Chapter 14
Induction Machines

The operating principles of induction machines and basic data concerning constructions of their stator and rotor are presented in this chapter. This chapter includes some basic information regarding construction of induction machines. Discussed and described are the stator windings, the rotor short-circuited cage winding, and slotted and laminated magnetic circuits of both stator and rotor. Fundamentals on creating the revolving magnetic field are reinstated for the three-phase stator winding. Basic operating principles of an induction machine are illustrated on simplified machine with one short-circuited rotor turn. The torque expression is developed and used to predict basic properties of mechanical characteristic. For the purpose of studying the electrical and mechanical properties of induction machines, corresponding mathematical model is developed in Chap. 15 and used within the next chapters. Chapter 16 deals with the steady-state operation, steady-state equivalent circuit and relevant parameters, mechanical characteristics, losses, and power balance. Variable speed operation of induction machines is discussed in Chap. 17, with analysis of constant frequency-supplied induction machines and introduction and analysis of variable frequency-supplied induction machines, fed from PWM-controlled three-phase inverters.

14.1 Construction and Operating Principles

Induction machines have stator comprising three-phase windings. Magnetic axes of the three phases are spatially shifted by $2\pi/3$. If the stator phase windings have sinusoidal currents of the same amplitude and the same angular frequency ω_e, and at the same time their initial phases mutually differ by $2\pi/3$, then the magnetic field within the machine revolves, maintaining the same amplitude. The speed of the field rotation is determined by the angular frequency ω_e of the source voltage. When an induction machine is fed from a network of industrial frequency $f = 50$ Hz, the field rotates at the speed of 100π rad/s. The rotor of an induction machine has a short-circuited cage winding. If the rotor revolves at the same

S.N. Vukosavic, *Electrical Machines*, Power Electronics and Power Systems, 365
DOI 10.1007/978-1-4614-0400-2_14,

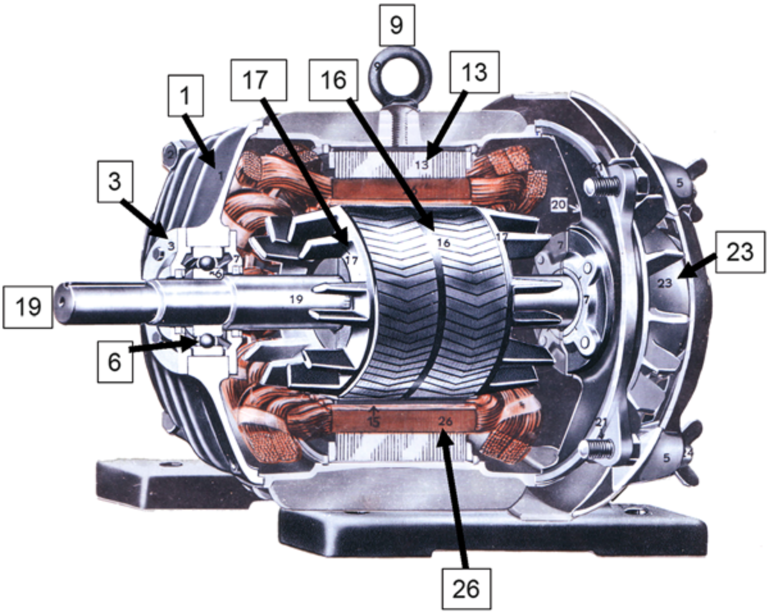

Fig. 14.1 Appearance of a squirrel cage induction motor

speed as the field does, they move synchronously, and there is no relative displacement between the two. In this case, there is no change of the flux in the rotor winding, and no electromotive force is induced. For that reason, there is no current in the rotor short-circuited winding. The speed of rotation of the magnetic field is called *synchronous speed*, and it is denoted by Ω_e. In the case when the difference $\Omega_{slip} = \Omega_e - \Omega_m$ exists between the speed of the field Ω_e and speed of the rotor, there is a change of flux in the rotor. An electromotive force is induced, and the electrical current is established in short-circuited rotor windings, which are usually made as squirrel cage. The frequency of rotor currents ω_{slip} depends on the speed difference Ω_{slip}, also called *slip speed*. The angular frequency ω_{slip} is called *slip frequency*. In machines with two magnetic poles (i.e., with $p = 1$ pair of poles), $\omega_{slip} = \Omega_{slip}$. Joint action of the rotor currents and the stator field results in electromagnetic torque T_{em}. This torque tends to bring the rotor into synchronism with the field. In the case when $\Omega_{slip} = \Omega_e - \Omega_m > 0$, the torque tends to increase the rotor speed and to bring the rotor closer to synchronism with the rotation of the field.

