# Chapter 3
# Magnetic and Electrical Coupling Field

Electromechanical conversion is based on forces and torques of electromagnetic origin. The force exerted upon a moving part can be the consequence of electrical or magnetic field. The field encircles and couples both moving and nonmoving parts of electromechanical converter. Therefore, the field is also called coupling field. In this chapter, some basic notions are given for electromechanical energy converters with electrical coupling field and converters with magnetic coupling field.

## 3.1 Converters Based on Electrostatic Field

Electromechanical conversion in electrostatic machines is based on electrical coupling field. The coupling field between moving parts is a prerequisite for electromechanical conversion. In an electrostatic machine, the field exists in the medium between mobile electrodes, and it causes electrical forces acting on the electrodes.

Preliminary insight in electromechanical energy conversion based on the electrical field can be obtained by considering the sample machine shown in Fig. 3.1, resembling the capacitor with two parallel metal plates. In the case when the plates are considerably larger compared to the distance between them ($S >> d^2$), the electrical field between the electrodes is homogeneous and equal to $E = U/d$ [V/m], where $U$ is the voltage between the electrodes. Electrical induction vector $\boldsymbol{D}$ [As/m$^2$] is obtained by multiplying the vector of electric field $\boldsymbol{E}$ by the permittivity of the medium $\varepsilon_0$. The force acting on the plates depends on the charge stored in the capacitor. If it is possible to move one of the plates, then the product of this force and the displacement gives mechanical work. The mechanical work can be obtained at the expense of the field energy or of energy of a source connected to the plates.

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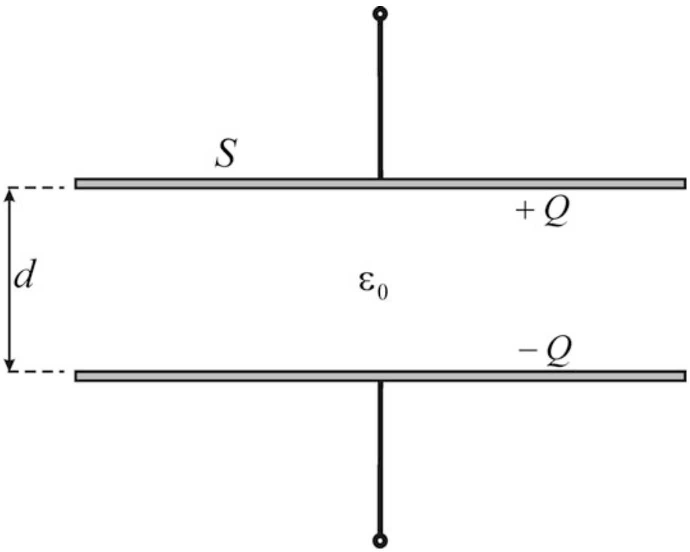


**Fig. 3.1** Plate capacitor with distance between the plates much smaller compared to dimensions of the plates

### *3.1.1 Charge, Capacitance, and Energy*

The electrical field in the interelectrode space is homogeneous. The field strength is determined by the ratio of the voltage and distance between the plates, $E = U/d$. Electrical induction $D$ is equal to the surface charge density $Q/S$. At the same time, the ratio $D/E$ is determined by permittivity (dielectric constant) $\varepsilon_0$.

$$E = \frac{U}{d}; D = \sigma = \frac{Q}{S} = \varepsilon_0 E; \Rightarrow Q = \varepsilon_0 ES = \varepsilon_0 S \frac{U}{d}. \tag{3.1}$$

Capacitance $C$ is determined by the ratio of charge $Q$ and voltage $U$. The capacitance depends on the plate surface $S$, distance $d$ between the plates, and permittivity of the dielectric material filling the interelectrode space:

$$C = \frac{Q}{U} = \varepsilon_0 \frac{S}{d}. \tag{3.2}$$

Total energy of the coupling electrical field can be obtained by integrating the energy density $w_e$ in the region where the electrical field extends. In the present case, the electrical field and the field energy exist within the interelectrode space. The energy density does not vary, and it is equal to $w_e = ½\varepsilon_0 E^2$. The volume of the

