Chapter 11
Introduction to DC Machines

Prior to commissioning the first electrical power stations, electrical energy was mostly obtained from batteries, chemical sources of electrical current. The batteries provide DC voltages and currents at their output terminals. It is for this reason that the first experiments and applications of electrical machines have been made with DC current electrical machines. Electrical engineers have studied the principles of operation of these machines and analyzed their characteristics, and they found the way of designing and manufacturing DC machines.

At first, the processes of production, transmission, and application of electrical energy were based on DC voltages and currents. All the tasks of electromechanical conversion were employing DC machines. Electric power stations were built in close vicinity of industrial facilities, cities, and other major consumers of electrical energy. The energy obtained from water or steam turbines used to be converted to electrical energy by means of electrical machines providing DC voltages and currents, also called DC generators. At their output terminals, most DC generators produced DC voltages of several hundred volts. By using a pair of conductors, electrical power was transmitted over short distances of 1–2 km and delivered to consumers. Early consumers of electrical energy have been designed to operate with DC voltages and currents. Electrical lighting bulbs have been made to convert electrical energy into light, while DC motors have been producing controlled mechanical work put to use in production processes.

Designing DC generators and motors for voltages in excess of 1,000 V involves technical difficulties that will be explained later on. As a consequence, DC generators and motors were manufactured and used for relatively low DC voltages U. Therefore, transmission and distribution of electrical power P involved very high currents due to $I = P/U$. Transmission of power of 1 MW required electrical currents in excess of 1,000 A. Electrical conductors in transmission lines were designed with very large cross sections in order to reduce the line resistance R. Such transmission was accompanied with considerable losses (RI^2) and large voltage drops (RI). Higher transmission voltage U leads to lower line current $I = P/U$ and, hence, lower losses and lower voltage drop. Yet, at that time, the maximum DC

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voltages available at generator terminals were rather limited. At the same time, there were no DC/DC power converters capable of transforming a low voltage, obtained from DC generators, into a high DC voltage, suitable for power transmission.

Contemporary systems for production, transmission, and distribution of electrical energy make use of alternating currents (AC) with frequencies of 50 or 60 Hz. By using power transformers, a relatively low voltage produced by AC generators is increased to several hundreds of thousands of volts. Most AC transmission lines are three-phase, with line-to-line voltages of 110, 220, 400, or 700 kV. Electrical currents in transmission lines are therefore reduced, along with the losses and voltage drops. With $P = 100$ MW and $U_l = 400$ kV, the line current is lower than 150 A. In the vicinity of consumers, high voltage at transmission lines is transformed by means of power transformers and scaled down to the level suitable for consumers (220 V). A power transformer changes voltage level according to transformation ratio $m = N_1/N_2$, defined by the number of turns of primary and secondary windings.

DC voltage cannot be transformed by power transformers. Recent developments in power electronics over the past couple of decades resulted in high power, high voltage static power converters required for DC power transmission. These devices were not available at the wake of electrical power systems. Therefore, DC voltage across generator terminals in the early power stations used to be fed to transmission lines without any conversion. The same voltage was made available to electrical loads, connected by distribution lines. Low DC voltages at transmission lines resulted in large currents, large voltage drops, and heavy losses. In order to keep the losses and voltage drops relatively low, transmission of electrical power over DC lines was feasible only at short distances.

Nowadays, electrical power generation, transmission, distribution, and consumption are based on AC voltages and currents. Processes of electromechanical conversion involve AC generators and motors. Therefore, the use of DC generators and motors is declining. DC electrical machines are being replaced by AC machines. Nevertheless, it is of interest to study DC machines as the first electrical machines that were widely used. Moreover, their relatively simple model makes them suitable for introducing basic principles, notions, and characteristics of electrical machines. While studying DC machines within the following three chapters, the reader gets acquainted with mechanical characteristics, steady-state operating area, transient characteristics, steady-state equivalent circuits, dynamic models, analysis of losses, power supply and controls, and other tasks, problems, and phenomena involved with application of electrical machines.

This chapter starts with description and principles of operation of DC motors and generators. Some basic information concerning the design of DC machines is presented as well. Analysis includes the operation of mechanical commutator, key component of DC machines, which converts DC currents into AC currents. This device receives DC current from electrical source and conducts them into rotor conductors. Due to mechanical commutation, rotor currents depend on rotor position. At constant speed, rotor conductors have AC current with their angular frequency determined by the speed. Some basic ways of forming the rotor winding and connecting the rotor conductors with the commutator are shown as well. This chapter ends with analytical expressions for electromagnetic torque and electromotive force.

Mathematical model of the machine is developed within the next chapters, describing the machine behavior during transients. On the basis of the steady-state analysis, the equivalent circuit is introduced and explained. It allows calculation of the current, flux, and torque of DC machine in the steady state, where the supply voltage and the rotor speed are constant and known. Within these chapters, mechanical characteristic is derived, expressing the steady-state relation of the speed and the torque. Losses in windings and magnetic circuits are analyzed along with processes of heating and the ways of heat removal (cooling). The maximum permissible steady-state current, power, and torque are introduced and explained, as well as the rated values of relevant. The field-weakening operation is introduced and explained, as well as relevant relations and characteristics. Transient and steady-state operating areas are determined from the torque-speed pairs attainable during transients and in the steady state.

11.1 Construction and Principle of Operation

DC machines consist of the stator magnetic circuit, rotor magnetic circuit, and rotor winding. The stator may have stator winding, called *excitation winding*, or permanent magnets. The stator flux is created either by permanent magnets in stator magnetic circuit or by DC currents in the stator winding. Currents in rotor conductors create the rotor magnetomotive force. In the preceding chapter, it has been shown that the vector product of two fluxes

$$\vec{T}_{em} = k\left[\vec{\Phi}_R \times \vec{\Phi}_S\right]$$

determines the electromagnetic torque of an electrical machine. Thus, the torque of a DC machine is determined by the vector product of the stator and rotor fluxes. The torque vector is collinear with the axis of the machine.

11.2 Construction of the Stator

The stator flux is called *excitation flux*, and it is obtained from direct electrical currents in the stator winding. The excitation flux can also be created by permanent magnets built in the stator magnetic circuit. The case when the excitation flux is obtained by the stator excitation winding is called *electromagnetic excitation*. Stator winding carries a direct current (DC) which creates stator magnetomotive force and stator flux. Since the stator carries a DC current, the stator flux does not move.

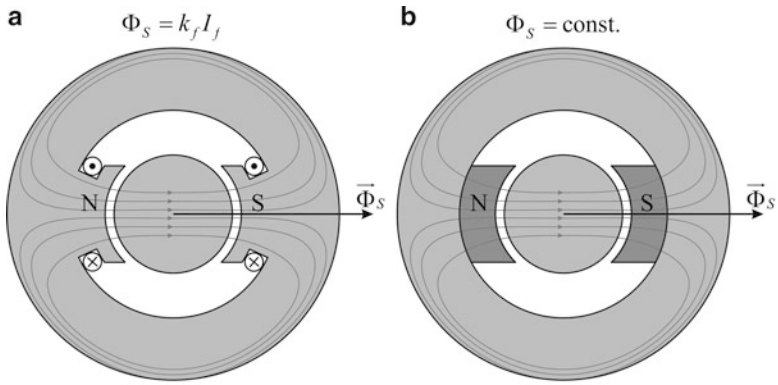

Fig. 11.1 Position of the stator flux vector in a DC machine comprising stator winding with DC current (**a**) and in DC machine with permanent magnets (**b**)

Instead of stator winding, DC machine can have permanent magnets built in the magnetic circuit of the stator. Permanent magnets have significant remanent induction B_r even in cases with no external field H. With permanent magnets, stator flux is obtained without any need to build a stator winding. In Fig. 11.1, lines of the stator field start from the north magnetic pole and propagate toward the south magnetic pole, passing along their way through the rotor.

11.3 Separately Excited Machines

Many DC machines have excitation that does not change with rotor currents. These machines are called *separately excited machines*. They include DC machines with permanent magnets, where the rotor currents do not affect the excitation flux caused by the magnets. They also include DC machines with the stator excitation winding fed from a separate electrical source, decoupled from the rotor supply.

In other types of DC machines, the stator excitation gets affected by the rotor currents. If the stator winding (i.e., the excitation winding) is connected in series with the rotor winding, the excitation current equals the rotor current. This type of electrical machine is called *series excited DC machine* or *series DC motor*. The excitation can depend on the rotor current in other ways. The excitation (stator) winding can be connected in parallel with the rotor winding. As the rotor voltage changes with the rotor currents, the excitation voltage and current would change with the rotor current as well. There are also DC machines where both series and parallel excitations are present.

Separately excited DC machines are the main subject of the study within this chapter.

11.4 Current in Rotor Conductors

Rotor of DC machine has axially set conductors carrying electrical currents. Through interaction with magnetic induction of the excitation field, the rotor conductors are exposed to electromagnetic force. A couple of forces create mechanical torque that acts on the rotor and incites motion. Due to specific construction of DC machines and the presence of mechanical commutator, which directs the electrical current into rotor conductors, direction of the current in conductors below the north magnetic pole (*N*) does not change. It remains the same despite the fact that the rotor revolves. Direction of the current in the conductors below the south pole (*S*) is opposite to the one in the conductors which are below the north pole.

Conductors 1 and 2 are built in the rotor slots, and they revolve at the same speed as the rotor does. With rotor making one half turn (Fig. 11.2b), conductors 1 and 2 exchange their places. In order to have a positive torque, it is necessary to change directions of the currents in conductors, as shown in the Figure. In conductor 1, the direction was $\otimes$ while it was in the zone of the north pole of the stator. When this conductor comes to the zone of the south pole, it has to carry a current of direction $\odot$ so that the torque remains positive. Similar conclusion may be drawn for conductor 2.

Conductors 1 and 2 constitute one contour (turn) of the rotor. When the rotor revolves at a speed Ω_m, it makes one revolution each $T = 2\pi/\Omega_m$. For the purpose of creating a torque which would not change sign but remain positive instead, current

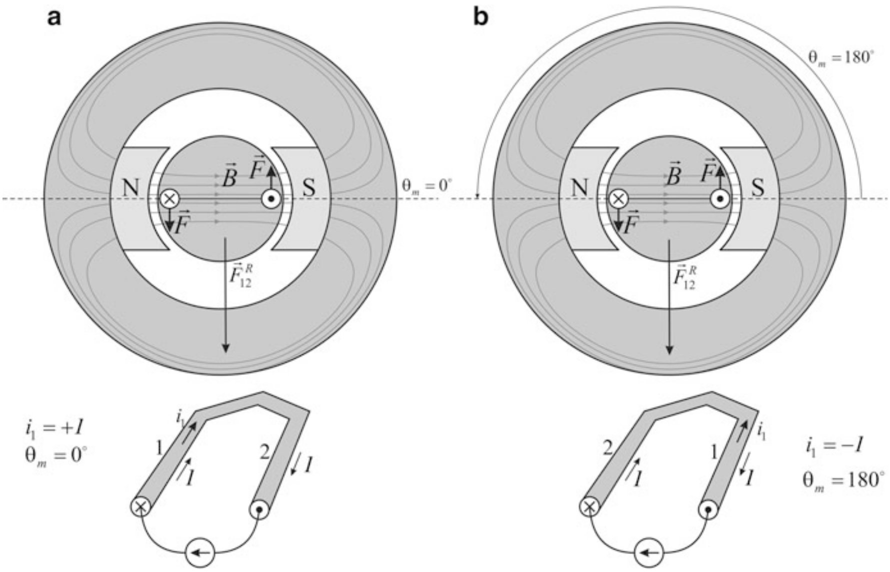

Fig. 11.2 Position of rotor conductors and directions of electrical currents. (**a**) Rotor at position $\theta_m = 0$. Rotor conductor 1 in the zone of the north pole of the stator and conductor 2 below the south pole of the stator. (**b**) Rotor shifted to position $\theta_m = \pi$. Conductors 1 and 2 have exchanged their places

through the considered contour should change sign synchronously with the rotor movement. The current should be positive during time interval $T/2 = \pi/\Omega_m$ and then negative during the next interval of $T/2$. Therefore, current in the rotor has to be periodic with the period T determined by the rotor speed.

The ways of directing the rotor currents into conductors so as to obtain periodic electrical currents will be put aside at this time. By considering Fig. 11.2 and assuming that, notwithstanding the rotor motion, electrical current in rotor conductor below the north pole retains direction ⊗, while the conductor below the south pole always has direction of the current ⊙, the rotor magnetomotive force and flux can be represented by vectors directed downward. On the other hand, vector of the stator field is horizontal, directed from left to right. It is concluded that the angle between the two fluxes is constant and equal to π/2, irrespective of the speed and direction of the rotor motion. This fulfills the optimal condition for creating constant torque.

Considered pair of rotor conductors is in the region under the stator poles. Coupled forces producing positive torque act upon conductors 1 and 2 (Fig. 11.2a). This torque supports counterclockwise movement. The electromagnetic torque is determined by (11.1) where l is length of one conductor, B is magnetic induction in the zones of the stator magnetic poles, and D is rotor diameter.

$$\vec{F} = i(\vec{l} \times \vec{B}),$$

$$M_{em} = l \cdot B \cdot i \cdot D. \tag{11.1}$$

11.5 Mechanical Commutator

Rotor currents in a DC machine are obtained from DC power sources. The source current is fed to a *mechanical commutator*. The rotating part of the commutator is called *collector*, while the stator part of the commutator has two carbon brushes usually designated by A and B. External power supply that feeds the rotor winding has its positive pole connected to the brush A and its negative pole to the brush B. Attached to the stator, the brushes do not move. The collector is fastened to the rotor shaft and rotates at the same speed as the rotor. Owing to this, the still carbon brushes, positioned diametrically, slide along collector ring. A collector ring is divided into a number of mutually insulated segments. The collector segments are called *commutator segments* or *collector segments*. The segments have electrical connection to rotor turns in the manner to be described later.