Figure 14.1 gives an insight to construction of an induction machine having rated parameters $U_n = 400$ V, $f_n = 50$ Hz, $P_n = 4$ kW, and $n_n = 1450$ rpm. Number (1) denotes the metal housing that accommodates the machine. The ring denoted by number (9) serves for lifting and transportation. Ball bearings are built-in at the two ends of the shaft (19). The bearings are denoted by numbers (6) and (7). The front bearing is housed in a cartridge (3). The rotor magnetic circuit is denoted by number (16). The rotor conductors are mostly made by casting aluminum into the rotor slots which, in the considered rotor, are not straight but are set obliquely. The rotor conductors are short circuited by aluminum rings (17) at the front and rear sides of the rotor cylinder. The aluminum rings (17) are extended to winglets intended to create an airstream for cooling. The stator conductors (26) are made of copper and covered by electrical insulation. They are laid in the slots of the stator magnetic circuit (13). At the rear side of the motor, the shaft may be equipped with a fan (23) that creates an airstream along the external sides of the housing. This method of assisting the heat transfer is called *self-cooling*. The self-cooling is not suitable for the motors rotating at high speeds, where the fan would create significant losses and acoustic noise.

14.2 Magnetic Circuits

The voltages and currents in the stator windings of an induction machine have angular frequency ω_e. Electromotive forces induced in the rotor have angular frequency ω_{slip}, and they cause electrical currents of the same frequency in short-circuited rotor winding. The flux and magnetic induction of the stator vary at angular frequency ω_e. Induction machines operated from the mains have angular frequency ω_e equal to 100π. The flux and magnetic induction of the rotor have angular frequency ω_{slip}. For induction machines of several kW, the slip frequency is of the order of 1 Hz. Hence, magnetic induction pulsates with respect to the stator magnetic circuit at the line frequency. It also pulsates with respect to the rotor magnetic circuits at lower frequency. In order to reduce iron losses due to eddy currents, both stator and rotor magnetic circuits are laminated, that is, they are made of iron sheets. The shape of these sheets is shown in Fig. 14.2.

By stacking iron sheets, cylindrical magnetic circuits of the stator and rotor are obtained. The stator magnetic circuit is a hollow cylinder. The rotor cylinder is placed axially within the stator. The two parts are separated by the air gap.

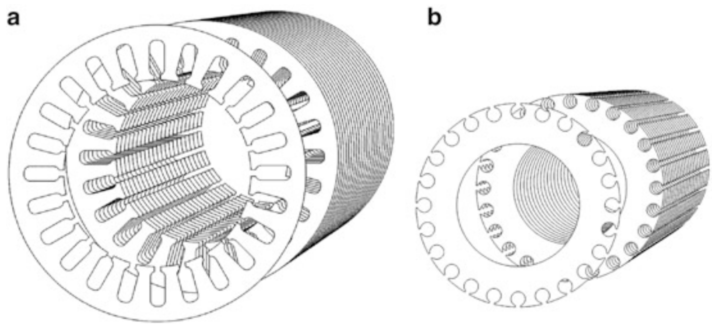

Fig. 14.2 (**a**) Stator magnetic circuit of an induction machine. (**b**) Rotor magnetic circuit of an induction machine

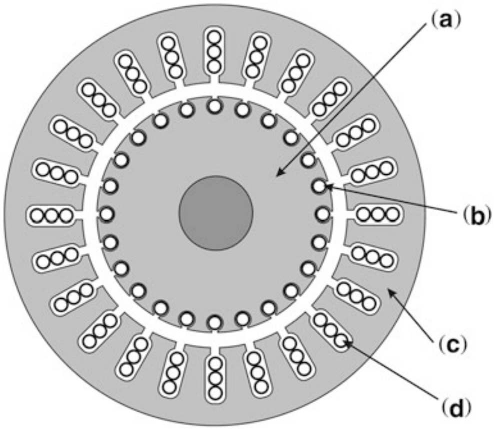

Fig. 14.3 Cross section of an induction machine. (**a**) Rotor magnetic circuit. (**b**) Rotor conductors. (**c**) Stator magnetic circuit. (**d**) Stator conductors

