flux is a scalar quantity having no direction. Flux through surface S is equal to the surface integral of the vector of magnetic induction $\boldsymbol{B}$, also called *flux density*. Surface S is encircled by contour C; thus, the said surface integral is called *flux through the contour*. Flux through a flat surface S placed in a homogeneous *external* magnetic field depends on its position relative to the field. With the *positive normal* on S aligned with the direction of the vector $\boldsymbol{B}$, the flux through S is equal to $\Phi = BS$. Although the flux Φ is a scalar, it is inherently related to spatial orientation of the surface S and/or the vector $\boldsymbol{B}$. The flux vector is obtained by associating the spatial orientation (i.e., direction) to the scalar Φ. In the given example, the spatial orientation is defined by the *positive normal* on S. Direction of the positive normal is determined by applying the right-hand rule to the reference circling direction for the contour C. The external magnetic field is the one which is not created by the electrical currents in the contour C.

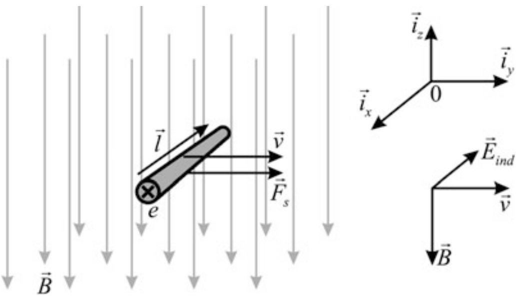

Fig. 2.3 The induced electrical field and electromotive force in the straight part of the conductor moving through homogeneous external magnetic field

charge Q. The considered domain does not contain any electrostatic field; hence the product QE in (2.5) is equal to zero.

$$\vec{F} = Q\vec{E} + Q(\vec{v} \times \vec{B}) = Q(\vec{v} \times \vec{B}) = Q\vec{E}_{ind}. \tag{2.5}$$

The force $\boldsymbol{F}$ acts on the moving charge Q due to its motion in homogeneous magnetic field. Notice in (2.5) that the same effect can be obtained by replacing the magnetic field with the electrostatic field of the strength $\boldsymbol{E}_{ind}$. Therefore, the induced electrical field can be determined by dividing force F by charge Q, which gives vector product of the speed and flux density $\boldsymbol{B}$:

$$\vec{E}_{ind} = \vec{v} \times \vec{B}. \tag{2.6}$$

It is of interest to determine the electromotive force e induced in the straight part of the conductor of length l. In general, the electromotive force induced in a conductor is determined by calculating the line integral of vector $\boldsymbol{E}_{ind}$ between conductor terminals. Since the induced electrical field does not vary along the conductor, the line integral reduces to the scalar product of vectors $\boldsymbol{l}$ and $\boldsymbol{E}_{ind}$. The electromotive force can be calculated from (2.7):

$$e = \vec{l} \cdot \vec{E}_{ind} = \vec{l} \cdot \left(\vec{v} \times \vec{B}\right). \tag{2.7}$$

In the present case, the conductor is aligned with x-axis, the magnetic field is in direction of z-axis, and the speed vector is aligned with y-axis. Therefore, the vector of the induced electrical field is collinear with the conductor (2.8); thus, the induced electromotive force is $e = lvB$. The sign of the induced electromotive force e depends on the adopted reference direction of the conductor. In the present case, it is the direction of vector l.

$$\vec{l} = -l\,\vec{i}_x, \quad \vec{B} = -B\,\vec{i}_z, \quad \vec{v} = v\,\vec{i}_y,$$
$$\vec{E}_{ind} = -vB\,\vec{i}_x, \quad e = \vec{l} \cdot \vec{E}_{ind} = lvB. \tag{2.8}$$