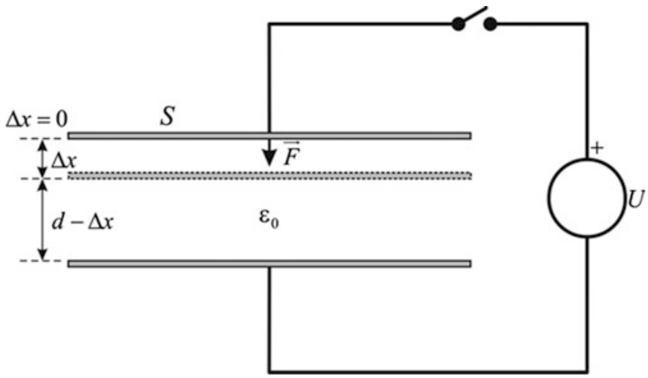


**Fig. 3.2** A capacitor having mobile upper plate

region is $V = Sd$. Therefore, total energy of the coupling electrical field is $W = ½CU^2 = ½Q^2/C$.

$$W_e = \int_V w_e \, \mathrm{d}V = \int_V \left( \int \vec{D} \cdot \mathrm{d}\vec{E} \right) \mathrm{d}V = \int_V \left( \frac{1}{2} \varepsilon_0 E^2 \right) \mathrm{d}V$$
$$= Sd \left( \frac{1}{2} \varepsilon_0 E^2 \right) = \frac{1}{2} C U^2 = \frac{Q^2}{2C}. \tag{3.3}$$

### 3.1.2 Source Work, Mechanical Work, and Field Energy

Figure 3.2 shows a charged capacitor having mobile upper plate. It can be shown that by moving the upper plate downward, electrical energy is converted to mechanical work. The electric charge on the plates is of opposite polarity. Therefore, they are subjected to a force of attraction $F$. If the upper plate moves downward and gets closer to the lower plate by $\Delta x$, mechanical work $F\Delta x$ is obtained. During the move, there is a change in the energy $W_e$ of the electrical coupling field. With the plates connected to the electrical source, the charge on the plates changes through an exchange of charges between the plates and the electrical source.

Electric force $\boldsymbol{F}$ acting on one plate of the capacitor can be determined by applying the method of *virtual works*, also called virtual disturbance method. It is necessary to envisage a very small displacement $\Delta x$ of the mobile plate toward the opposing plate. In such case, the direction of the force $F$ corresponds to the direction of the hypothetical displacement $\Delta x$. The method of virtual works proceeds with calculation of changes in the field energy and determines the work of the electrical source. The work $\Delta W_{meh} = F\Delta x$ is made by the electric force $\boldsymbol{F}$ during displacement $\Delta x$. The force can be calculated by dividing the work increment

$\Delta W_{meh}$ by the displacement $\Delta x$. The same virtual work method can be applied in cases when the mobile part of the electromechanical converter performs rotation. In such cases, the displacement $\Delta x$ is replaced by the angular shift $\Delta\theta$, while the mechanical works $\Delta W_{meh}$ assume the form $T_{em}\Delta\theta$. The symbol $T_{em}$ designates the torque generated by the electrical forces. The torque $T_{em}$ acts upon the moving (revolving) part and affects its speed.

A source of the constant voltage $U$, shown on the right-hand side of Fig. 3.2, can be connected to the plates by closing the switch. Reduction of the distance $d$ between the plates increases capacitance $C$. While the source is connected, the voltage between the plates is constant. Due to an increase of the capacitance, the charge on the plates $Q = CU$ increases. Therefore, the source supplies an additional charge $\Delta Q$. The work of the source is equal to $\Delta W_i = U\Delta Q$, while the obtained mechanical work is $\Delta W_{meh} = F\Delta x$. The work of the source increases total energy of the system, that is, the sum of the electrical energy and mechanical work. With a constant voltage, the electrical energy is given by (3.4).

In the next considerations, it will be shown that work of the source is divided in two equal parts, that is, $\Delta W_e = \Delta W_{meh} = ½\Delta W_i$.

If the switch in Fig. 3.2 is open, the source is separated from the plates, and the work of the source is equal to zero. Electrical charges on the plates cannot be changed, as well as the field $D$ between the plates ($Q$ = const., $D$ = const.). Therefore, the density of the field energy $w_e = ½D^2/\varepsilon_0$ remains unchanged.