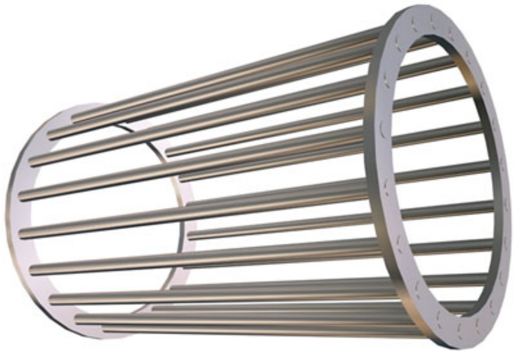

Fig. 14.4 Cage winding

On the internal surface of the stator magnetic circuit, there are axial slots facing the air gap. The same way, the outer surface of the rotor has axial slots facing the air gap. The slots are used for placing conductors that constitute stator and rotor windings. The stator slots comprise copper conductors. In most cases, they have a round cross section. The stator conductors can be isolated by resin. They are connected so as to make three stator coils, also called three phases or three parts of the stator winding. The rotor slots are mostly filled by aluminum bars, often made by casting which consists of pouring liquid aluminum into the rotor slots. These bars are short circuited by the front and rear aluminum rings, forming in this way short-circuited rotor winding called *squirrel cage*. A cross section of the machine, shown in Fig. 14.3, indicates positions of conductors with respect to the magnetic circuits.

Figure 14.4 shows the shape of rotor conductors and the short-circuiting rings. To visualize the rotor cage, it is assumed that the magnetic circuit is removed. Similarity to the cage and the circumstance that the turns are short circuited gave the name *cage rotor*. Sometimes, machines having aluminum-cast short-circuited rotors are called *squirrel cage rotor* machines. In high-power machines, where it is significant to increase the energy efficiency, rotor conductors are made of copper bars. Copper has lower specific resistance compared to aluminum, which reduces specific and total losses in the rotor cage.

Rotor conductors have their electrical insulation made differently. An electrical contact between the rotor bars, cast into the rotor slots, and iron sheets that constitute the rotor magnetic circuit gives a rise to sparse electrical currents that can jeopardize performances of the machine. These currents can be avoided by applying an acid solution to internal surfaces of the rotor slots before casting the aluminum bars or by making the inner surfaces nonconductive in some other way. The insulation layer created in this way separates aluminum bar from the magnetic circuit and prevents any uncontrollable currents. Exceptionally, the rotor of induction machine can have a three-phase winding made of round, insulated copper wire, in the way quite similar to the one used in manufacturing the stator winding. In such cases, the three rotor terminals are made available to the user. Such rotor is also called *wound rotor*, and it is briefly explained within the next paragraphs. It has been used prior to deployment of three-phase, variable frequency static power converters. Wound rotor machines are rarely met nowadays, as a vast majority of induction machines have a cage rotor.

Prescribed method of manufacturing the rotor cage is rather simple, and it does not require high-precision processing nor any special technologies or materials. Manufacturing of the rotor of DC machine is considerably more involved and complicated. It requires mechanical commutator, device that requires rather precise production process and which contains a number of different materials.

Compared to DC machines, induction machines have a number of advantages. They include rather simple manufacturing procedure, robustness, higher specific power $\Delta P/\Delta m$, lower mass and volume, as well as possibility to operate at considerably higher rotor speeds compared to DC machines. Therefore, induction machines are the most widespread machines nowadays. Absence of the brushes and collector eliminates the maintenance and prolongs the lifetime. Robust construction of induction motor results in an improved reliability, which is usually expressed by the mean time between failures (MTBF).

Over the past century, most induction machines were operated from the mains, namely, supplied by AC voltages of fixed amplitude, having the line frequency. These machines were mostly running with constant speed. Speed regulation was possible only with wound rotor.

With recent developments in the area of power converters, semiconductor power switches, digital signal processors (DSP), and digital controls of power converters and drives, it is possible nowadays to design, manufacture, and deploy reliable and affordable systems based on induction motors supplied from static power converters providing variable frequency AC voltages. Variable frequency supply allows for efficient and reliable operation of induction machines over wide range of speeds. Digitally controlled induction motors are frequently used as the torque actuators in motion control systems.

14.3 Cage Rotor and Wound Rotor

In addition to short-circuited rotor, it is possible to encounter *wound rotor* induction machines with *slip rings* used for external access to the rotor winding. Induction motors with wound rotors have been used at times when there was no possibility to change the amplitude and frequency of the stator voltages. The stator used to be fed from the mains, with the amplitude and frequency which could not be changed. At that time, there were no suitable static power converters capable of converting the electrical energy of line-frequency voltages and currents into the energy of variable frequency voltages and currents. Under these conditions, application of induction machines with wound rotors was used to alter the rotor speed of line-frequency-supplied inductions machines. Today, the use of wound rotor motors is declining, and the use of squirrel cage motors is prevailing.