2.4 Generator Mode

Terminals of the conductor shown in Fig. 2.3 could be connected to the terminals of an immovable resistor, forming in this way a closed current circuit containing the induced electromotive force e, interconnections, and resistor R. This circuit is shown in Fig. 2.4. By neglecting the resistance and inductance of interconnections, the current established in the circuit is $i = e/R$. The moving conductor performs the function of a generator, whereas resistor R is a consumer of electrical energy. Since direction of the current in the conductor corresponds to the direction of electromotive force, the conductor is a source of electrical power and energy. Existence of the current in the conductor creates force $\boldsymbol{F}_m$ which opposes the movement (2.2). The external force $\boldsymbol{F}_s$ acts in the direction opposite to $\boldsymbol{F}_m$, overcoming the resistance $\boldsymbol{F}_m$. It is of interest to analyze the operation of the system in Fig. 2.4 with the aim of establishing the relation between the invested mechanical power $F_s v$ and obtained electrical power ei.

The electromotive force $e = lvB$, induced in the conductor, is equal to the voltage $u = Ri$, which appears across resistance R. The electromagnetic force acting on the conductor, shown in Fig. 2.4, acts from right to left and is given by (2.9):

$$\vec{F}_m = i\left(\vec{l} \times \vec{B}\right), \quad \left|\vec{F}_m\right| = ilB. \tag{2.9}$$

By maintaining the movement, the external force $\boldsymbol{F}_s$ performs the work against magnetic force $\boldsymbol{F}_m$ and delivers it to the moving conductor. Transfer of the mechanical work to electrical work is performed through electromagnetic induction. Electromotive force e, induced in the moving conductor, maintains the current $i = e/R$ in the circuit and delivers electrical energy to the resistor.

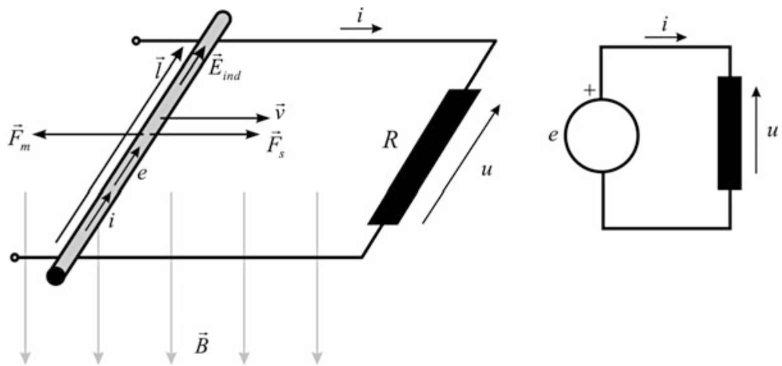

Fig. 2.4 Straight part of a conductor moves through a homogeneous external magnetic field and assumes the role of a generator which delivers electrical energy to resistor R

The sum of forces acting on the conductor is equal to zero:

$$\vec{F}_s + \vec{F}_m + m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = 0.$$

In the state of dynamic equilibrium, the speed v is constant. With the acceleration $\mathrm{d}v/\mathrm{d}t$ equal to zero, the inertial force $F_i = m\ \mathrm{d}v/\mathrm{d}t$ is equal to zero as well. Therefore,

$$\vec{F}_s + \vec{F}_m = 0; \quad \vec{F}_s = -\vec{F}_m; \quad \left|\vec{F}_s\right| = \left|\vec{F}_m\right| = ilB. \tag{2.10}$$

Mechanical power of external force F_s is equal to $P_m = F_s v = i\ l\ vB$. The induced electromotive force e develops power $P_e = ei = i\ lvB = P_m$ and delivers it to the rest of the electrical circuit. With $P_e = e^2/R > 0$, the considered system converts mechanical work to electrical energy. In the course of this analysis, energy losses have been neglected; thus, there is equality between the input (mechanical) power and output (electrical) power ($P_e = P_m$).

Question (2.1): In the case when the resistor shown in Fig. 2.4 moves together with the conductor, what will be the electrical current in the circuit?