By reducing the distance between the plates, the volume of the region comprising the electrical field is reduced as well. Therefore, the total field energy $\Delta W_e$ is also reduced. With the source separated from the system, reduction in the field energy yields the mechanical work $\Delta \mathrm{W}_{meh} = -\Delta \mathrm{W}_e$. In the case of a constant charge, the field electrical energy is given by (3.5):

$$W_e(\Delta x) = S(d - \Delta x)\left(\frac{1}{2}\varepsilon_0 E^2\right) = \frac{1}{2}CU^2 = \frac{U^2}{2}\frac{\varepsilon_0 S}{d - \Delta x}, \tag{3.4}$$

$$W_e(\Delta x) = S(d - \Delta x)\left(\frac{1}{2}\varepsilon_0 E^2\right) = \frac{Q^2}{2C} = \frac{Q^2}{2}\frac{d - \Delta x}{\varepsilon_0 S}. \tag{3.5}$$

### *3.1.3 Force Expression*

The machines operating with the electrical coupling field are called electrostatic machines. Domain with the electrical field is filled with dielectric material. Dielectric is called *linear* if the vector of electrical induction $\boldsymbol{D}$ is proportional to the vector $\boldsymbol{E}$, $\boldsymbol{D} = \varepsilon\boldsymbol{E}$. Electrostatic machine with linear dielectric is called *linear machine*. The structure shown in Fig. 3.2 represents a linear electrostatic machine with negligible energy losses. Therefore, in the case with $Q$ = const., the mechanical work $\Delta W_{meh} = F\Delta x$ is determined by $\Delta W_{meh} = -\Delta W_e$, whereas in the case of a constant voltage relation, $\Delta W_{meh} = +\Delta W_e = +½\Delta W_i$ applies. According to these expressions, the force can be determined as partial derivative of the coupling field energy $W_e$ with respect to coordinate $x$. This coordinate represents displacement of the mobile electrode along the motion axis of the system.

In the case when the source is disconnected, the system in Fig. 3.2 has a constant charge, and the work of the source $U$ is equal to zero. Applying the method of virtual works, the change in the field energy and the mechanical work are obtained from (3.6).

$$\begin{aligned} &\Delta W_i = U\Delta Q = 0 \\ &\Delta W_i = \Delta W_{meh} + \Delta W_e \quad \Rightarrow \quad \Delta W_{meh} = -\Delta W_e. \end{aligned} \tag{3.6}$$

When the source is disconnected, the force $F$ acting on the mobile electrode is given by (3.7). In the case when the changes $\Delta W_e$ and $\Delta x$ are very small, the ratio $\Delta W_e/\Delta x$ assumes the value of the first derivative of $W_e(x)$,

$$\begin{aligned} &F = -\frac{\Delta W_e}{\Delta x}, \\ &F = -\frac{\mathrm{d}W_e}{\mathrm{d}x} = -\frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{Q^2}{2}\frac{d-x}{\varepsilon_0 S}\right\} = \frac{Q^2}{2S\varepsilon_0}. \end{aligned} \tag{3.7}$$

If the source is connected, the considered system has a constant voltage. By applying the method of virtual works, the work of the source $U$, the change in the field energy, and the mechanical work are obtained in (3.8):

$$\begin{aligned} &\Delta W_i = U\,\Delta Q; \quad \Delta W_e = \Delta\left(\frac{CU^2}{2}\right) = \frac{1}{2}U\Delta Q = \frac{1}{2}\Delta W_i, \\ &\Delta W_i = \Delta W_{meh} + \Delta W_e \quad \Rightarrow \quad \Delta W_{meh} = \Delta W_i - \Delta W_e = \Delta W_e. \end{aligned} \tag{3.8}$$

With the source connected, the force $F$ acting on the mobile electrode is given by (3.9). With infinitesimally small changes $\Delta W_e$ and $\Delta x$, the ratio $\Delta W_e/\Delta x$ assumes the value of the first derivative $\mathrm{d}W_e\,(x)/\mathrm{d}x$,

$$\begin{aligned} F &= +\frac{\mathrm{d}W_e}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{U^2}{2}\frac{\varepsilon_0 S}{d-x}\right\} = \frac{U^2}{2}\frac{\varepsilon_0 S}{(d-x)^2} \\ &= \frac{E^2}{2}\varepsilon_0 S = \frac{D^2}{2\varepsilon_0}S = \frac{Q^2}{2S\varepsilon_0}. \end{aligned} \tag{3.9}$$

Expressions for electrical force, given by (3.7) and (3.9), are applicable only when the medium is linear, that is, when the permittivity of the dielectric material does not depend on the field strength. In cases when the source $U$ is not connected, displacement of the mobile electrode does not cause any change in charge $Q$.