Part (a) in Fig. 14.5 shows a short-circuited cage rotor of induction machine. Part (b) shows a *wound* rotor which has a three-phase winding similar to that of the stator. The three-phase windings are usually star connected, while the remaining three terminals of the rotor winding are connected to metal rings called *slip rings*, mounted at the front end of the machine. When the motor is in service, there are *sliding brushes* pressed against the rings, providing electrical contact and making the rotor terminals accessible to external uses. Sliding brushes are elastic metal-graphite plates which slide, as the rotor revolves, along peripheral surface of the rings. By connecting three external resistors to the rotor circuit, the equivalent resistance of the rotor circuit changes, and this alters the mechanical characteristic of the motor, allowing for desired speed changes.

The need for applying wound rotor machines has disappeared along with the appearance of static power converters which allow continuous change of the supply frequency and, hence, continuous change of the rotor speed.

14.4 Three-Phase Stator Winding

Stator of induction machines has three-phase windings, namely, three separate coils making the system of stator windings. Each phase winding has two terminals. The three-phase windings can be star connected or delta connected. Star connection

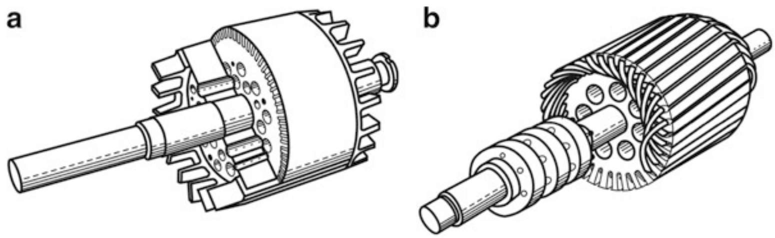

Fig. 14.5 (**a**) Cage rotor. (**b**) Wound rotor with slip rings

is denoted by Y and delta connection by Δ. With star connection, one terminal of each phase winding is connected to three-phase AC source, while the other phase terminals are connected to a common node. This common node is called *star point*. With the star point having no other connections (floating), the sum of the three-phase currents must be equal to zero at every instant, $i_a(t) + i_b(t) + i_c(t) = 0$. Most star-connected stator windings have their star point floating. Exceptionally, some large-power, high-voltage[1] induction machines may have their star point connected.[2]

In some cases, phases of the three-phase stator winding are connected to *delta* connection, wherein the three-phase windings are connected into triangle, restricting the phase voltages by $u_a(t) + u_b(t) + u_c(t) = 0$. For the given power rating, delta connection has lower current in stator conductors with respect to star connection. Therefore, delta connection is advantageously used in electrical machines with exceptionally low stator voltages, such as the motors in battery-fed traction drives, where the stator current is very large.

There are advantages of star connection which make it more frequently used.[3] Without the lack of generality, it is assumed throughout this book that the stator phases are star connected.

Magnetic axes of the stator phase windings are spatially shifted by 2π/3, as shown in Fig. 14.6. Desired magnetic field within three-phase induction machines is established by establishing the phase currents of the same amplitude I_m and the same angular frequency ω_e. Their initial phases have to be shifted by 2π/3, the angle that corresponds to the spatial shift between magnetic axes of the three phases. With prescribed currents in the phase windings, magnetic field is established in magnetic circuits and the air gap of the machine. The field revolves at angular speed $\Omega_e = \omega_e$.[4] The phase currents of the same amplitude and frequency, and with the initial phase

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[1] Voltages in excess to 1 kV are called *high voltages*. The term *medium voltage* is also in use, and it refers to lower end of *high voltages* and corresponds to voltages from 1 up to 10 kV. The upper limit of *medium voltages* is not strictly defined. There are also terms *very high voltages* and *ultrahigh voltage*, both lacking a clear definition.

[2] High-voltage induction machines have an increased insulation stress. Due to transient phenomena, floating star point may have considerable overvoltages. In some cases, star point of high-voltage machines is grounded by means of impedance connected between the star point and the ground.

[3] The sum of the phase voltages of delta-connected phase windings is equal to zero, $u_a(t) + u_b(t) + u_c(t) = 0$. Practical AC machines have imperfect, nonsinusoidal electromotive forces that include harmonics such as the third, which has the same initial phase in all the three-phase windings. This is the property of all 3*n*th harmonic, also called *triplian* harmonics. Within the three phases of the stator winding, the waveforms of a triplian harmonics have the same amplitude and phase. Therefore, with star connection, triplian harmonics cannot produce any current due to $i_a(t) + i_b(t) + i_c(t) = 0$. On the other hand, delta connection provides the circular path for triplian harmonics of the stator current. With delta connection, any distortion in electromotive forces that results triplian harmonics contributes to circular currents which compromise the operation of induction machine by increasing losses.