Answer (2.1): During a parallel movement of the conductor and resistor, equal electromotive forces will be induced within each of them, thus compensating each other. Therefore, the electrical current will be equal to zero.

2.5 Reluctant Torque

Electromechanical energy conversion can be accomplished by exploiting the tendency of ferromagnetic material placed in a magnetic field to get aligned with the field and take position of minimum magnetic resistance. Figure 2.5 shows an elongated piece of ferromagnetic material of high permeability ($\mu_{Fe} >> \mu_0$), inclined with respect to the lines of magnetic field. The electromagnetic forces tend to bring the piece in vertical position where it will be collinear with the field.

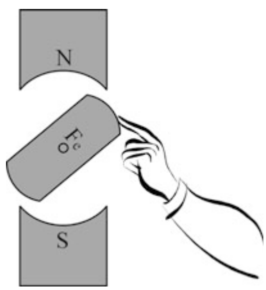

Fig. 2.5 Due to reluctant torque, a piece of ferromagnetic material tends to align with the field, thus offering a minimum magnetic resistance

In Fig. 2.5, it is assumed that the magnetic field exists owing to a permanent magnet. The moving part (rotor) of ferromagnetic material can rotate and will tend to take vertical position where magnetic resistance along the field lines (magnetic resistance along the flux path) is lower. When the rotor assumes vertical position, the flux passes from the magnet poles into the rotor whose permeability is high. The ferromagnetic rotor always tends to align with the field. The torque which appears in the considered (inclined) position tends to bring the ferromagnetic to vertical position. This torque is called *reluctant*, and the considered principle of the torque generation is called *reluctant principle*. This name stems from *reluctance*, also called magnetic resistance. Reluctant torque depends on changes in magnetic resistance due to spatial displacement of the moving part. *The reluctant torque tends to bring rotor to position where magnetic resistance is minimal*. The rotor can be connected to a work machine to deliver mechanical power.

Question (2.2): What is the value of reluctant torque acting on the rotor when it is in horizontal position?

Answer (2.2): The reluctant torque tends to bring the rotor to the position of minimal magnetic resistance. In horizontal position, magnetic resistance assumes its maximum value. A hypothetical shift of the rotor in any direction will lead to a decrease of magnetic resistance. Unless moved from horizontal position, there is no tendency to move the rotor in any direction, and the reluctant torque is equal to zero. In the considered case, there is an unstable equilibrium. Any movement of the rotor to either side would result in the reluctant torque which speeds up the initial movement.

2.6 Reluctant Force

Figure 2.6 shows a system where the *reluctant force* stimulates a translatory movement. Electromagnetic force acts on the piece of ferromagnetic material placed in a nonhomogeneous magnetic field. The force tends to bring the piece of ferromagnetic to the place where the flux density *B* is high.

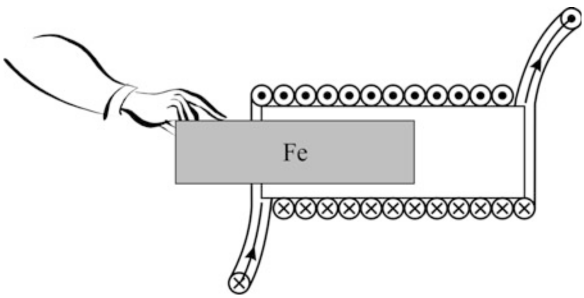

Fig. 2.6 The electromagnetic forces tend to bring the piece of ferromagnetic material inside the coil

The coil shown in Fig. 2.6 is made of circular wound conductors carrying a DC current. This system of conductors (*coil*, *winding*, or *bobbin*) creates a magnetic field that extends along the coil and has maximum intensity inside the coil. Hence, the flux path goes through the cylindrical coil. A piece of mobile ferromagnetic material can be inserted in the coil or extracted from the coil.