DC current from an external DC source is fed to diametrically positioned brushes A and B. Immobile brushes lean on the collector segments, directing in this way current into the rotor conductors. When the rotor moves, the brushes slide from one pair of the collector segments to the next pair. As a consequence, a change occurs in distribution of electrical current in rotor conductors. The end result is that, notwithstanding the rotor motion, the rotor currents create the magnetomotive force vector $\boldsymbol{F}^R{}_{12}$ which does not move with respect to the stator, as shown in Fig. 11.2. The rotor magnetomotive force gives a rise to the rotor flux, which does not move either.

Mechanical commutator converts DC currents, obtained from the power supply of the rotor winding, to periodic currents carried by rotor conductors. Frequency of these currents is determined by the speed of rotation. The role of the mechanical commutator is similar to that of static power converters called inverters, which employ power transistor switches and convert DC voltages and currents into AC voltages and currents. The change of voltages and currents in conductors of the rotor equipped with mechanical commutator is similar to the change of voltages and currents in a system comprising DC supplied inverter which feeds AC currents and supplies asynchronous or synchronous machines.

In modern applications of electrical machines, DC machines with mechanical commutator are replaced by static power converters (transistor inverters) feeding asynchronous or synchronous machines, also called AC machines.

The method of making of rotor winding as well as the method of connecting this winding to collector may be relatively involved. A detailed study of different methods of realization of rotor windings of DC machines is beyond the scope of this book. For the purpose of understanding the operation of mechanical commutator, further discussion presents an analysis of some relatively simple examples, intended for illustration of basic functions of mechanical collector.

11.6 Rotor Winding

Figure 11.3 shows a rotor having only one turn. It is made of conductors 1 and 2 connected in series. The conductors are in electrical connection with collector which has segments S1 and S2. Such collector can be made from a metal cylinder by cutting it in two mutually insulated halves. The front end of conductor 1 is connected to segment S1, while the front end of conductor 2 is connected to segment S2. At the rear end of the rotor, the ends of conductors 1 and 2 are brought together by the end turn. Brushes A and B are connected to a current source supplying electrical current $i(t)$. In the given position, brushes A and B lean on segments S1 and S2, respectively. Therefore, there is electrical contact between the brush and the segment that gets in touch with it. At position presented in the figure, current in conductor 1 has direction $\otimes$, while current in conductor 2 has direction $\odot$. When the rotor moves by π, conductors exchange their places. At the same time, the segments S1 and S2 change their places as well, as they are fastened to the rotor and revolve along with the rotor. Now conductor 1 gets under the south pole (left), but direction of the electrical current in this conductor is changed. Therefore, the electrical current in rotor conductor under the north pole retains direction $\otimes$, while the current in rotor conductor under the south pole retains direction $\odot$. Mechanical commutator insures that the

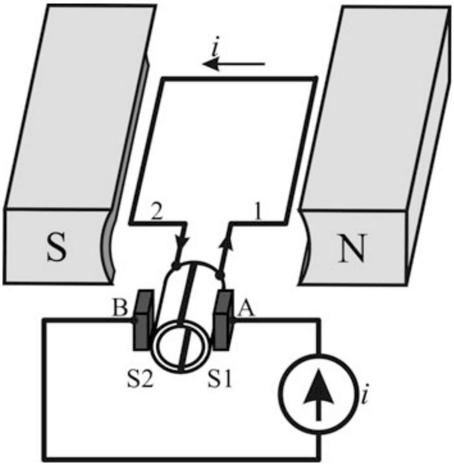

Fig. 11.3 Mechanical collector. *A*, *B*, brushes; *S1*, *S2*, collector segments

current distribution under the stator poles remains the same for any position of the rotor, notwithstanding its relative motion.

Current in each of conductors changes direction synchronously with the rotor motion. The commutator with its brushes and two-segment collector converts DC current of the source into periodic current in the rotor conductors. One revolution of the rotor corresponds to one period of alternating currents in rotor conductors. As a consequence, the observer at the stator side (i.e., the observer which does not move with respect to the stator) does not see any motion of the rotor magnetomotive force. Namely, the rotor conductors below the north magnetic pole, designated by N in Fig. 11.4, retain the current direction, while the conductors below the south pole S retain direction ⊙. This distribution remains unaltered in spite of the fact that the rotor and rotor conductors move.

In practice, rotor winding has a number of turns evenly distributed along the rotor perimeter. The conductors are connected to respective segments of the collector. A collector ring may have a number of mutually insulated segments which are galvanically connected to two or more conductors. Current i_a is fed to the rotor by means of a couple of carbon brushes which are in touch with the collector and which pass the electrical current to the segments. The commutator directs the current to the rotor conductors in such way that distribution of the rotor currents corresponds to the one shown in Fig. 11.4. Said distribution does not change notwithstanding the rotor motion. It should be noted that the rotor turns along with the rotor conductors. Due to the action of mechanical commutator, the rotor has alternating currents and they create the rotor current sheet which does not revolve but remains still with respect to the stator. In Fig. 11.4, direction of the rotor magnetomotive force and the rotor flux is vertical, irrespective of the rotor position. Therefore, the rotor flux remains still with respect to the stator, and the angle between the stator and rotor flux vectors is $\pi/2$. DC machines usually have a relatively large number of the rotor conductors and corresponding number of collector segments. Appearance of the rotor of a typical DC machine is shown in Fig. 11.5.

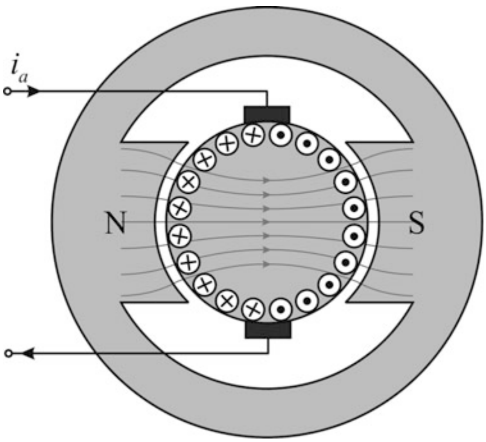

Fig. 11.4 Position of the rotor current sheet with respect to magnetic poles of the stator

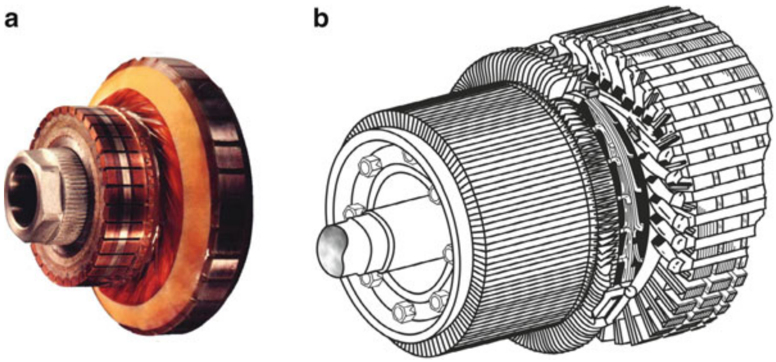

Fig. 11.5 Appearance of the rotor of a DC machine. (**a**) Appearance of the collector. (**b**) Appearance of the magnetic and current circuits of a DC machine observed from collector side

Figure 11.6 shows the method of connecting rotor conductors to the collector segments for DC machine with 4 rotor slot, 4 collector segments, and 8 conductors. Given example is seldom seen in practice. The number of rotor slots is usually much higher. Yet, the case in Fig. 11.6 is selected as an introduction to making the rotor winding. Conductors P1–P8 are placed in slots. Each slot houses two conductors. The rotor is observed from the side where the mechanical collector is mounted on the shaft. The ends of conductors P1–P8 are connected as well on the rotor side opposite to mechanical collector, the rare side of the machine. This side is not visible. Therefore, relevant wire connections at the rare side are shown in the right-hand side of Figure 11.6 by dotted lines. Connected by rare side connections, conductors make four turns, P1–P2, P3–P4, P5–P6, and P7–P8. At the front side of the rotor, where the mechanical collector is mounted, wire connections are represented in the left-hand side of Fig. 11.6 by solid lines.

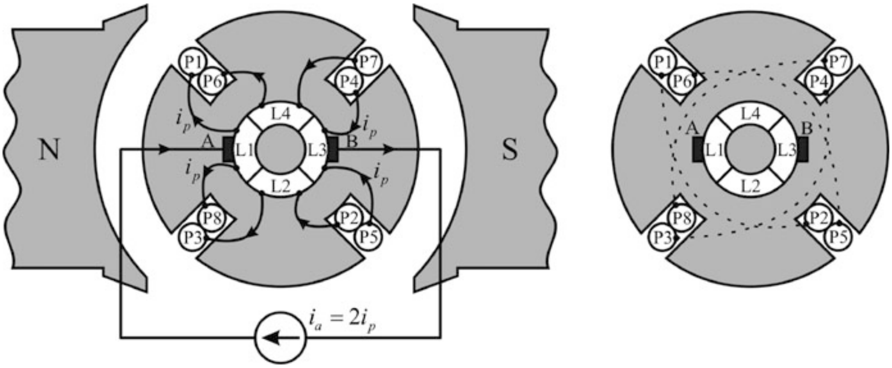

Fig. 11.6 Connections of rotor conductors to the collector segments in the case when 4 rotor slots contain a total of 8 conductors

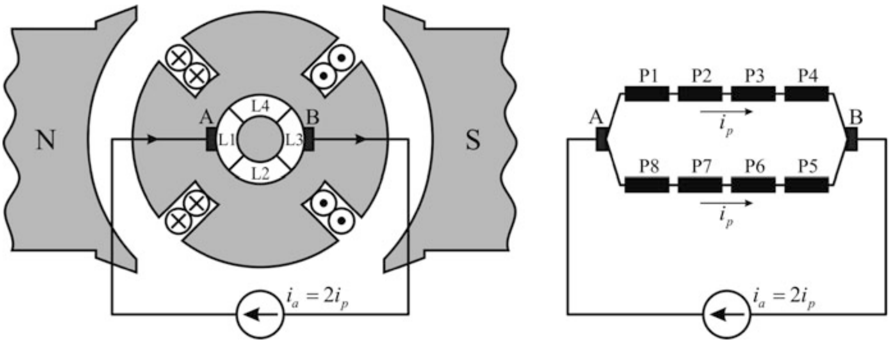

Fig. 11.7 Direction of currents in 8 rotor conductors distributed in 4 slots

Segment L1 of the collector is connected to conductors P8 and P1, segment L2 to conductors P2 and P3, segment L3 to conductors P4 and P5, and segment L4 to conductors P6 and P7. Connections of the segments with relevant conductors are at the front side, represented by solid lines, while the connections of conductor ends at the rear side of the rotor are drawn by dotted lines. In the considered rotor position, brushes A and B are in touch with segments L1 and L3; thus, the current of the source i_a is split in two parallel paths, as shown in Fig. 11.7. Each of the rotor conductors carries electrical current of $i_a/2$.

In all the conductors below the north magnetic pole of the stator, direction of electrical current is ⊗, while in conductors below the south magnetic pole, direction is ⊙. If the rotor is turned by π/2, brush A comes in touch with segment L4, while brush B touches segment L2. Connections between the conductors and segments shown in Fig. 11.6 can be used to determine direction of currents in rotor conductors after the rotor moves. Due to rotation, conductors will change their position. At the

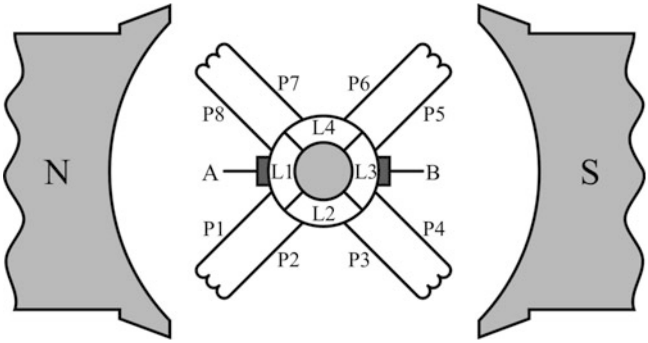

Fig. 11.8 Wiring diagram of the rotor current circuit

same time, direction will change in some of them due to mechanical commutator. Finally, direction of currents in conductors below the north magnetic pole remains $\otimes$ even after the rotor has moved, while in conductors below the south pole direction remains $\odot$. The way of making the rotor winding and connecting the segments ensures that direction $\otimes$ is preserved in all conductors under the north magnetic pole, notwithstanding the rotor motion. In every single conductor, direction changes and becomes $\odot$ as the conductor passes from the zone under the north pole into the zone below the south magnetic pole. In the course of rotation, the rotor conductors have alternating currents with frequency determined by the speed of rotation.