[4] With induction machines having $p > 1$ pairs of magnetic poles, $\Omega_e = \omega_e/p$. Machines with multiple pairs of magnetic poles are explained further on.

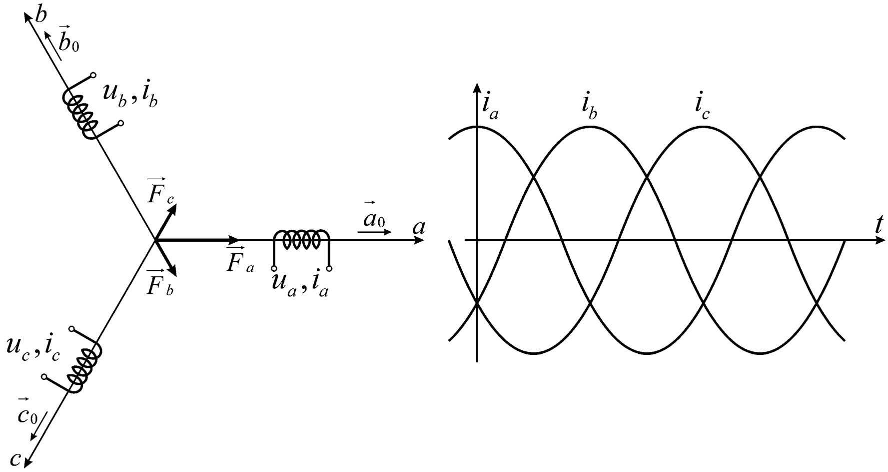

Fig. 14.6 Magnetomotive forces of individual phases

difference of 2π/3, constitute a *symmetrical three-phase system* of electrical currents. Instantaneous values of these currents are given by (14.1), while their change is depicted in the right-hand part of Fig. 14.6.

$$F_a = N_s I_m \cos \omega_e t$$

$$F_b = N_s I_m \cos\left(\omega_e t - \frac{2\pi}{3}\right)$$

$$F_c = N_s I_m \cos\left(\omega_e t - \frac{4\pi}{3}\right) \tag{14.1}$$

In Fig. 14.6, the phase windings are denoted by coils. Each of the three coil signs represents one phase winding. The phase windings have their conductors distributed in a number of stator slots along the machine circumference, next to the inner surface of the stator magnetic circuit. It is understood that conductor density has sinusoidal change along the air-gap circumference, as shown in Fig. 14.7. In this figure, the coil sign denotes the winding and lies on its magnetic axis. Shortened representation of phase windings places a coil sign instead of introducing a number of distributed conductors. These signs are used for clarity. An attempt to represent all the three phases by drawing their individual conductors would result in a drawing which is of little practical value.

Magnetomotive force of phase winding has an amplitude determined by the phase current, while the corresponding vector extends along the magnetic axis of the winding. The winding flux vector has the same direction. Hence, the winding represented by coil symbol has the magnetomotive force and flux vectors directed along the axis indicated by the symbol of coil that represents the winding.

14.5 Rotating Magnetic Field

Each phase winding creates a magnetomotive force along the magnetic axis of the winding. Amplitudes of magnetomotive forces F_a, F_b, and и F_c are dependent on currents $i_a(t)$, $i_b(t)$, and $i_c(t)$. Vector sum of these magnetomotive forces gives the resultant magnetomotive force of the stator $\boldsymbol{F_s}$ (Fig. 14.8). The quotient of the vector $\boldsymbol{F_s}$ and magnetic resistance R_μ gives the stator flux vector. The lines of the stator flux pass through the air gap and encircle the rotor magnetic circuit (Fig. 14.9). Passing through aluminum cast, short-circuited rotor turns, the stator flux contributes to the rotor flux. The coefficient of proportionality is determined by the mutual inductance L_m between the stator winding and the rotor cage. In cases when the rotor revolves in synchronism with the field ($\Omega_m = \Omega_e$), there is no change of the rotor flux. Therefore, maintaining the synchronism, the rotor electromotive force is equal to zero as well as the rotor current.