If the ferromagnetic piece is in the coil, the magnetic resistance (*reluctance*) along the flux path is low. When the ferromagnetic piece is outside the coil, the reluctance is high.

Taking into account that the mobile part of ferromagnetic material tends to take position where the magnetic resistance is minimal, the force will appear tempting to bring the mobile piece of ferromagnetic material in the coil.

2.7 Forces on Conductors in Electrical Field

Thanks to the action of electrical field E, one can obtain force, power, and work from the setup shown in Fig. 2.7. In the space between two parallel, charged capacitor plates, there is an electrostatic field E. In the case when the distance between the plates is small compared to their dimensions, the field can be considered *homogeneous*. Namely, the field lines are parallel, while the field strength does not change between the plates.

The charges are distributed on the interior surfaces of the plates. The field between the plates acts on the surface charges by a force tending to bring the plates closer. Force $\boldsymbol{F}$ may cause the plates to move. If one of the plates shifts by Δx, a mechanical work $F\Delta x$ is achieved.

Based on this principle, it is possible to operate electromechanical converters with electrical coupling field, also called *electrostatic machines*.

2.8 Change of Permittivity

Electromechanical conversion can be based on electrical force acting on a mobile part of dielectric material with permittivity (dielectric constant ε) different from the permittivity of the environment. Figure 2.8 shows two charged plates and a mobile

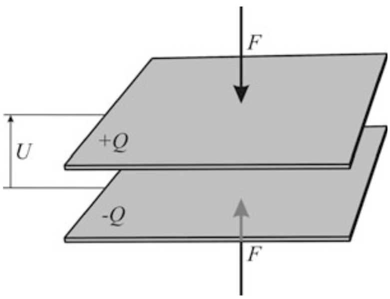

Fig. 2.7 Electrical forces act on the plates of a charged capacitor and tend to reduce distance between the plates

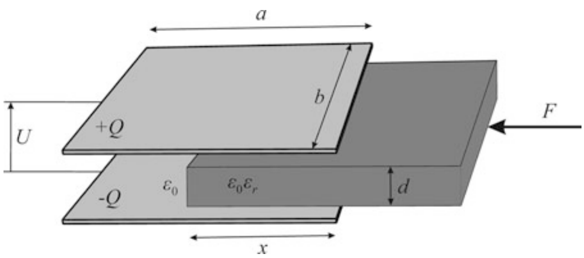

Fig. 2.8 Electrical forces tend to bring the piece of dielectric into the space between the plates. The dielectric constant of the piece is higher than that of the air

piece of dielectric material of permittivity $\varepsilon = \varepsilon_r\ \varepsilon_0$. Free space between the electrodes is filled by air of permittivity ε_0.

The piece of dielectric material of relative permittivity $\varepsilon_r > 1$ can move along a horizontal direction. By moving to the left, it comes to position $x = a$, when it fills completely the space between the plates. By moving to the right, the dielectric comes to position $x = 0$, when the space between the plates is completely filled by air. The following analysis will show that an electrical force F acts on the piece of dielectric in position $0 < x < a$, tempting to bring it into the space between the plates.

With voltage U across the plates, electrical field E in the space between the plates is $E = U/d$, where d is distance between the plates. The conductive plates represent equipotential surfaces; thus, relation $U = Ed$ applies in the air as well as in the dielectric, while the strength of the electrical field is the same in both media. Electrical induction within the dielectric is $D{=}\varepsilon_r\varepsilon_0 U/d$, whereas in the air, it is $D{=}\varepsilon_0 U/d$. Total energy of the electrical field is given by (2.11), where $S = ab$ is surface of the plates:

$$W_e = \frac{1}{2}\varepsilon_0\left(\frac{U}{d}\right)^2 \cdot \frac{a-x}{a}Sd + \frac{1}{2}\varepsilon_r\varepsilon_0\left(\frac{U}{d}\right)^2 \cdot \frac{x}{a}Sd$$
$$= \frac{1}{2}\varepsilon_0\left(\frac{U}{d}\right)^2 \cdot \frac{Sd}{a}\left[(a-x) + x\varepsilon_r\right]. \tag{2.11}$$