Wiring diagram of the rotor current circuit can be presented in the manner shown in Fig. 11.8. The figure shows the rotor with rather simple construction, having 4 slots, 8 conductors, and mechanical commutator with 4 segments. Commutator allows creation of the rotor current sheet which does not rotate with respect to the stator, producing in this way vectors of the rotor magnetomotive force and flux which do not move with respect to the stator. The power supply to the rotor winding is shown in Figs. 11.6 and 11.7 as a constant current source connected to brushes A and B.

In the course of the rotor motion, the brushes direct the current to collector segments and subsequently to the rotor conductors. As the rotor conductor moves below the north pole and passes under the south pole, direction of electrical current changes. For that reason, each rotor conductor has alternating current with a frequency determined by the speed of rotation. Observed from the stator side, distribution of the rotor currents remains unaltered. Therefore, the rotor currents create a current sheet which does not move with respect to the stator. Rotor currents are shown by signs $\otimes$ and $\odot$ in Fig. 11.7, and they create the rotor flux which can be represented by the vector of vertical direction, standing at an angle of $\pi/2$ with respect to the stator field. According to the right-hand rule, the rotor flux is directed downward.

11.7 Commutation

As shown in the previous example, the rotor conductors below the north magnetic pole of the stator have electrical current in direction ⊗, while conductors below the south magnetic pole have direction ⊙. In the course of rotation, each of the rotor conductors resides below north magnetic pole during one half turn and below the south magnetic pole under the second half turn. Therefore, direction of electrical current in each individual conductor changes with a frequency determined by the speed of rotation. Carbon brushes A and B are fastened to the stator, and they touch the segments of the collector, passing the DC current received from the power supply i_a. During rotation, the brushes touch the segments which are below them at each particular instant. Hence, the segments slide under the brushes. Brush transition from one segment to the next is followed by change in electrical current in rotor conductors attached to relevant segments. Directing DC current i_a by collector action results in alternating currents in the rotor conductors. Transition of the brush from one segment to the other and consequential change in electrical current in rotor conductors is called *commutation*. In the course of transition, one brush touches two segments at the same time, bringing them into short circuit and short circuiting the rotor turns connected to relevant segments. The case when brush A simultaneously touches segments L1 and L2 is shown in Fig. 11.9.

Advancing from position given in Fig. 11.8 in clockwise direction, toward position given in Fig. 11.9, the rotor moves by π/4. Since the brush A in Fig. 11.9 makes a short circuit between segments L1 and L2, while the brush B makes a short circuit between segments L3 and L4, turns P1–P2 and P5–P6 are short-circuited during commutation. If the rotor makes further move by π/4 in the same direction, it arrives at position shown in Fig. 11.10. With respect to Fig. 11.8, the rotor position is changed by π/2 in clockwise direction, and now the brush A has contact with segment L2. Wiring diagram of Fig. 11.10 should be compared with the wiring diagrams in Figs. 11.6 and 11.7.

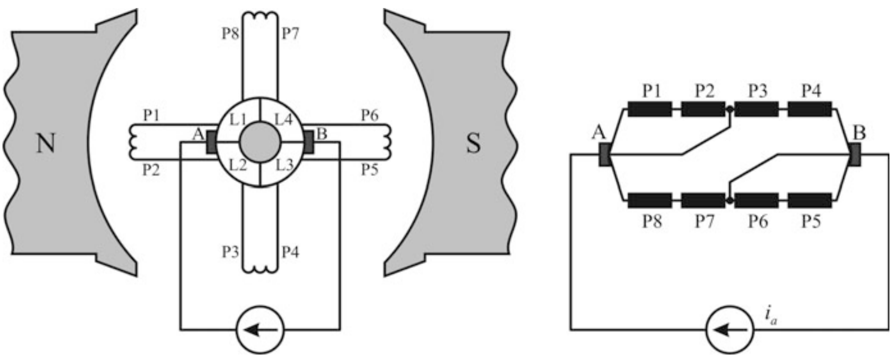

Fig. 11.9 Short circuit of rotor turns during commutation

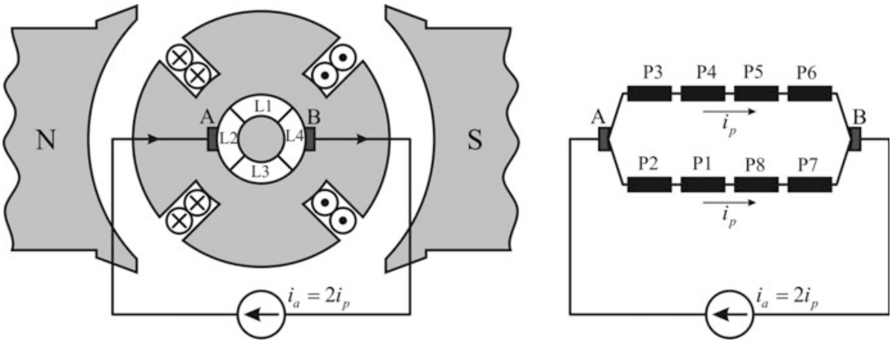

Fig. 11.10 Rotor position and electrical connections after the rotor has moved by π/4 + π/4 with respect to position shown in Fig. 11.8

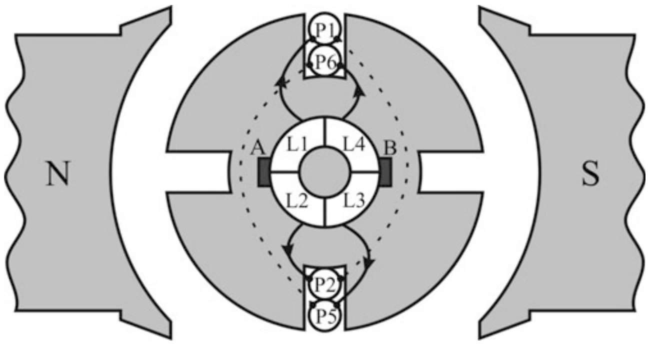

Fig. 11.11 Position of short-circuited rotor turns during commutation. The turns P1–P2 and P5–P6 are brought into short circuit by the brushes *A* and *B*, respectively

For the considered rotor, commutation repeats four times per each mechanical turn. It is necessary to analyze the problems associated with periodic short circuits of the rotor turns in the process of commutation. Starting from the scheme of placing the rotor conductors into slots, shown in Fig. 11.6, it is concluded that, during commutation shown in Fig. 11.9, the turns P1–P2 and P5–P6 get short-circuited. At the same time, conductors P1, P2, P5, and P6 pass through the zone between the stator magnetic poles, halfway between the north and south pole, where the radial component of magnetic induction is small and changes sign. The place where the short-circuited turns are found during commutation is shown in Fig. 11.11, where the remaining turns are omitted. Contribution of stator excitation to magnetic induction in the air gap is the highest in the middle of stator magnetic poles. At places where conductors P1, P2, P5, and P6 are found in Fig. 11.11, the radial component of magnetic induction is close to zero. Therefore, the electromotive forces induced in these conductors are close to zero. As a consequence, short circuiting these turns does not produce any significant short circuit currents.

In practice, DC machines usually have considerably higher number of segments, rotor slots, and rotor conductors. Collector segments are then connected to rotor conductors in the manner explained later on. Whatever the number of segments, the process of commutation occurs when carbon brushes A and B pass from one collector segment to the other. The collector revolves along with the rotor, and its segments slide below the brushes at the speed determined by the rotor motion.

The number of commutations during one mechanical turn of the rotor is determined by the number of segments, and there are usually several tens of them. In all versions of the rotor construction, collector segments are connected to rotor conductors. The latter are connected in series, and they form turns. Several series-connected turns can be made, and they are also called *section*. Hence, one part of the rotor winding is connected between each pair of neighboring segments, and this part can be a single turn or a multiple turn section. Whenever the brush touches two adjacent segments, considered part of the rotor winding gets short-circuited. The short circuit current between the adjacent segments is established through the brushes. The current depends on the electromotive force induced in the short-circuited turns and the equivalent impedance of these turns. For this reason, it is of uttermost importance to have a very low or none electromotive force in short-circuited windings. For that to achieve, DC machines are designed and made so that there is no electromotive force in the rotor turns while they get short-circuited by brushes. An electromotive force in a short-circuited turn would lead to short circuit currents through the brushes, sparking, electric arc, and eventually damage of both brushes and collector.

Figure 11.11 illustrates the commutation process in a machine with 4 segments and 4 rotor slots. The turns brought into short circuit by the brushes are in the region between stator magnetic poles, where the radial component of the magnetic induction has negligible values. The same effect should be accomplished in all DC machines. In machines having a large number of rotor segments, brushes may be wider and extend over two or more segments. In this case, several segments are brought into short circuit by one brush. All conductors belonging to short-circuited turns in the course of commutation have to be away from the stator magnetic poles, in the region between the poles, where the induction is negligible. The area between the magnetic poles is called *neutral zone*. This will be dealt with in the subsequent sections.

11.8 Operation of Commutator

Mechanical commutator of DC machines performs the function of converting DC currents, supplied from DC power source via brushes, into periodic currents which exist in rotor conductors. Change of the current in rotor conductors is shown in Fig. 11.12b. Direction of this current changes synchronously with the rotor motion. Between the commutation intervals, denoted by shaded areas in the figure, the current i_c is equal either to $+i_a/2$ or $-i_a/2$. The commutation intervals are relatively

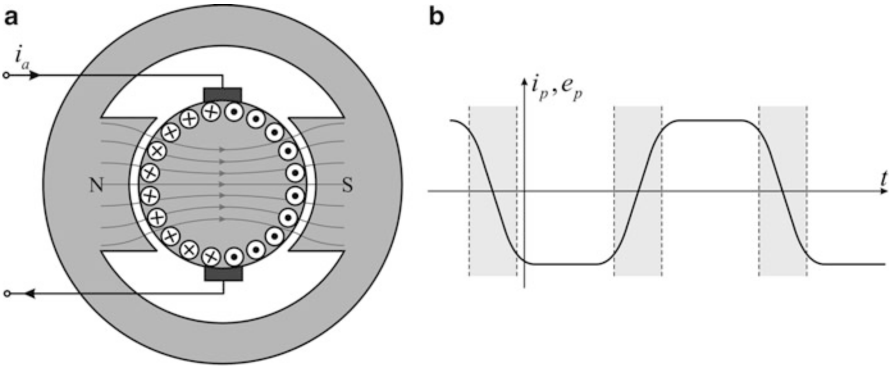

Fig. 11.12 Commutator as a DC/AC converter. (**a**) Distribution of currents in rotor conductors. (**b**) Variation of electromotive force and current in a rotor conductor. *Shaded* intervals correspond to commutation

short with respect to the period. Thus, the shape of rotor currents is close to a train of rectangular pulses having amplitude $i_a/2$. Rotor currents are not sinusoidal. Current i_p in Fig. 11.12b is a symmetrical, periodic current with average value equal to zero.[1]

Therefore, the commutator shown in Fig. 11.12a is a mechanical converter which converts DC currents to alternating currents (AC). Frequency of currents carried by rotor conductors is determined by the rotor speed.

In cases where DC machine is used as generator, rotor is put to motion by means of driving torque obtained from a water turbine or steam turbine. Rotor conductors revolve in magnetic field created by stator excitation. Relevant magnetic induction is proportional to the stator flux. Electromotive forces induced in rotor conductors are proportional to magnetic induction and peripheral speed. Under the north pole, magnetic induction has opposite direction with respect to that under the south pole. For this reason, electromotive forces induced in conductors below the two poles have different signs. While the rotor turns, each rotor conductor passes below stator poles with a period determined by the rotor speed. Therefore, electromotive force induced in a single conductor changes periodically. Its average value is equal to zero while its frequency depends on the rotor speed. Sample electromotive force induced in one conductor due to rotor motion is shown in Fig. 11.12b. It is shown in following sections that AC electromotive forces in rotor conductors result in a DC electromotive voltage measured between brushes A and B. This AC/DC conversion of voltages takes place due to the action of mechanical commutator which adds the AC electromotive forces in individual rotor conductors in such way that a DC electromotive force appears between the brushes. Hence, the commutator acts as a rectifier.

[1] In a broader sense, it is possible to call it *alternating current*. Strictly speaking, only sinusoidal functions of time are understood as alternating currents, called sinusoidal or harmonic currents.

11.9 Making the Rotor Winding

For better understanding of DC machines and for studying the function of the mechanical commutator, this section studies a sample rotor winding with 8 slots and 8 conductors connected to mechanical commutator with 4 collector segments and two brushes. To facilitate understanding of the way the conductors are connected to the collector, it is necessary to present rotor winding in *unfolded form*, quite similar to unwrapping the rotor cylindrical surface and presenting it in the form of a flat rectangle. This presentation can be obtained by a thought experiment wherein the rotor cylinder is cut along the radius denoted by dotted line in Fig. 11.13a. It is necessary to imagine that the cylindrical rotor surface is being unfolded in the way shown in Fig. 11.13b. Finally, by bringing the cylindrical surface to a plane, *unfolded form* is obtained, given in Fig. 11.13c. Rotor conductors are shown on this unfolded drawing with magnetic poles of the stator shown on the top. It should be noted that conductors P1–P4 are shown under the north magnetic pole, as they are in Fig. 11.13a.