When the rotor speed is lower than the synchronous speed ($\Omega_m < \Omega_e$), the rotor is lagging with respect to the field. The speed difference $\Omega_{slip} = \Omega_e - \Omega_m > 0$ is called *slip speed*. For the observer residing on the rotor, in the frame of reference of short-circuited rotor cage, the stator flux revolves with relative speed of Ω_{slip}, determined by $\Omega_{slip} = \omega_{slip}$.[5]

Change of flux leads to induction of electromotive force in short-circuited rotor turns. Rotor current is an AC, and it has angular frequency ω_{slip}. It is proportional to the induced electromotive force e and inversely proportional to rotor impedance

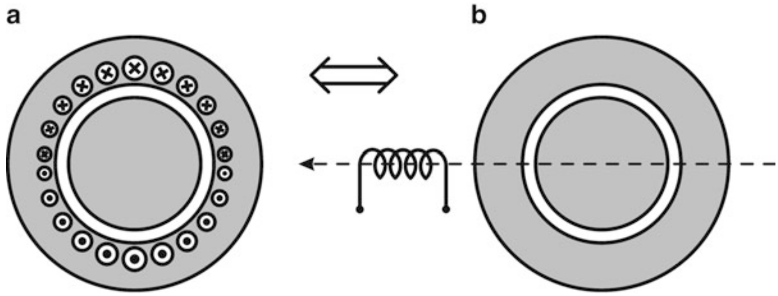

Fig. 14.7 (**a**) Each phase winding has conductors distributed along machine perimeter. (**b**) A winding is designated by coil sign whose axis lies along direction of the winding flux

[5] In the preceding part of the book, the electrical machines are considered having two-pole magnetic field. They have one north magnetic pole and one south magnetic pole. These machines are called *two-pole* machines, and they have $p = 1$ pair of magnetic poles. Machines with multiple pairs of magnetic poles will be explained as well. As an example, distribution of magnetic field in the air gap may have two north and two south magnetic poles. The number of pairs of poles is denoted by p. It will be shown later that the magnetic field created by AC currents of angular frequency ω_e rotates at angular frequency $\Omega_e = \omega_e/p$. Therefore, for two-pole machines, angular frequency ω_e is equal to the angular speed of rotation Ω_e.

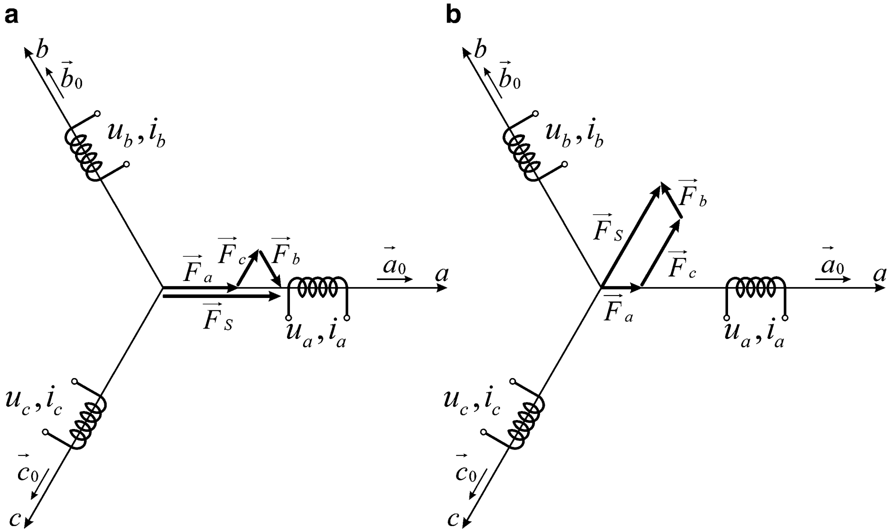

Fig. 14.8 Resultant magnetomotive force of three-phase winding. (**a**) Position of the vector of magnetomotive force at instant $t = 0$. (**b**) Position of the vector of magnetomotive force at instant $t = \pi/3/\omega_e$

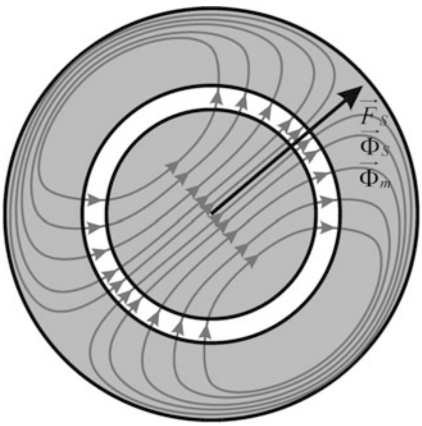

Fig. 14.9 Vector representation of revolving field. ($\boldsymbol{F_s}$)-vector of the stator magnetomotive force. ($\boldsymbol{\Phi_s}$)-vector of the flux in one turn of the stator. ($\boldsymbol{\Phi_m}$)-vector of mutual flux encircling both the stator and the rotor turns

$R + j\omega_{slip}L$, where R and L are parameters of short-circuited rotor winding. If the slip ω_{slip} is rather small, the rotor current is approximately equal to e/R. Joint action of the magnetic field and currents in rotor conductors creates electromagnetic torque which tends to bring the rotor into synchronism with the field. Namely, in cases where $\Omega_e - \Omega_m > 0$, the torque acts upon the rotor so as to increase the rotor speed Ω_m and bring it closer to the synchronous speed Ω_e.