If the plates are connected to a source of constant voltage U, a small displacement Δx will change the field energy accumulated in the space between the plates. The source U will provide an amount of electrical work, while the force F will contribute to delivered mechanical work $\Delta W_{meh} = F\Delta x$ obtained along the displacement Δx. The equilibrium between the work of the source ΔW_i, change in the field energy ΔW_e, and mechanical work is given by relation $\Delta W_i = \Delta W_e + \Delta W_{meh}$. Equation 3.8 in the following chapter proves that $\Delta W_e = \Delta W_i/2$ and $\Delta W_e = \Delta W_{meh}$. Therefore, the force acting on the moving piece of dielectric is obtained from (2.12):

$$F = \frac{\mathrm{d}W_e}{\mathrm{d}x} = \frac{1}{2}\varepsilon_0\left(\frac{U}{d}\right)^2 \cdot \frac{Sd}{a}(\varepsilon_r - 1). \tag{2.12}$$

It is possible to determine electrical force F by using the equivalent pressure on the surfaces separating the media of different nature. On the basis of a conclusion from electrostatics, electrical force acting on a dividing surface that separates the spaces filled with two different dielectric materials can be determined from the equivalent pressure $p = w_{e1} - w_{e2}$. The values w_{e1} and w_{e2} are specific energies of electrostatic fields in the two separated media. They are also called the spatial energy densities of the electrostatic field. The energy of electrical field energy in the air has density of $w_0 = ½\ \varepsilon_0(U/d)^2$, whereas in the dielectric it is $w_d = ½\ \varepsilon_r\varepsilon_0(U/d)^2$. The force F can be determined from (2.13), where $S_d = bd = Sd/a$ is rectangular surface separating the two domains:

$$F = (w_d - w_0)S_d = \frac{1}{2}\varepsilon_0\left(\frac{U}{d}\right)^2 \cdot (\varepsilon_r - 1) \cdot \frac{Sd}{a}. \tag{2.13}$$

Question (2.3): Determine the direction of force when the source is disconnected. It should be noted that total charge Q existing on the plates is then constant, whereas the voltage between the plates is variable depending on position of the dielectric.

Answer (2.3): In the space between the plates, there is a homogeneous electrical field. Conductivity $1/\rho$ of the metal plates is very high, and potential of all the points on one plate is the same. Therefore, voltage between the plates is U in the part filled by the dielectric as well as in the part filled by air. Since the field is homogeneous and orthogonal to the plates, product Ed is equal to voltage U; thus, electrical field $E = U/d$ is the same in both air and the dielectric. Since permittivity of the dielectric is higher, electrical induction D_d in the dielectric is higher than induction D_0 in the air:

$$D_0 = \varepsilon_0\frac{U}{d}, \quad D_d = \varepsilon_r\varepsilon_0\frac{U}{d}.$$

Surface charge density σ at the surface of a conductor is determined by the scalar product of the vector of electrical induction and normal to the surface at a given point:

$$\sigma = \vec{n} \cdot \vec{D}.$$

In the case being considered, the vector of electrical induction is perpendicular to the surface of the conductor and collinear with the normal n. As a consequence, the density of surface charge σ is equal to the induction D. Therefore, it will be higher in the parts of the plates which are against the dielectric. By using notation shown in Fig. 2.8, total charge Q can be expressed in terms of the shift x and values D_d and D_0,

$$Q = (a - x)b \cdot D_0 + xb \cdot D_d = (a - x)b \cdot \varepsilon_0\frac{U}{d} + xb \cdot \varepsilon_r\varepsilon_0\frac{U}{d}$$
$$= b\varepsilon_0\frac{U}{d}(a - x + x \cdot \varepsilon_r),$$