In Fig. 11.15, the segments are denoted by L1–L4, while the conductors are denoted by P1–P8. Conductors P1–P4 carry electrical currents in direction ⊗, and they reside under the north stator pole. Conductors P5–P8 carry electrical currents in direction ⊙, and they reside under the south stator pole. While denoting direction of electrical currents, it is assumed that the reader is at the front side of the rotor, looking at the mechanical collector, as shown in Fig. 11.14. Sign ⊗ designates electrical current directed from front part (collector) to the rear part of the rotor, while sign ⊙ designates electrical current directed from the rear part of the rotor toward the front side and toward the reader.

The four segments shown in the figure can be obtained by splitting a metal ring into four equal arcs and by putting electric insulation layers in between. The segments can also be presented in unfolded form, using the same approach (Fig. 11.15). It should be noted that the width of the unfolded drawing corresponds to the rotor circumference, namely, the unfolded drawing is 2π wide. One can also consider that the horizontal axis in Fig. 11.15 corresponds to angular change from 0 (left) to 2π (right). Unfolded presentation in Fig. 11.15 shows the conductors, brushes, and collector segments. On the top is the position of the magnetic poles. The poles (N, S)

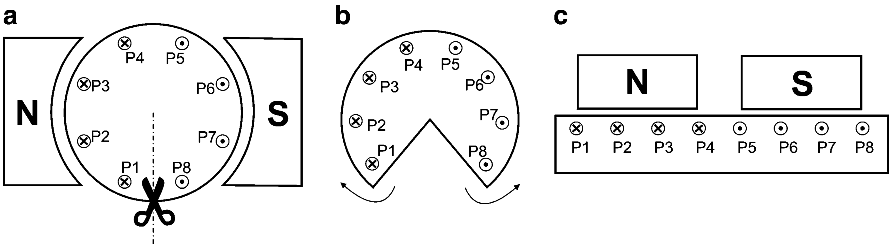

Fig. 11.13 Unfolded presentation of the rotor

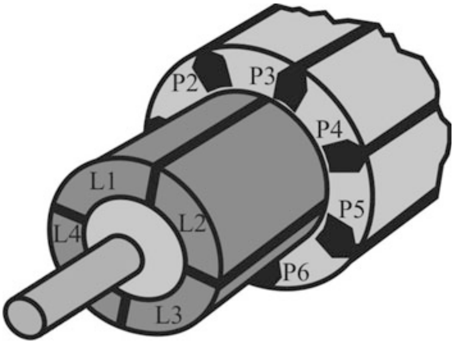

Fig. 11.14 Rotor of a DC machine observed from the front side

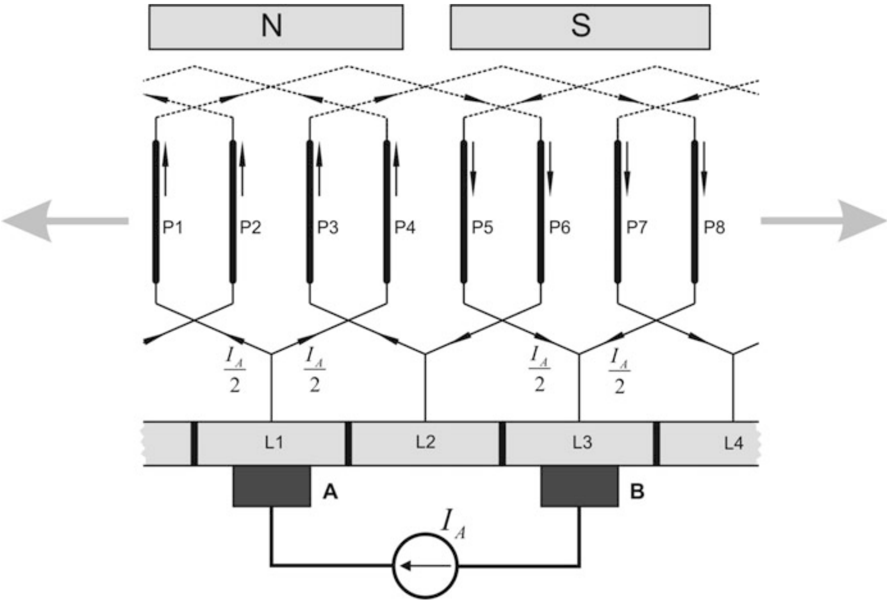

Fig. 11.15 Unfolded presentation of rotor conductors and collector segments. The brushes *A* and *B* touch the segments *L1* and *L3*

and the brushes (A, B) do not move. In Fig. 11.15, the rotor motion can be envisaged as a parallel transition of conductors P1–P8 and segments L1–L4 toward left or right.

At considered position, conductors P1–P4 are below the north magnetic pole. The brushes A and B touch the segments L1 and L3, respectively. When the rotor moves, there is relative movement of the collector segments and rotor conductors with respect to the stator. Magnetic poles of the stator and brushes are fastened to the stator, and they do not move. The effects of the rotor motion in Fig. 11.15 are manifested as translation of the rotor conductors and collector segments.

Direction of such translation depends on the rotor speed. The rotor motion observed from the front side as CW (clockwise) moves the conductors and segments in Fig. 11.15 toward right.

The figure presents the instant when brush A touches segment L1, while brush B touches segment L3. Conductors P1÷P8 are drawn by thick lines while their internal connections and connections to the segments are drawn by thin lines. The change in the line thickness is used to enhance clarity of the drawing, and it does not imply any change in the cross section of relevant conductors and wires. The dotted lines indicate connections effectuated at the rear side of the rotor and hence invisible from the front.

Brushes A and B are connected to a source of constant current I_a. In the considered position, brush A touches segment L1 which is connected to conductors P1 and P4. Starting from brush A, the current splits in two parallel branches. Each of these branches carries one half of the source current, $I_a/2$.

In the first branch, current $I_a/2$ passes to rotor conductor P1 and has direction from collector (front side) toward rear side of the rotor. At the rear side, conductor P1 is connected to conductor P6 by end turn which is marked by the dotted line. Direction of current through conductor P6 is toward the reader. Reaching the front side, conductor P6 gets connected to segment L2. In the present rotor position, the segment L2 is not connected to any of the brushes. Therefore, the current of P6 is passed to conductor P3, proceeding in direction from the front to the rear. At the rear side of the rotor, conductor P3 is connected to conductor P8 which carries current $I_a/2$ toward the collector and gets connected to the segment L3 and brush B.

In the second branch, current $I_a/2$ passes through conductors P4 and P7, gets connected with the segment L4, continues through conductors P2 and P5, and ends up in the segment L3 and brush B. Hence, the two parallel branches meet at the segment L3, adding up into current I_a which passes to brush B and returns to the source. Taking into account positions of conductors given in Fig. 11.13, it can be concluded that the collector directs source current to rotor conductors in such way that all conductors below the north magnetic pole of the stator have electrical current $I_a/2$ directed from the front side (collector) to the rear side of the machine. All the conductors below the south pole have the current of the same intensity in the opposite direction, from the rear to the front.

As the rotor turns, conductors P1–P8 and segments L1–L4 on unfolded drawing in Fig. 11.5 move toward left or right, while the brushes and the stator poles remain still. When the rotor moves by $\pi/4$ toward left, the brush A gets in touch with segment L2, while the brush B gets in touch with segment L4. At this new position of the rotor, distribution of currents in rotor conductors is given in Fig. 11.16. Conductors P3, P4, P5, and P6 are below the north pole of the stator. Direction of current in these conductors is from the reader toward the rear side of the rotor. Conductors P7, P8, P1, and P2 are under the south magnetic pole, and they carry currents in the opposite direction. By comparing this with the previous case (Fig. 11.15), it can be concluded that rotation of the rotor leads to variation of electrical currents in individual rotor conductors, but it does not change distribution of rotor currents observed from the stator side. In other words, irrespective of the

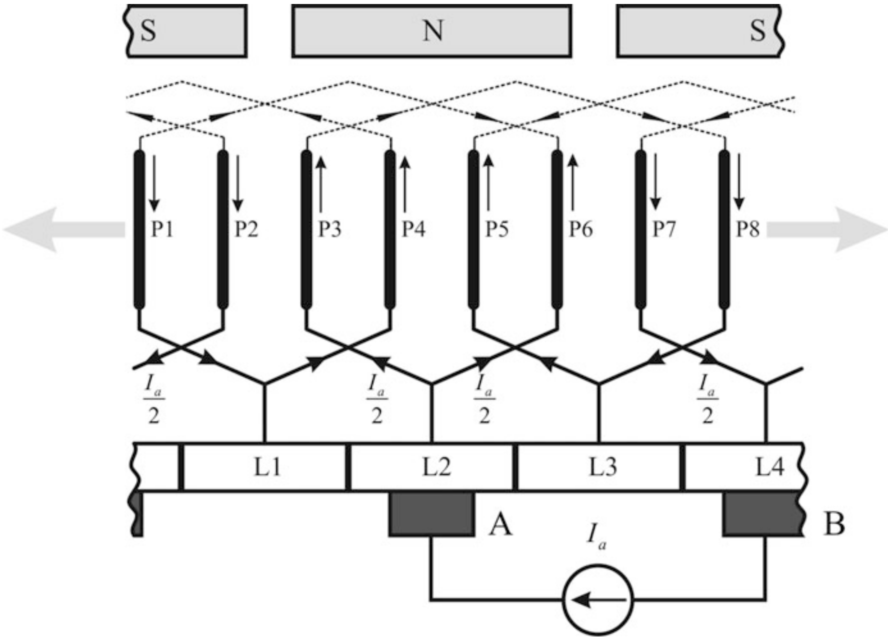

Fig. 11.16 Directions of currents in rotor conductors at position where brush *A* touches the segment *L2*

rotor motion, currents in conductors below the north magnetic pole retain direction ⊗, while currents in conductors below the south magnetic poles retain direction ⊙. In this way, rotation of the rotor does not change the course and direction of the rotor magnetomotive force and the rotor flux which remain unmoving with respect to the stator. Hence, the mechanical commutator insures that the rotor flux vector remains still with respect to the stator flux vector.

Selecting one of rotor conductors and tracking the change in its electrical current as the rotor completes one mechanical turn, one comes to conclusion that the conductor has alternating current and that one mechanical turn corresponds to one period of the current. Hence, mechanical commutator can be envisaged as a device which converts DC source current I_a into an alternating current.

The method of making the rotor winding is shown in Fig. 11.15. It starts by connecting the conductor P1 with the segment L1 at the front side of the machine and proceeds with putting an end turn at the rear side which connects P1 to P6. It is of interest to notice that the connection P1–P6 at the rear of the machine is realized by connecting the end of P1 to the *fifth conductor to the right*. Further on, the front side of the conductor P6 is connected to the segment L2, and this connection involves the front end of the conductor P3, the *third to the left*. The making of the winding proceeds in the same manner until all the conductors are connected. All the rear side connections are made by jumping to the *fifth conductor to the right*. All the front side connections are made by jumping to the *third conductor to the left*.

Besides, all the front side connections between the two conductors involve as well connection to one collector segment. It should be noted that the turns made in the prescribed way are fractional pitch turns. Full pitch turn would involve jumping to the fourth conductor (i.e., slot), whether to left or right.

Explained is the basic principle of making the rotor winding. In practice, there are a number of schemes being used. Most DC machines have more than eight slots. The approach can be generalized. With rotor having $2N$ slots, connections at the rear side are made with conductor in $(N+1)$-th slot to the right, while connections at the collector side are made with conductor in $(N-1)$-th slot to the left.

One slot usually accommodates several conductors. Therefore, the process of making the rotor winding and connecting the winding to collector segments is more complicated than the one illustrated in Fig. 11.15. Instead of placing only one turn in two slots, it is possible to prepare a *section* made of several turns and placing the two sides of this section in two slots. It is also possible to place two groups of conductors, each belonging to different sections, into the same slot. In such cases, the winding is said to have *two layers*. The way of connecting the rotor conductors shown in Fig. 11.15 results in a *lap winding*. There are also other ways, such as wave windings.[2]

Figure 11.17 shows the front view of the rotor which is also given in Fig. 11.15 in its unfolded form. Signs $\otimes$ and $\odot$ are associated with the rotor conductors. Full lines marked by arrows show connections of rotor conductors at the front side. Dotted lines show the connections between rotor conductors at the rear side of the machine. Designation P1P6 next to the dotted line marks that this is connection between conductors P1 and P6 made at the rear side of the rotor. A comparison of this presentation with the one in Fig. 11.15 illustrates the merits of the unfolded scheme.

11.10 Problems with Commutation

By considering the example of a rotor winding having 8 conductors and 4 collector segments analyzed in the preceding section, it is concluded that at each instant, there are two parallel branches between brushes A and B. In each of them, there are 4 conductors connected in series. Current in the rotor conductors is equal to one half of the current taken from the source which is connected to the collector brushes and which feeds the rotor winding.