14.6 Principles of Torque Generation

Principle of operation of an induction machine can be explained by using Fig. 14.10. Represented flux $\boldsymbol{\Phi_m}$ revolves at the synchronous speed Ω_e. It is assumed that the rotor revolves at the speed of $\Omega_m < \Omega_e$, meaning that the rotor is lagging behind the flux by the amount of slip,

$$\omega_{slip} = \Omega_{slip} = \Omega_e - \Omega_m > 0 \tag{14.2}$$

Angle θ_{slip} between the flux vector $\boldsymbol{\Phi_m}$ and the rotor is equal to the integral of the slip; thus, it increases gradually. The figure shows only one short-circuited contour of the rotor in order to explain the principle of the torque generation. The flux within the rotor contour changes with the angle θ_{slip},

$$\theta_{slip}(t) = \theta_{slip_0} + \int_0^t \Omega_{slip}\, d\tau \tag{14.3}$$

Angle between the reference axis of the contour and the flux vector is $\theta_{slip} + \pi/2$. The part of the stator flux which encircles the rotor contour is equal to

$$\Phi_{Rm} = -\Phi_m \cdot \sin\,\theta_{slip}. \tag{14.4}$$

The total rotor flux includes the effects of rotor current which contribute to the rotor flux in proportion to the coefficient of self-inductance L_R. In the course of gaining an insight into the operating principles, these effects are neglected for the time being. This assumption is justified by the fact that with relatively low slip frequencies, the reactance $L_R\omega_{slip}$ can be neglected. Hence, it is considered instead that the mutual flux $\Phi_{Rm} = -\Phi_m{\cdot}\sin(\theta_{slip})$ corresponds to the total rotor flux Φ_R.

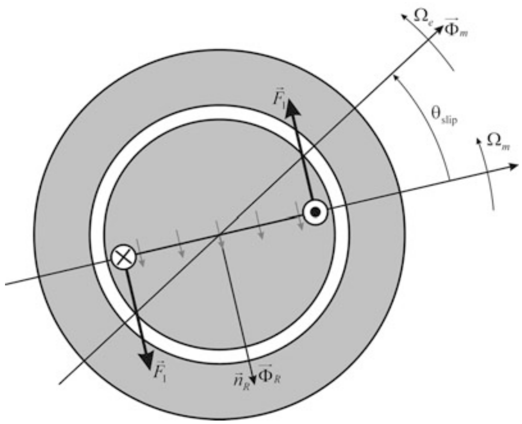

Fig. 14.10 An approximate estimate of the force acting on rotor conductors

Thus, the electromotive force induced in the rotor winding is proportional to the flux Φ_m and to the slip frequency ω_{slip}

$$e = \frac{\mathrm{d}\Phi_R}{\mathrm{d}t} = -\Phi_m \cdot \Omega_{slip} \cos \theta_{slip} \tag{14.5}$$

In steady state, $\theta_{slip} = \Omega_{slip} t$. Therefore, rotor current is an AC current. Its amplitude is inversely proportional to the rotor impedance $R + \mathrm{j}\omega_{slip}L$. For small values of slip, resistance R is considerably higher than reactance $\omega_{slip}L$. Hence, the rotor current i_R is approximately equal to $e/R \sim \Phi_m \omega_{slip}$

$$u = 0 = R_R i_R + e\,, \quad i_R = +\frac{\Phi_m \cdot \Omega_{slip}}{R_R} \cos \theta_{slip} \tag{14.6}$$

In the case under consideration, $i_R > 0$. Direction of the rotor current corresponds to direction shown in Fig. 14.10. Validity of this conclusion is checked by the following reasoning. Since the flux Φ_m advances with respect to the rotor, the flux in the rotor contour in Fig. 14.10 changes its value. It increases in direction opposite to the reference direction $\boldsymbol{n}_R$.