while capacitance C is determined by expression

$$C = \frac{Q}{U} = \frac{b\varepsilon_0}{d}(a - x + x \cdot \varepsilon_r).$$

Since the plates are separated from the source, the mechanical work $\Delta W_{meh} = F\Delta x$ is obtained by subtracting this amount from the field energy, $\Delta W_{meh} = -\Delta W_e$. Therefore, the electrical force can be determined according to expression $F = -\mathrm{d}W_e/\mathrm{d}x$. Electrical energy of the coupling field can be expressed as $W_e = ½Q^2/C$ or $W_e = ½CU^2$. In the present case, charge Q is constant, whereas voltage U is variable, and the electrical force can be determined according to expression

$$F = -\frac{\mathrm{d}W_e}{\mathrm{d}x} = -\frac{Q^2}{2}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{C}\right).$$

By differentiating the reciprocal value of capacitance, the following expression for the electrical force is obtained:

$$F = -\frac{Q^2 d}{2b\varepsilon_0}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{a - x + x \cdot \varepsilon_r}\right) = \frac{Q^2 d}{2b\varepsilon_0}(\varepsilon_r - 1)\left(\frac{1}{a - x + x \cdot \varepsilon_r}\right)^2.$$

The above expression is positive, so the direction of action of the force is the same as if the source was connected to the plates. By introducing substitution $Q = CU$ in the above expression, the electrical force is determined by

$$F = \frac{U^2}{2}(\varepsilon_r - 1)\frac{b\varepsilon_0}{d} = \frac{\varepsilon_0}{2}\frac{U^2}{d^2}(\varepsilon_r - 1)\frac{Sd}{a},$$

the expression which is fully equivalent to (2.12) and (2.13). It can be concluded that the force will not change by switching the source on or off, provided that the charge Q is the same in both cases.

Question (2.4): Consider a charged capacitor made of the plates shown in Fig. 2.8 and assume that the plates are not connected to the source. Is there any difference between E and D in the part filled by air and part filled by dielectric? Will the total energy be increased or decreased in the case that the dielectric is pushed further into the space between the plates?

Answer (2.4): In the space between the plates, the electrical field E is equal in all points, whereas the electrical induction D is ε_r times higher in the space filled by dielectric compared to induction in the space filled by air. The spatial density of the field energy in the dielectric is $w_{ed} = ½\varepsilon_r\varepsilon_0 E^2$, and it is ε_r times higher than the density $w_{ea} = ½\varepsilon_0 E^2$ in the air. Total field energy is $w_{ea}V_a + w_{ed}V_d$, where w_{ea} and w_{ed} are the densities of field energy in the air and in the dielectric, whereas V_a and V_d are the volumes of the interelectrode space filled by air and dielectric.

When the piece of dielectric moves toward inside of the capacitor, volume V_a decreases, whereas volume V_d increases. Since $w_{ea} < w_{ed}$, there are indications that the total field energy increases. However, filling the space between the plates by dielectric material increases the equivalent capacitance $C = Q/U$, as it is proportional to the permittivity of the dielectric material filling the space between the plates. Since the charge Q is constant, an increase in the capacitance will cause a decrease of the voltage. As a consequence, the fields E will reduce. Spatial density of the field energy depends on the square of the field strength. Therefore, it can be concluded that a deeper insertion of the dielectric reduces the total energy of the electrical field. These considerations can be verified by an analysis of the expression for field energy $W_e(x)$,

$$W_e = \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2}{2}\frac{d}{(a - x + x \cdot \varepsilon_r)b\varepsilon_0},$$

which shows that in the case of a constant Q, total field energy decreases when the value of x rises, that is, when a piece of dielectric is pushed further into the space between the plates.