When brush A passes from segment L1 to segment L2, brush B passes from segment L3 to segment L4. Passage of brushes from one segment to another leads to changes of direction of electrical currents in individual rotor conductors. In the course of commutation, shown in Fig. 11.18, brush A makes a short circuit between

[2] More details on windings of electrical machines can be found in publication Pyrhonen J, Jokinen T, Hrabovcova V (2008) Design of rotating electrical machines. Wiley, ISBN: 978-0-470-69516-6

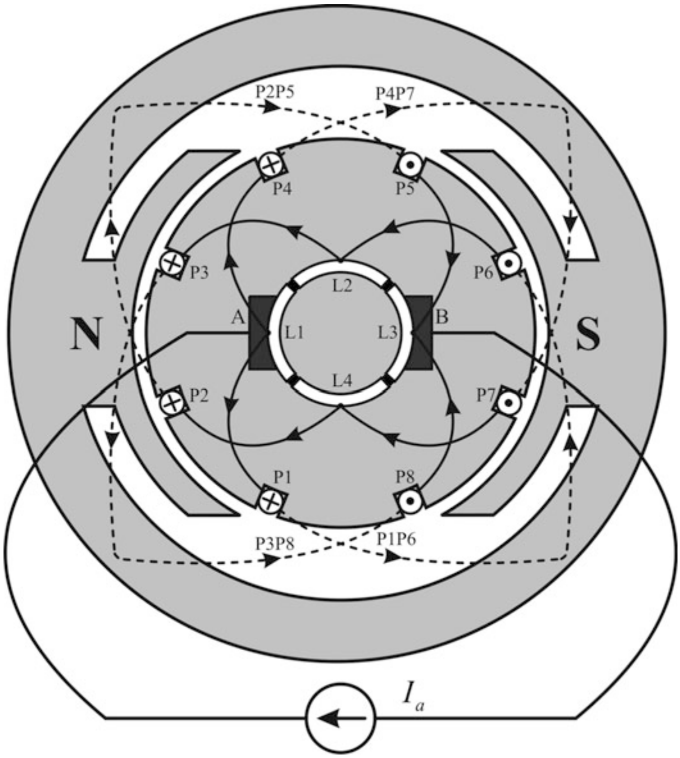

Fig. 11.17 Front side view of the winding whose unfolded scheme is given in Fig. 11.15

segments L1 and L2 while brush B makes a short circuit between segments L3 and L4. Owing to the short circuit between the adjacent collector segments, one rotor turn made of conductors P1 and P6 is short-circuited by the brush A, while the turn P2–P5 is short-circuited by the brush B.

At position presented in Fig. 11.15, direction of electrical current in conductors P1 and P2 is ⊗, while in conductors P5 and P6 direction of current is ⊙. When the rotor moves by π/2 and arrives at position presented in Fig. 11.16, direction of electrical current in conductors P1 and P2 is changed to ⊙, while direction of current in conductors P5 and P6 is changed to ⊗. Therefore, during commutation, direction of current is changed in those parts of the rotor winding which are short-circuited by the brushes. In Fig. 11.18, conductors of short-circuited turns are drawn by thicker lines.

In the case where, at the same time, the electromotive force in turn P1–P6 assumes significant value, a short circuit current will be established through the brush A, limited only by the impedance of the turn. In the same way, electromotive force in turn P2–P5 results in short circuit current through the brush B. The short circuit current is determined by the ratio of the electromotive force and the equivalent impedance of the short-circuited turns. Short circuit currents in the turns that

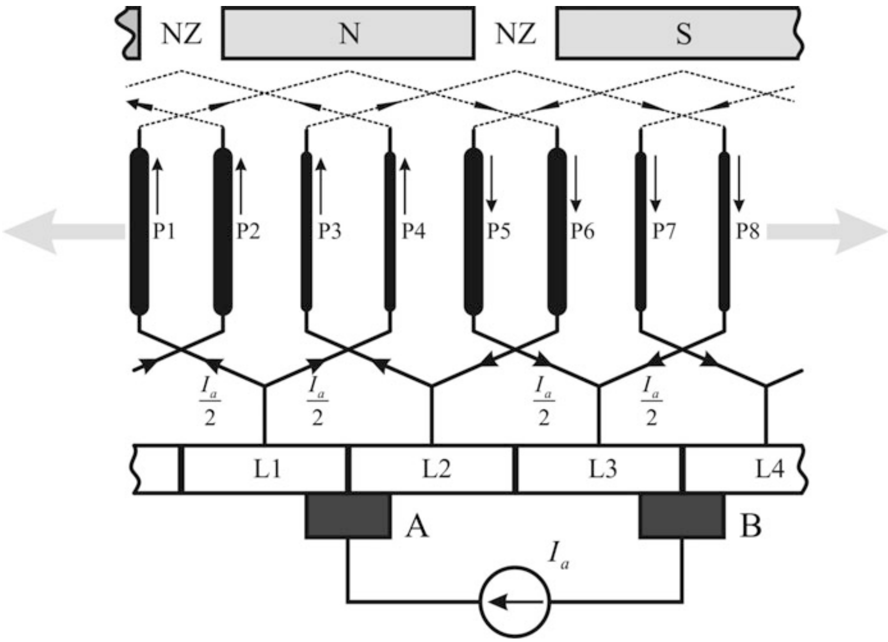

Fig. 11.18 Short-circuited segments *L1* and *L2* during commutation

commutate increase the total currents in the brushes and, therefore, increase the current density at the contact surface between the brushes and the segments. Excessive overheating of the brushes can result in an electric arc between the brushes and the segments, as well as between the adjacent segments. Sparking and arc can result in accelerated ware of the brushes and eventual damage to the collector. If the commutation is inadequate, there is a permanent electric arc between the brushes and collector segments. In such case, the collector and brushes overheat. Sparking and arc produce considerable quantity of ionized particles in close vicinity of brushes. Particles of ionized gas created under the brush A adhere to the collector surface. Due to rotation, they get carried away toward the brush B. In cases where the commutation is severely impaired, the electric arc may extend between brushes A and B. In this state, called *circular arcing*, the brushes and the rotor power supply I_a are in short circuit. At the same time, the rotor winding gets short-circuited. Prolonged operation in this mode leads to permanent damage to the winding and to mechanical collector and presents a fire risk.

Inadequate commutation leads to an increase of losses, damages collector and brushes, and may result in circular arcing and permanent damage to the machine. For this reason, it is significant that the electromotive forces in short-circuited turns are kept close to zero during their commutation (contours P1–P6 and P2–P5 in Fig. 11.18). In the position shown in this figure, relevant conductors are found between the stator magnetic poles in the neutral zones denoted by NZ. In the cross section of the machine, shown in Fig. 11.17, the neutral zones are in the upper part

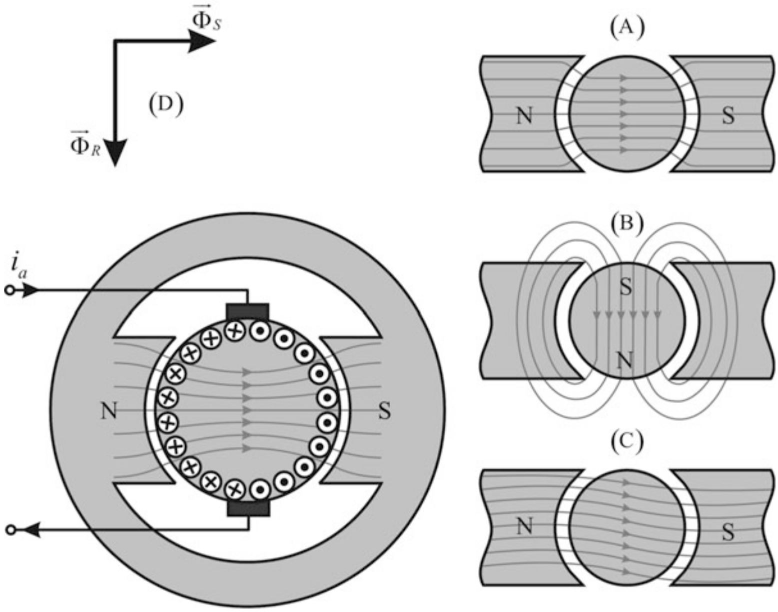

Fig. 11.19 Armature reaction and the resultant flux

of the rotor cylinder, at an angle of π/2 with respect to the brushes. The distance between the rotor and stator magnetic circuit is very large in the neutral zone. The radial component of the magnetic induction faces a very large magnetic resistance. At the same time, magnetomotive force of the stator excitation does not contribute to radial fields in the neutral zone. For this reason, it can be considered that radial component of magnetic induction in neutral zones is very small. With negligible magnetic induction, electromotive forces induced in rotor conductors passing through neutral zones are of little consequence as well. Therefore, the assumption is justified that electromotive forces in rotor turns involved in commutation process can be neglected.

Cross section of the machine is shown in Fig. 11.19, along with the lines of the stator magnetic field. The lines come out of the north magnetic pole, pass through the rotor magnetic circuit, and enter the south magnetic pole of the stator. It is justified to assume that there are some lines of the field passing by the conductors which are located in the neutral zone. They extend in tangential direction which is collinear with the vector of the peripheral speed. Electrical field induced due to motion depends on the vector product of the speed and magnetic induction. As these two vectors are collinear, the vector product is equal to zero, as well as induced electrical field. In the absence of induced electrical field, electromotive force in relevant conductors is equal to zero. Hence, tangential component of the magnetic field in the neutral zone does not induce any electromotive force in rotor turns that are short-circuited by brushes in the course of commutation.

Conductors P3, P4, P7, and P8 are in the zones of magnetic poles, where significant component of the radial magnetic induction is present; thus, the induced electromotive forces in these conductors are proportional to the speed of rotation, magnetic induction, and length of the conductors. The conductors of the turn P3–P8 are not below the same magnetic pole, and the electromotive forces induced in these conductors are of opposite directions. Yet, the way of connecting them in series (Fig. 11.18) leads to actual adding of their electromotive forces. Neglecting the voltage drop on resistances, the voltage which appears across segments L2 and L3 is double the electromotive force induced in one conductor.

Commutation without excessive short circuit currents and with no sparks requires that DC machine has sufficiently wide *neutral zone* between the stator magnetic poles where the magnetic induction is close to zero. The way of connecting the rotor conductors to the commutator segments must ensure that all the rotor turns involved by the commutation process and short-circuited by the brushes have their conductors in the neutral zone. The short circuit is created by the brushes during intervals when they touch two adjacent segments. Therefore, angular width of the stator magnetic poles should be less than π in order to allow for two neutral zones between the poles. As a consequence, one part of rotor conductors will always be in the neutral zone, where the magnetic induction has very low value. Increasing the width of neutral zones reduces the problems associated with commutation. At the same time, it reduces the number of rotor conductors which are encircled by the stator magnetic field and which contribute to the electromechanical conversion and torque generation.

Question (11.1): In Fig. 11.18, short-circuited turns P2–P5 and P1–P6 contain the conductors placed at the edges of neutral zones, in the vicinity of magnetic poles. Discuss the risk that, due to vicinity of magnetic poles, electromotive forces are induced in short-circuited windings.

Answer (11.1): Figure 11.18 is drawn in the way that conductors P1, P2, P5, and P6 are at the edges of neutral zones, in the vicinity of magnetic poles. It is justified to expect that, at this position, the radial component of magnetic induction is higher than in the middle of the neutral zone. Conductors P2 and P5 make one turn which is short-circuited by the brush B (Fig. 11.18). They are laid at the edges of the north magnetic pole, symmetrically with respect to the pole. Vicinity of the magnetic pole contributes to an increased magnetic induction. Because of the symmetry, in positions where conductors P2 and P5 are placed, radial component of the magnetic induction has the same value. For this reason, the electromotive forces induced in conductors P2 and P5 are of equal amplitude and direction. Conductors P2 and P5 are series connected and make short-circuited turn P2–P5. Connections of conductors P2 and P5 are such that their electromotive forces subtract and cancel. Therefore, the electromotive force of the turn P2–P5 in position shown in Fig. 11.18 is equal to zero. This is due to the fact that conductors P2 and P5 are symmetrical with respect to the north magnetic pole. In all cases where short-circuited conductors, such as P2 and P5, come at the very edge of the neutral zone, in close vicinity of the stator magnetic poles, it is possible to have considerable electromotive forces induced in such conductors. Yet, their symmetrical placement with respect to the pole ensures cancelation of their electromotive forces. The electromotive force in short-circuited turn P2–P5 is equal to zero. The same conclusion can be drawn for conductors P1 and P6.

11.11 Rotor Magnetic Field

Maintaining low intensities of the magnetic induction in neutral zones is hindered by the presence of the rotor magnetomotive force. Figure 11.19 shows a simplified presentation of the stator field, which propagates horizontally (a), and the rotor field, created by the current sheet, which propagates in vertical direction (b). It can be concluded that the rotor currents create the rotor flux which has its north and south poles in neutral zones, in the area comprising the rotor conductors involved in commutation and short-circuited by the brushes. The resultant magnetic field of the machine is the sum of the stator and the rotor field. It can be presented in the way shown in Fig. 11.19c. The stator and rotor fluxes can be represented by the two mutually orthogonal vectors designated by Φ_S and Φ_R.

Rotor of DC machine is also called *armature*, owing to the appearance of the rotor conductors which are shown in Fig. 11.5b. In the relevant literature, the term *induct* is also used to designate rotor of DC machines. In rotor conductors, electromotive forces are *induced,* proportional to the angular speed of rotation and to the stator flux, hence the term *induct*. Electrical current I_a fed to the brushes is often called *armature current* while the voltage U_a between the brushes A and B is called *armature voltage*. The magnetomotive force and flux created by the rotor currents are called *reaction of induct* or *armature reaction*. The term *reaction* is used due to the fact that the rotor electromotive force comes as a consequence of the stator flux. At the same time, the rotor currents get affected by this electromotive force. Since the rotor currents create the rotor flux, such flux is considered to be a *reaction* to the excitation coming from the stator. In a way, the stator flux *induces* electromotive forces in rotor conductors and affects the rotor current. For this reason, the stator is also called *inductor*.