The induced electromotive force in short-circuited rotor turn and the consequential rotor current have direction opposite to the change of the flux, wherein the flux change is the origin of the electromotive force. Hence, direction of the rotor current pretends to establish the flux which opposes to the original flux change. With reference directions as shown in Fig. 14.10, the rotor current causes the flux change which is directed downward. The rotor current varies in proportion to function cos (θ_{slip}) = cos($\theta_0 + \omega_{slip}t$). It is an AC current of frequency ω_{slip}. In rotors with multiple turns, AC currents create the flux which revolves with respect to the rotor at the slip speed of $\Omega_{slip} = \omega_{slip}/p$. With the rotor running at the speed of Ω_m, the speed of the rotor flux rotation with respect to the stator is equal to $\Omega_m + \Omega_{slip} = \Omega_m + (\Omega_e - \Omega_m) = \Omega_e$. Hence, the stator and rotor flux vectors of induction machine revolve in synchronism, at the same speed of Ω_e, determined by the supply frequency ω_e. In steady state, the stator and rotor flux vectors maintain their relative position.

14.7 Torque Expression

Joint action of the magnetic field and rotor currents creates the electromagnetic torque which tends to bring rotor into synchronism with the field. Simplified structure in Fig. 14.10 can be used to derive some basic relations between the torque, flux, and the slip frequency.

In the region of the rotor conductor denoted by $\odot$, there is radial component of the magnetic induction equal to $B_m \cdot \cos(\theta_{slip})$, where B_m is the maximum induction in the air gap. This analysis assumes that the air-gap flux is created primarily by the stator currents and that the induced rotor currents have a negligible effect on the magnetic inductance in the air gap. Magnetic induction assumes the maximum value of B_m in the air-gap regions along the flux vector $\boldsymbol{\Phi_m}$. In other directions, its value is smaller, and it changes according to the expression for sinusoidal distribution of the radial component of magnetic induction in the air gap. Hence, for the air-gap region displaced by $\Delta\theta$ from direction of the vector $\boldsymbol{\Phi_m}$, magnetic inductance assumes the value of $B = B_m\cos(\Delta\theta)$. Electrical current of the rotor is equal to $i_R = \Phi_m \cdot \omega_{slip} \cdot \cos\theta_{slip}/R_R$. The force acting on the conductor denoted by $\odot$ is equal to the product Li_RB, where L is the axial length of the machine. An equal force is acting on the conductor denoted by $\otimes$. Direction of both forces is positive with respect to the reference tangential direction. Direction of the current in the second conductor is changed. Direction of the magnetic induction is also changed. In the region of conductor $\otimes$, magnetic field comes out of the stator magnetic circuit, passes through the air gap, and enters into the rotor magnetic circuit.

Expression $\Phi_m = (2/\pi)B_m\pi RL = 2B_mRL$ relates flux Φ_m to the maximum value of magnetic induction B_m. In this expression, $R = D/2$ is the radius of the rotor cylinder. Therefore, the expression for magnetic induction obtains the form

$$B = k_1 \cdot \Phi_m \cos\theta_{slip} = B_m \cos\theta_{slip} \tag{14.7}$$

The electromagnetic torque is equal to

$$T_{em} = D\,L\,i_R B = D\,L\,\frac{\Phi_m \omega_{slip}\cos\theta_{slip}}{R_R} B_m \cos\theta_{slip} \tag{14.8}$$

that is,

$$\begin{aligned} T_{em} &= DL\frac{\Phi_m\left(p\Omega_{slip}\right)\cos\theta_{slip}}{R_R}\left(k_1\Phi_m\right)\cos\theta_{slip} \\ &= k_2\Phi_m^2\Omega_{slip}\cos^2\left(\theta_{slip}\right)\cdot \end{aligned} \tag{14.9}$$

Therefore, the torque delivered by an induction machine is directly proportional to the slip frequency and to the square of the flux. It is inversely proportional to the rotor resistance.

Question (14.1): In the case considered above, the torque is proportional to $\cos^2(\theta_{slip}) = \cos^2(\theta_0 + \omega_{slip}t)$. Therefore, the torque pulsates from zero up to twice the average value. Is it possible to alter the structure of Fig. 14.10 so as to obtain a constant, ripple-free torque?

Answer (14.1): By adding another short-circuited contour on the rotor, shifted by $\pi/2$ with respect to the existing one, the torque pulsations can be suppressed. The torque acting on conductors of the second contour will be proportional to $\sin^2(\theta_{slip})$. When added to previously obtained torque, proportional to $\cos^2(\theta_{slip})$, the sum of the two becomes $T_{em} = \Phi_m{}^2\omega_{slip}/R_R = \text{const}$.