2.9 Piezoelectric Effect

Applying pressure on a crystal of silicon will induce charges on its surfaces and give rise to voltage between surfaces (Fig. 2.9). This phenomenon is known as *piezoelectric effect*. In a piezoelectric microphone, sound waves cause variable pressure of air against the surface of a crystal. As a consequence, variable forces act upon the crystal. A voltage which represents electrical image of the sound appears across the ends of the crystal. This voltage can be amplified and processed further.

It is possible to manufacture a crystal with linear dependence between the voltage across the crystal and the applied force. Such crystal can be used for designing precise electronic scales (weight-measuring devices).

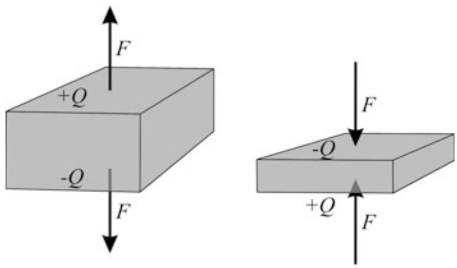

Fig. 2.9 Variation of pressure acting on sides of a crystal leads to variations of the voltage measured between the surfaces

The inverse piezoelectric effect can be used in electromechanical conversion. If the surfaces of the crystal are covered by conducting plates connected to a variable voltage, a force varying in accordance with the variable voltage will appear. This effect can be used for creating very small displacements controlled by the applied voltage. If the connected voltage represents a record of sound, variations of the force will cause vibrations of the crystal surfaces and change the pressure of air against the surfaces, thus operating the piezoelectric loudspeakers.

In a piezoelectric device, the crystal surface moves by a fraction of millimeter. Motors based on piezoelectric effect are used in motion control applications with very small displacements and with very high precision, such as positioning the reading heads in hard disk drives.

2.10 Magnetostriction

One of the principles applicable for electromechanical conversion is *magnetostriction*. In general, magnetization of ferromagnetic materials can change their shape and dimensions. This phenomenon is called magnetostriction. The length of the ferromagnetic rod shown in Fig. 2.10 will change with the applied magnetic field. The effect gives a rise to a force. Multiplied by mechanical displacement, the force produces mechanical work. Yet, few electromechanical converters are based on magnetostriction because of rather small displacements and a poor power-to-weight ratio. Conventional electrical machines and power transformers usually have magnetic circuits made of iron sheets, wherein magnetic field pulsates at the line frequency (50 Hz/60 Hz). The effect of magnetostriction causes magnetic circuits to vibrate. With the magnetostrictive forces proportional to the square of the magnetic field strength, the vibration frequency is twice the line frequency (100 Hz/120 Hz). These vibrations cause waves of variable air pressure and sound which are experienced as humming, frequently encountered with electrical equipment.

The phenomenon reciprocal to magnetostriction is the change of permeability in ferromagnetic materials subjected to mechanical stress. Namely, the stress due to external forces will change magnetic properties of the material. When an external force is applied to an iron rod, the same magnetic field strength H will result in an increased magnetic induction (flux density) B. This phenomenon is called the *Villari* effect. By applying the described principle, it is possible to measure the stress in the elements of steel constructions such as the bridges or skyscrapers.

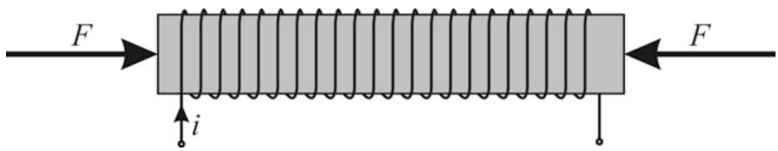

Fig. 2.10 The magnetization varies as a function of force which tends to constrict or stretch a piece of ferromagnetic material

This chapter discussed the principles of developing electromagnetic forces that act on moving parts of electromechanical converters and provide the means for the process of electromechanical conversion. The following chapter introduces some basic principles of electromechanical converters with electrical coupling field and electromechanical converters with magnetic coupling field.