The rotor flux (i.e., armature flux) is relatively small. The lines of the rotor field come out of the rotor magnetic circuit and enter a very large air gap in neutral zones. Therefore, the rotor flux passes through regions of very low permeability (μ_o) and very high magnetic resistance. For this reason, the value of magnetic induction in neutral zone is relatively small. Nevertheless, even a relatively small field in neutral zone may have undesirable influence on commutation. The presence of magnetic induction in neutral zone results in induced electromotive forces in rotor conductors passing through the neutral zone. These conductors are involved in the process of commutation. The turns made of such conductors are connected to adjacent collector segments, and they get short-circuited by the brushes. Short-circuited loop created in the prescribed way involves the rotor conductors, collector segments, and brushes. Induced electromotive forces create short circuit currents which may cause an electric arc at the contact between brushes and segments. For this reason, DC machines make use of additional elements intended to reduce magnetic induction in the neutral zones. DC machines may have *compensation winding* and *auxiliary poles* which are designed and made to suppress the armature reaction. They reduce magnetic induction in the neutral zone and ensure that commutation takes place with no sparks and no arcing.

11.12 Current Circuits and Magnetic Circuits

Structural elements of any electrical machine can be divided into magnetic and current circuits, the latter also called windings. In general, it is possible to identify four main items:

- Stator magnetic circuit
- Rotor magnetic circuit
- Stator current circuits
- Rotor current circuits

Figure 11.20 shows cross section of a DC machine presenting basic elements of current circuits and magnetic circuits of DC machines. The figure does not show the commutator which is described in the preceding sections. The rotor magnetic circuit (A) contains an opening in the center, intended for the shaft, and it has axial slots along the perimeter. Parts of the stator magnetic circuit are the main poles (B), yoke (C), and auxiliary poles (D). Rotor current circuit (F) includes conductors which are laid in slots on the rotor. They are connected in the way described in the preceding sections. Stator current circuits comprise the excitation winding (G), compensation winding (E), and auxiliary poles winding (H). A more detailed description and functions of these elements will be presented further on.

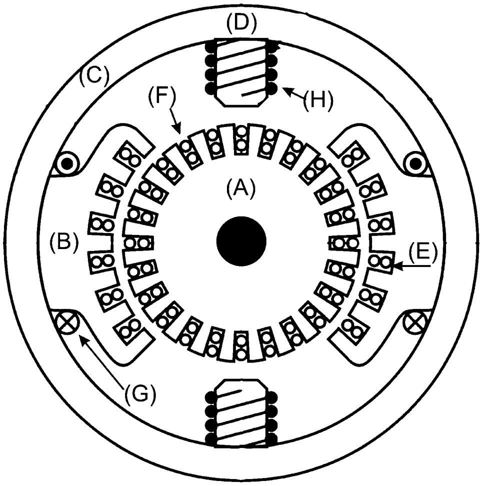

Fig. 11.20 Construction of a DC machine

11.13 Magnetic Circuits

The stator magnetic circuit contains the main poles, auxiliary poles, and yoke. The main poles direct the stator flux, also called *excitation flux*. Starting from the north magnetic pole of the stator, the flux passes through the air gap, goes through the rotor magnetic circuit, makes another passage through the air gap, enters the south magnetic pole of the stator, and then, via yoke, returns to the north pole. Within the stator magnetic poles, the flux does not change the course and direction; thus, the magnetic induction in the stator iron is constant. For that reason, there are no losses in iron. The stator magnetic circuit does not have to be *laminated*, that is, it does not have to be made by stacking iron sheets. Instead, it can be made of solid iron. The auxiliary stator poles are used to reduce the magnetic induction in the neutral zone, which will be explained later.

The rotor magnetic circuit is of cylindrical shape. The rotor flux does not move with respect to the stator. It remains still with respect to the stator and the stator flux. The rotor revolves in magnetic field created by the stator and rotor windings. As a consequence, direction of the magnetic field relative to the rotor magnetic circuit varies as the rotor turns. Namely, observer that revolves with the rotor experiences revolving magnetic field. As the field pulsates with respect to the rotor magnetic circuit, there are eddy currents and iron losses in the rotor. The frequency of the field pulsations depends on the rotor speed. Variable magnetic field produces both hysteresis and eddy current losses in rotor iron. In order to reduce these losses, the rotor is built by stacking iron sheets (lamination). The shape of these sheets is given in Fig. 11.20. Along the rotor perimeter, there are slots where the rotor conductors are placed. At the center of the rotor sheets, there is a round opening intended for fastening the shaft. Cylindrical magnetic circuit of the rotor is formed by stacking a large number of iron sheets of thickness less than 1 mm.

11.14 Current Circuits

Current circuits of the stator include the excitation winding, compensation winding, and winding of the auxiliary poles. The excitation winding has N_f turns around the main poles. Excitation current creates the magnetomotive force of excitation $F_f = N_f I_f$. All the quantities and parameters related to the excitation winding have the subscript *f* for *field*. Namely, the excitation winding provides the magnetic field of DC machine. Dividing the magnetomotive force by magnetic resistance of the magnetic circuit, one obtains the stator flux Φ_f, also called excitation flux. This flux is equal to the surface integral of magnetic induction B over cross section of the main poles. At the same time, it is equal to the surface integral of the magnetic induction over the surface extending in the air gap below the main poles. In Figs. 11.19 and 11.20, the excitation flux is directed from left to right and passes twice through the air gap of width δ. By neglecting H_{Fe} and the magnetic voltage drop in iron, the magnetic induction in the air gap below the main poles can be estimated as $B = \mu_o N_f I_f/(2\delta)$. Therefore, one can change the excitation flux Φ_f by varying the excitation current. In addition to excitation winding, stator has compensation winding and winding of auxiliary poles.

Compensation winding consists of conductors laid in slots made on the inner side of the main poles of the stator. This winding is connected in such way that its conductors carry electrical current $I_a/2$, the same current that is carried by rotor conductors. Direction of the current in conductors pertaining to compensation winding is opposite to direction of rotor currents. Since conductors of the rotor winding and those of the compensation windings are in close vicinity, their magnetomotive forces mutually cancel due to opposite directions of their electrical currents. Therefore, current in the compensation winding *compensates* and cancels the magnetomotive force of the rotor. This is done in order to reduce the magnetic induction in the neutral zone and to avoid short circuit currents in rotor turns involved in the process of commutation. The compensation winding cancels the magnetomotive force of all the rotor conductors located under the main stator poles. This does not include all the rotor conductors, as some of them are found in neutral zones, away from the main poles. The magnetomotive force of these conductors is not fully compensated by action of the compensation winding. Therefore, the auxiliary poles are also built and placed against neutral zones with the purpose to restrain the magnetic induction in these zones. Auxiliary poles have corresponding winding.

Winding of auxiliary poles has turns around the auxiliary poles magnetic core, denoted by (D) in Fig. 11.20. The air gap below the auxiliary poles is considerably wider compared to the air gap below main magnetic poles. This is done to increase the magnetic resistance encountered by the armature reaction. Current in the winding of the auxiliary poles is made proportional to the rotor current. Direction of this current and the number of turns are adjusted to achieve compensation of the magnetomotive force of the rotor which has not been compensated by action of the compensation winding. By joint action of the compensation winding and auxiliary poles, it is possible to control and suppress the magnetic induction in neutral zones, that is, below auxiliary poles. Essentially, the magnetic induction in the neutral zones should be reduced to a value close to zero. Detailed analysis of the process of commutation is not included in this book. Further study shows that the stress and ware of the brushes and collector segments is reduced in cases with *linear commutation*, where the current in short-circuited turns involved in commutation process varies from $+I_a/2$ to $-I_a/2$ in linear fashion. Prerequisite for linear commutation is establishing magnetic induction in neutral zones which has a very small value that varies in proportion to armature current I_a. Detailed analysis of the process of commutation and method of designing the compensation winding and the winding of auxiliary poles are beyond the scope of this book.

Rotor winding is formed by connecting rotor conductors which are placed in corresponding slots to the collector segments in the manner prescribed earlier. With N_R rotor conductors connected to segments, there are two parallel branches between brushes A and B at each instant, each branch having $N_R/2$ conductors. As stated before, the rotor winding is also called *armature winding*, while current I_a, fed to the brushes from an external source, is also called *armature current*. The terms *inductor* (for the stator) and *induct* (for the rotor) are also in use, since the electromotive force in the rotor is *induced* due to the stator flux. Thus, excitation current I_f is also called inductor current, while armature current I_a is called *induct current*. The magnetomotive force of the rotor and the rotor flux can be represented by the vectors of vertical direction (Fig. 11.19d) and are called the *magnetomotive force of induct*, *flux of induct* but also *reaction of induct* or *armature reaction*. Term *armature reaction* can be explained by taking example of a DC machine operating as a generator. If the brushes A and B of the generator are connected to a resistive load, the armature current and current in the rotor conductors are obtained by dividing the rotor electromotive force by the equivalent resistance of the electrical circuit. Hence, the armature current is proportional to the induced electromotive force. The electromotive force is proportional to the rotor speed and to the inductor (stator) flux. As a consequence, the rotor currents, magnetomotive force, and flux are all proportional to the stator excitation. Therefore, the rotor current and flux are apparent reaction to the stator excitation, which brings up the term *armature reaction*.

Question (11.2): Determine the excitation flux Φ_f of a DC machine. The excitation current is I_f, the number of turns in the excitation winding is N_f, the axial length of the machine is L, the rotor radius is R, the main north pole of the machine is seen from the center of the rotor at an angle α, while the air gap under the main poles is δ.

Answer (11.2.): The excitation current creates magnetomotive force $F_f = N_f I_f$. The excitation flux passes through the yoke and main poles, through rotor magnetic circuit, and it passes twice through the air gap under the main poles. Since magnetic field H_{Fe} in iron is negligible due to a very high permeability of iron, it is justified to assume that significant values of magnetic field H exist only in the air gap. Therefore, radial component of magnetic field in the air gap is $H = N_f I_f/(2\delta)$. Radial component of the magnetic induction in the air gap is $B = \mu_o N_f I_f/(2\delta)$. Under circumstances, magnetic induction under the main poles has a constant value. Therefore, the excitation flux is obtained multiplying the magnetic induction by the surface of the main poles. The surface of each main magnetic pole is $S = \alpha R \cdot L$. Eventually, $\Phi_f = \alpha R \cdot L \cdot \mu_o N_f I_f/(2\delta)$.

Question (11.3): For the machine described in the previous question, it is known that the rotor conductors carry electrical current $I_a/2$. There are 10 rotor conductors under the north pole of the stator and 10 conductors under the south pole of the stator. Calculate the electromagnetic torque of the machine.

Answer (11.3): The vector of magnetic induction is of radial direction; it is orthogonal to the conductor. The electromagnetic force acting on the straight conductor depends on the vector product of the magnetic induction $\boldsymbol{B}$ and the conductor length $\boldsymbol{l}$. Thus, the force acts in tangential direction. Relevant vectors are perpendicular, and the force $F = LBI_a/2$ acts on each of conductors. Contribution of each conductor to the total electromagnetic torque is $T_1 = RF = R\,LBI_a/2$.

The electromagnetic force acts only upon the conductors below the main poles. Namely, there is no force on conductors in the neutral zone where magnetic induction has negligible values. Therefore, the electromagnetic torque is $T = 20\ T_1 = 20{\cdot}R{\cdot}L{\cdot}B{\cdot}I_a/2$.

Question (11.4): The machine described in the preceding questions rotates at a constant speed of Ω_m. Assume that the brushes are disconnected from the power supply and that the voltage between the brushes is measured by a voltmeter. If the rotor winding is made to have two parallel branches between the brushes, determine the voltmeter reading.

Answer (11.4): The electromotive force is $E_1 = L{\cdot}v{\cdot}B$, where $v = R{\cdot}\Omega_m$ is the peripheral rotor speed. It is induced in conductors which are under the main stator poles. In neutral zones, magnetic induction is negligible, and the electromotive force induced in rotor conductors passing through neutral zones is very small and should not be taken into account. The rotor conductors are connected in series, and their electromotive forces add up. In Fig. 11.15, it is shown that all the conductors are split into two parallel branches. Series connection of conductors P1, P6, P3, and P8 has its ends connected to the brushes. At the same time, series connection of conductors P4, P7, P2, and P5 is connected between the brushes as well. Both branches with series-connected conductors are made in such way that the electromotive forces of individual conductors are added. The question concerns the machine having 10 conductors under each of the stator magnetic poles. Therefore, the total number of conductors having electromotive force E_1 is equal to 20. Since the conductors are split in two parallel branches, the electromotive force E_a is equal 10 E_1, namely, $10\ L{\cdot}R{\cdot}\Omega_m{\cdot}B$.

11.15 Direct and Quadrature Axis

Stator flux is also called excitation flux, or flux of the inductor. Magnetic axis that corresponds to the excitation flux is called *direct axis*. Within previous figures, the direct axis is set horizontally. As a rule, direct axis is determined by the position of the main stator poles. In Figs. 11.19 and 11.20, the armature reaction is directed along vertical axis. The rotor current sheet has electrical currents in the left-hand side of the cross section, directed away from the reader. In the right-hand side of the cross section, the currents are of the opposite direction, toward the reader. For this reason, the rotor magnetomotive force and flux, also called the armature reaction, can be represented by vectors in vertical direction. The axis of the armature reaction is called *quadrature axis*.

The stator auxiliary poles act along the quadrature axis, compensating the effects of the armature reaction. The same is the role of the compensation winding, whose conductors create a magnetomotive force along the lateral axis in the opposite direction of the reaction of induct. The purpose of the auxiliary poles and compensation winding is reducing the magnetic induction in the neutral zone, along the

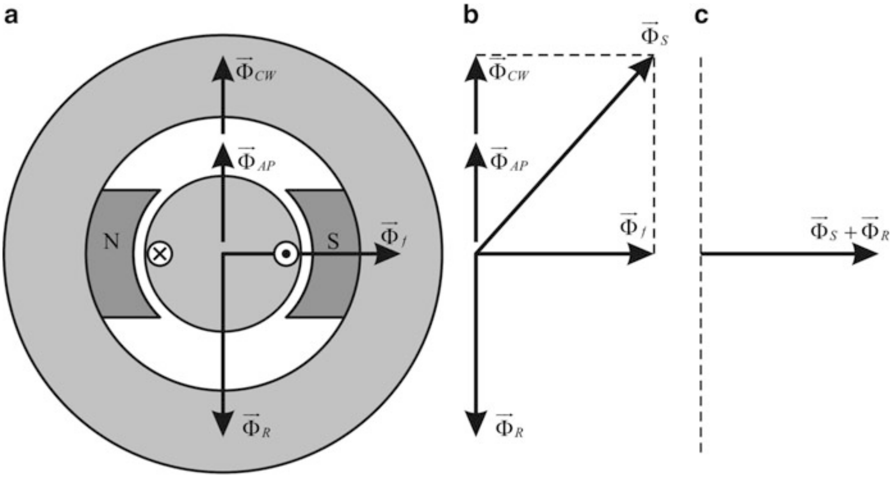

Fig. 11.21 Vector representation of the stator and rotor fluxes. (**a**) Position of the flux vectors of individual windings. (**b**) Resultant fluxes of the stator and rotor. (**c**) Resultant flux of the machine

quadrature axis. The compensation winding cancels the magnetomotive force of the rotor conductors passing under the main poles, while the auxiliary poles are built in neutral zones where they affect the magnetic induction and electromotive force induced in the turns involved in the process of commutation. By joint action of the compensation winding and auxiliary poles, the neutral zone has very low values of the resultant magnetic induction. The flux vectors representing the stator and rotor windings are shown in Fig. 11.21. Quadrature axis flux of the rotor is compensated by the quadrature axis flux of the stator.

11.15.1 Vector Representation

The rotor flux of a DC machine is also called armature reaction and it is represented by the flux vector Φ_R in Fig. 11.21. Along the quadrature axis there are vectors Φ_{AP} and Φ_{CW}, which represent fluxes of auxiliary poles and compensation winding. Direct axis of the machine is set horizontally, while quadrature axis is vertical. Directions of fluxes Φ_{AP} and Φ_{CW} are opposite to direction of the rotor flux Φ_R. This is due to the need to reduce the magnetic induction in neutral zones, accomplishing in this way an efficient commutation. In other words, it is necessary to reduce the resultant flux along the quadrature axis. Ideally, the compensation winding and auxiliary poles completely eliminate the armature reaction Φ_R, making the resultant flux along the quadrature axis equal to zero. As already stated, the compensation winding has conductors laid in the immediate vicinity of the rotor conductors, and they carry electrical currents of the same intensities but of opposite directions. The conductors are separated by a relatively small air gap; thus, the compensation winding efficiently cancels the magnetomotive force created by rotor conductors that are passing below the main poles. All the remaining rotor conductors are in neutral zones, between the main poles, out of the reach of the main poles, passing against the auxiliary poles. The action of the auxiliary poles is intended for further reduction of the magnetic field in neutral zones.

11.15.2 Resultant Fluxes

Figure 11.21a shows vectors representing the fluxes of individual stator and rotor windings. Part (b) of the figure shows the resultant flux vectors. The rotor has only one winding, the armature winding. Therefore, the resultant rotor flux is equal to the armature reaction Φ_R. The stator has three windings, the excitation winding, the compensation winding, and the winding on auxiliary poles. The resultant stator flux Φ_S is equal to the sum of the three fluxes: the excitation flux, the flux of compensation winding, and the flux of auxiliary poles.

11.15.3 Resultant Flux of the Machine

Resultant flux of the machine is equal to the vector sum of the fluxes in all the windings of the machine. Therefore, the resultant flux vector of the machine is equal to the sum of the resultant stator flux Φ_S and the rotor flux Φ_R. In cases where fluxes Φ_{AP} and Φ_{CW} compensate the armature reaction in full, the resultant flux along the quadrature axis is equal to zero, while the resultant flux along the direct axis is equal the excitation flux Φ_f, as shown in Fig. 11.21c.

Question (11.5): Figure 11.21 shows the case when the compensation winding and auxiliary poles eliminate the rotor flux Φ_R in full. By adding these two fluxes, one obtains the stator flux along the quadrature axis $\Phi_{AP} + \Phi_{CW}$ which has the same amplitude as the rotor flux Φ_R, but it has the opposite direction. For this reason, the total flux of the machine along the quadrature axis is equal to zero. From previous chapters, it is known that the electromagnetic torque of the machine is determined by the vector product of the stator and rotor fluxes. Does the fact that equivalent quadrature flux of the machine equals zero leads to conclusion that the electromagnetic torque of the machine is equal to zero as well?

Answer (11.5): It is necessary to note that the electromagnetic torque depends on the vector product of vectors Φ_R and Φ_S. Vector Φ_R represents the flux created by all the rotor windings, while vector Φ_S represents the flux created by all the stator windings. In the case of a DC machine, the rotor has only one winding, the armature winding, and the flux Φ_R is equal to the armature reaction flux. The stator has three windings. Therefore, the vector Φ_S represents the sum of the fluxes in these three stator windings, namely, the excitation winding, the compensation winding, and the winding of auxiliary poles. Electrical currents in conductors of the compensation winding and the winding of auxiliary poles create a quadrature component of the stator flux. In the considered case, the compensation winding and the winding of auxiliary poles create the flux along the quadrature axis which is of the same amplitude as the rotor flux but of the opposite direction. The sum of the fluxes created by the compensation winding and the auxiliary poles is $-\Phi_R$. Therefore, the resultant flux of the machine in quadrature axis is equal to zero. It is necessary to notice that the resultant flux of the machine comes as the sum of the fluxes of all the machine windings, residing on both stator and rotor. Although the resultant flux along the quadrature axis is zero, there still exists the rotor flux along the quadrature axis. For that reason, there still exists the possibility for the machine to generate the electromagnetic torque. The torque is determined by the vector product of the stator and rotor flux vectors. It depends on flux vectors Φ_R and Φ_S, shown in Fig. 11.21b. The angle between these two flux vectors is not equal to $\pi/2$, but the sine of this angle has a non zero value. The torque is determined by the product of amplitudes Φ_R and Φ_S and the sine of the angle between them. Therefore, the torque assumes a nonzero value in the case under consideration. The torque can be calculated as the product of the direct component of the stator flux Φ_f and quadrature component of the rotor flux Φ_R, both different than zero in the considered case. Therefore, despite the fact that the resultant quadrature flux of the machine is equal to zero, the electromagnetic torque of the machine is different from zero.

11.16 Electromotive Force and Electromagnetic Torque

Further study of DC machines requires the expressions for calculating the electromotive force and electromagnetic torque from the machine flux, current, and speed. For purposes of modeling, deriving the steady-state equivalent schemes, and constructing mechanical characteristics, it is necessary to derive the torque expression and the electromotive force expression for DC machines. The electromotive force E_a in armature winding is also called the rotor electromotive force and denoted by E_a, and it can be measured between the brushes A and B in conditions where the armature current I_a is equal to zero (no load condition). The electromagnetic torque and electromotive force should be expressed in terms of the armature current, excitation flux, angular speed of the rotor, and the machine parameters.

11.16.1 Electromotive Force in Armature Winding

In each of the rotor conductors passing under the main poles of the stator, there is an induced electromotive force $E_1 = R \cdot \Omega_m \cdot B \cdot L$, where Ω_m is angular speed of the rotor, R is the rotor radius, L is length of the rotor cylinder, while B is the radial

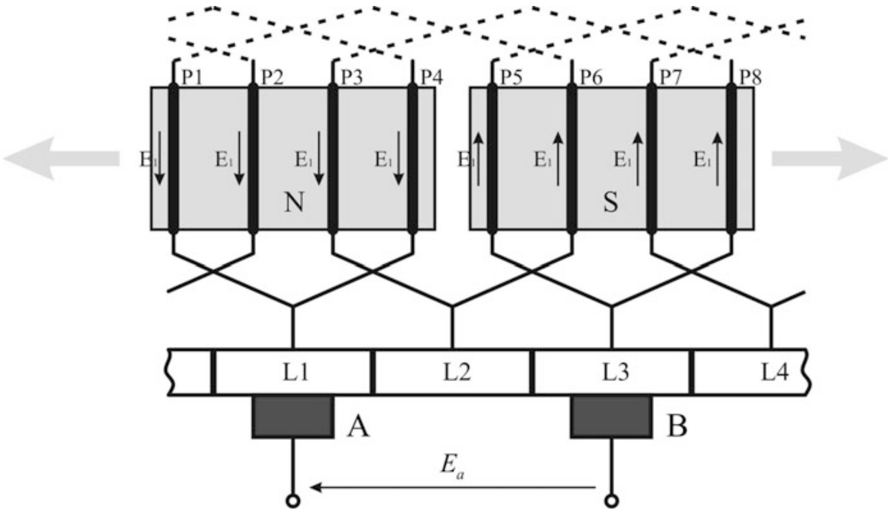

Fig. 11.22 Calculation of electromotive force E_a

component of the magnetic induction under the main poles. In conductors passing through the neutral zone, there are no electromotive forces because magnetic induction in neutral zones is negligible.

Conductors passing under the north magnetic pole have an electromotive force of the opposite sign with respect to conductors passing under the south magnetic pole. Therefore, as the rotor revolves, each conductor slides under opposite magnetic poles and has an electromotive force that changes its sign periodically, in synchronism with rotor mechanical turns. The frequency of the sign changes is determined by the rotor speed. An example of the change in electromotive force induced in a single conductor is given in Fig. 11.12b. The change of this electromotive force resembles a train of pulses. The pulse amplitude is E_1 while the sign changes in synchronism with the rotor motion. The sign changes as the considered conductor leaves the region under the north magnetic pole of the stator and enters the region under the south pole. It is shown hereafter that connections of rotor conductors and collector segments results in adding individual electromotive forces and provides a DC voltage between the brushes A and B. In the prescribed way, the mechanical commutator converts the AC electromotive forces of individual conductors into DC voltage available between the brushes.

Rotor conductors are divided in two parallel branches, and each of the branches is connected between the brushes. As the rotor revolves, individual conductors slide under the stator magnetic poles. At the same time, they pass from one of the branches to another. Namely, conductors do not appertain to any of the parallel branches for more than one half of the rotor mechanical turn. During the next half turn, the same conductor belongs to the other parallel branch. This occurs due to mechanical commutator and the process of commutation. Figure 11.23a shows wiring diagram of rotor conductors in rotor position given in Fig. 11.22, where

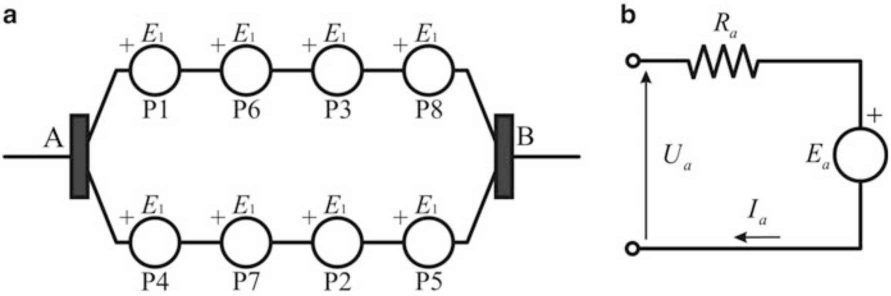

Fig. 11.23 (**a**) Addition of electromotive forces of individual conductors. (**b**) Representation of armature winding by a voltage generator with internal resistance

the brush A touches the segment L1. The two parallel branches are connected between the brushes A and B. These branches are P1-P6-P3-P8 and P4-P7-P2-P5. The electromotive force induced in conductors belonging to each of these branches is added to produce E_a, the electromotive force of the armature (rotor) winding. This electromotive force is equal to voltage U_a, measured between the brushes in no load condition, when the armature current is $I_a = 0$. In the position shown in Figs. 11.22 and 11.23, the electromotive forces in conductors P1, P2, P3, and P4 are of the same direction. These conductors pass under the north magnetic pole. Conductors P5, P6, P7, and P8 are passing under the south magnetic pole. In these conductors, the electromotive force has direction which is opposite with respect to the one induced in P1, P2, P3, and P4. When considering one of parallel branches, such as P1-P6-P3-P8, and taking into account the way of making their series connection in Figs. 11.22 and 11.23, it is concluded that electromotive forces of the four conductors are added, resulting in $E_a = 4E_1$.

It is of interest to show that E_a is a DC quantity and that the rotor motion does not lead to variation of E_a sign. When the rotor turns by $\pi/4$ with respect to position in Fig. 11.22, the brushes A and B get in touch with segments L2 and L4, as shown in Fig. 11.16. In this new position, conductors P3, P4, P5, and P6 pass below the north magnetic pole and therefore have their induced electromotive forces of the same direction. Conductors P7, P8, P1, and P2 pass below the south magnetic pole. Their electromotive forces have the opposite direction with respect to conductors passing under the north pole. At the same time, distribution of conductors between the two parallel branches changes. According to Fig. 11.16, conductors P3, P8, P5, and P2 make one parallel branch, while conductors P6, P1, P4, and P7 make the other parallel branch. The figure shows that the conductors are connected in the way that their electromotive forces are actually added; thus, the electromotive force E_a remains equal to $4E_1$, with brush A being at the higher potential than brush B. In other words, a DC voltage exists between the brushes A and B, notwithstanding the rotor motion. This shows that the mechanical commutator has the role of a rectifier which converts the alternating electromotive forces into DC electromotive force E_a. This electromotive force is called *rotor* or *armature* electromotive force.

If the brushes A and B are connected to an external resistor or other consumer of electrical energy which operates with DC currents, the machine shown in Fig. 11.23 will run as a generator, supplying electrical energy to the consumer. It will be shown later that in such case, it is required to drive the rotor by an external torque obtained from a hydro turbine, steam turbine, or internal combustion motor. The armature winding then represents a voltage generator whose terminals are available at brushes A and B and which produces a DC voltage U_a. The equivalent voltage generator is shown in Fig. 11.23b. Under no load conditions, voltage between the brushes is equal to the electromotive force $E_a = 4E_1 = 4R\Omega_m BL$. Internal resistance of the equivalent voltage generator R_a includes resistance of the brushes A and B, resistance of rotor conductors connected in two parallel branches, and a relatively small resistance of collector segments. It can be determined by measuring resistance between the brushes in conditions where the rotor speed is equal to zero ($\Omega_m = 0$). The electromotive force is then equal to zero, and the equivalent voltage source is reduced to internal resistance R_a. The value of R_a can be calculated for DC machine with eight rotor conductors, divided in two parallel branches with four conductor each, as shown in Fig. 11.23. If resistance of one rotor conductor is R_1 while the equivalent resistance of the brushes and collector is ΔR, then the internal resistance of the equivalent source is $R_a = 2R_1 + \Delta R$. Previous considerations show that the armature winding of DC machine can be represented by a voltage generator having no load electromotive force E_a and internal resistance R_a.

11.16.2 Torque Generation

It is required to determine relation between the electromagnetic torque, excitation flux, and *armature* current. The excitation flux from the main poles passes to the rotor magnetic circuit via air gap, where the magnetic induction is of radial direction. The surface separating internal side of the main pole from the air gap is of the form of a bent rectangle which represents a sector of the cylinder. The surface of this sector is $S = WL$, where L is length of the machine while $W = \alpha R$ is the width of the main pole, measured along its internal side which faces the air gap. The width W is one section of the circle having the radius R. To an observer positioned at the rotor center, the surface S is seen at the angle α. Magnetic induction B in the air gap below the main poles is equal to the ratio of flux Φ_f and surface S. The current in rotor conductors is $I_a/2$, where I_a is the *armature* current, fed to the brushes from an external source. Figure 11.24a gives an unfolded scheme of the rotor winding and shows forces acting upon its conductors. Part (b) of this figure shows directions of electrical currents in rotor conductors, seen from the collector side. At the given position of the rotor, conductors P1, P2, P3, and P4 are below the north pole and carry currents of direction $\otimes$. Lines of the field of magnetic induction come out of the main pole, denoted by N; they pass through the air gap, come across the rotor conductors, and enter the rotor magnetic circuit. In the considered zone below the north pole, the vector product of radial component

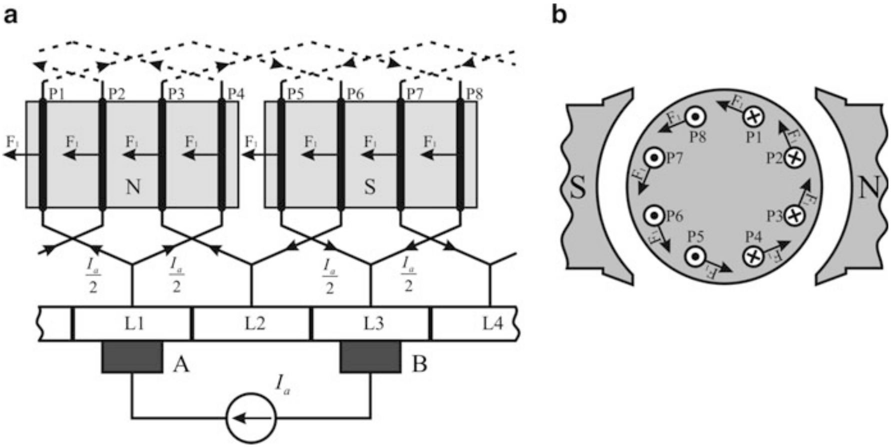

Fig. 11.24 (**a**) Forces acting upon conductors represented in an unfolded scheme. (**b**) Forces acting upon conductors. Armature winding is supplied from a current generator

of magnetic induction and coaxially directed current (⊗) gives tangential force, denoted by F_1. Below the south pole, direction of the current in conductors changes. At the same time, direction of the magnetic induction changes as well. It comes out of the rotor magnetic circuit, passes through the air gap, and enters the stator magnetic circuit (S). Change in directions of both the current and magnetic induction results in tangential force $F1$ which retains its original direction and acts in counterclockwise direction.

Individual forces contribute to electromagnetic torque. The arm of each force is equal to R. The torque contribution of each individual conductor is $T_1 = RF_1 = RL(I_a/2)B$. Since the total number of conductors is 8, the electromagnetic torque is equal to $T_{em} = 8RL(I_a/2)B = 2DLI_a\Phi_f/S$, where D, L, I_a,Φ_f, and S denote the rotor diameter, axial length of the machine, armature current, excitation flux, and cross-section area of the main poles, respectively.

11.16.3 Torque and Electromotive Force Expressions

In this section, induced electromotive force of the armature winding E_a is expressed in terms of the rotor speed and the excitation flux. Further on, the electromagnetic torque T_{em} is expressed in terms of the excitation flux and the armature current. To begin with, it is necessary to establish relation between the magnetic induction in the air gap, excitation current, and the excitation flux.

Figure 11.25 provides the form and dimensions of the main poles. The electromotive force and forces upon conductors depend on position where the conductors are located. Rotor conductors are positioned at the external surface of the rotor magnetic circuit; thus, it is necessary to determine magnetic induction in

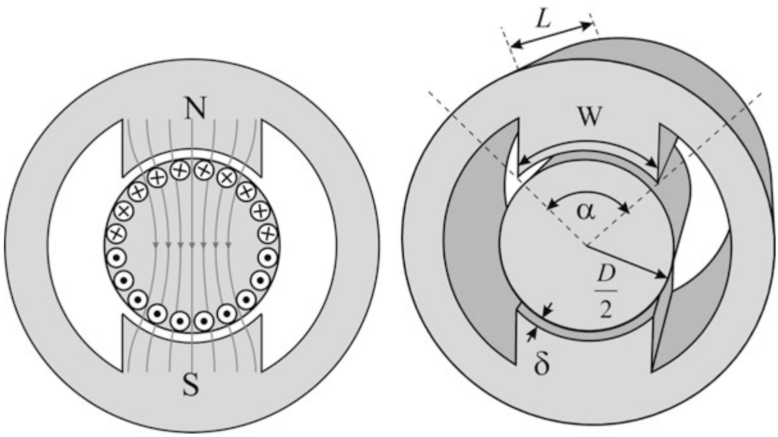

Fig. 11.25 Dimensions of the main magnetic poles

the air gap. The width δ of the air gap is much smaller than the radius R of the rotor. Boundary conditions for magnetic field can be applied to the surface separating the air gap from the ferromagnetic material. According to boundary conditions, magnetic induction on the air gap side of the surface is oriented in radial direction. As already shown, the cross-section S of the main poles is WL. Variable $W = \alpha D/2$ is the width of the main poles, while α is the angle covered by the main pole to an observer positioned at the rotor center. The excitation flux passes through the stator and rotor magnetic circuits where the magnetic field H_{Fe} is very small. The flux passes twice through the air gap; thus,

$$F_f = N_f I_f = 2 \cdot \delta \cdot H_f, \tag{11.2}$$

where $F_f = N_f I_f$ is the magnetomotive force of the excitation winding, while H_f is the intensity of magnetic field below the main poles. Magnetic induction B_f created by the excitation winding in the air gap, under the main poles is equal to

$$B_f = \mu_0 H_f = \mu_0 \frac{N_f I_f}{2 \cdot \delta}. \tag{11.3}$$

Magnetic induction in the air gap has the same value throughout the main pole cross section. Therefore, the excitation flux is equal to the product of the magnetic induction and surface area

$$\Phi_f = S B_f = L W \mu_0 H_f = \mu_0 \frac{L W N_f I_f}{2 \cdot \delta}, \tag{11.4}$$

where $W = \alpha D/2$ is the width of the main pole. Magnetic resistance along the path of the excitation flux is equal to the ratio of magnetomotive force F_f and flux Φ_f. Magnetic resistance R_μ is

$$R_\mu = \frac{F_f}{\Phi_f} = \frac{2 \cdot \delta}{\mu_0 \cdot L \cdot W}. \tag{11.5}$$

Total flux of the excitation winding is $\Psi_f = N_f \Phi_f$. Inductance L_f of the excitation winding is the ratio of the total flux and the excitation current, and it is determined according to expression

$$L_f = \frac{\psi_f}{I_f} = \mu_0 \frac{LWN_f^2}{2 \cdot \delta} = \frac{N_f^2}{R_\mu}. \tag{11.6}$$

Coefficient of proportionality between the excitation flux Φ_f, which represents the flux in one turn of the excitation winding, and excitation current I_f is equal to

$$L_f' = \frac{\Phi_f}{I_f} = \frac{\psi_f}{N_f I_f} = \mu_0 \frac{LWN_f}{2 \cdot \delta} = \frac{L_f}{N_f}. \tag{11.7}$$

The rotor comprises a total of N_R conductors, but some of them are not under the main poles, and they pass through the neutral zone between the main poles. The remaining rotor conductors are in the zone of the main poles, within the reach of the magnetic induction B_f. In the neutral zone, the conductors are not subject to any force and no electromotive force is induced in them. The rotor conductors are evenly distributed along the machine perimeter. Below the north pole of the width W, there are $N_R W/(\pi D)$ rotor conductors. The same number of conductors is found below the south magnetic pole.

11.16.4 Calculation of Electromotive Force E_a

In the rotor conductors influenced by the field of the main poles, the induced electromotive force E_1 is

$$E_1 = B_f \cdot L \cdot v = B_f \cdot L \cdot R \cdot \Omega_m, \tag{11.8}$$

where v is rotor peripheral speed, Ω_m is angular speed of rotor rotation, and R is radius of the rotor. Rotor conductors are connected in series in the way that the induced electromotive forces are added. Total number of conductors containing induced electromotive force is equal to the sum of conductors below the north and south poles, $2N_R W/(\pi D)$. Since all rotor conductors are connected in two parallel branches, while the branch terminals are brought to brushes A and B, electromotive force E_a is equal to the sum of electromotive forces in one of the branches. Induced electromotive forces exist in conductors under the main poles. Therefore, E_a is calculated as the product of the electromotive force E_1 in one conductor, and the number of conductors positioned below one of magnetic poles,

$$E_a = \frac{1}{2}\left(\frac{2N_R W}{\pi D}\right)E_1 = \frac{N_R W}{2\pi R} B_f L R \Omega_m = \frac{N_R}{2\pi}\left(LWB_f\right)\Omega_m. \tag{11.9}$$

By using relation (11.4) between the magnetic induction and excitation flux, previous expression takes the form

$$E_a = \frac{N_R}{2\pi}\Phi_f \Omega_m = k_e \Phi_f \Omega_m, \tag{11.10}$$

where coefficient $k_e = N_R/(2\pi)$ is determined by the number of rotor conductors and is called *coefficient of electromotive force*.

11.16.5 Calculation of Torque

Each conductor which passes through the zone of the main poles is subject to the force

$$F_1 = L\,\frac{I_A}{2} B_f = \frac{\Phi_f}{W \cdot L} \cdot \frac{I_A}{2} \cdot L = \frac{\Phi_f}{W} \cdot \frac{I_A}{2}. \tag{11.11}$$

The arm of the considered force is $R = D/2$, and its contribution to the total torque is equal to $T_1 = F_1\ D/2$. Since the force F_1 acts only on conductors positioned below the main poles, there are $2N_R W/(\pi D)$ conductors contributing to the torque. The sum of their contributions is

$$\begin{aligned} T_{em} &= \left(\frac{2N_R W}{\pi D}\right) \cdot T_1 = \frac{2N_R W}{\pi D} \cdot \frac{D}{2} \cdot \frac{\Phi_f}{W} \cdot \frac{I_A}{2} \\ &= \frac{N_R}{2\pi} \cdot \Phi_f \cdot I_A = k_m \cdot \Phi_f \cdot I_A, \end{aligned}$$

where $k_m = N_R/(2\pi)$ is the *coefficient of electromagnetic torque*.

Equation (11.10) shows that the electromotive force of the armature winding is determined by the product of the excitation flux and the rotor angular speed, while (11.12) shows that the electromagnetic torque of the machine is determined by the product of the excitation flux and the armature current.