Instead, it leads to changes in the capacitance $C$ and the voltage across the plates. The expression for electrical force when the source is disconnected takes the following form:

$$F = -\frac{\mathrm{d}W_e}{\mathrm{d}x} = -\frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{Q^2}{2}\frac{1}{C}\right\} = -\frac{Q^2}{2}\frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{1}{C}\right\}.$$

If the source is connected, the voltage across the plates is constant. Therefore, the shift of the mobile electrode changes the capacitance $C$ and the charge $Q$. The force expression assumes the following form:

$$F = +\frac{\mathrm{d}W_e}{\mathrm{d}x} = +\frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{U^2}{2}C\right\} = +\frac{U^2}{2}\frac{\mathrm{d}C}{\mathrm{d}x}.$$

**Question (3.1):** Equation 3.7 gives force $F$ acting on the mobile electrode in the case when the source is disconnected, whereas (3.9) gives this force when the source is connected. Note that in both cases, the same result is obtained, proportional to $Q^2$. Is it possible that the force acting on the mobile electrode does depend on source $U$ being connected or disconnected? Provide an explanation.

**Answer (3.1):** The electrical force acting on the mobile plate can be represented as a sum of forces acting on electrical charges distributed over the plate surface. Individual forces are dependent on the density of electrical charge and the field strength in the vicinity of the plate. It is necessary to compare the force obtained with the source $U$ connected to the force obtained with the source detached from the plates. If the plates accommodate the same electrical charge $Q$ in both cases, the surface charge density remains the same. The surface charge density determines the electrical induction $D$. Therefore, in both cases, the electrical field strength $E = D/\varepsilon_0$ is the same. From this, it can be concluded that in both cases the same force acts on the mobile plate.

### *3.1.4 Conversion Cycle*

In the preceding section, it has been shown that the electromechanical conversion can be performed in two different modes. With the source disconnected, mechanical work $\Delta W_{meh}$ is obtained on account of the energy accumulated in the coupling field, $\Delta W_{meh} = -\Delta W_e$. If the source is connected, the work of the source $\Delta W_i$ is divided in two equal parts, that is, $\Delta W_e = \Delta W_{meh} = ½\ \Delta W_i$. Graphical representation of electromechanical conversion is shown in Fig. 3.3. It is of interest to note that none of the two presented modes can last continuously.

If the source is connected, the electromechanical conversion is performed by turning one part of the source work into mechanical energy, whereas the rest of the

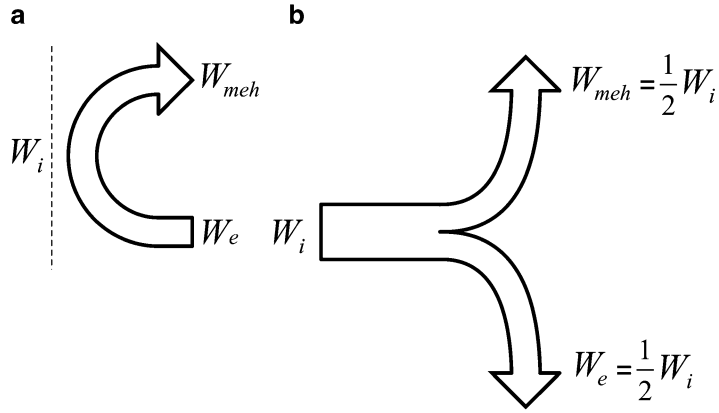


**Fig. 3.3** One cycle of electromechanical conversion includes phase (**a**) when the plates of the capacitor are disconnected from the source $U$ and phase (**b**) when the plates are connected to the source

source work increases the energy stored within the coupling field. The field energy $W_e$ is dependent of the density $w_e = ½\varepsilon_0 E^2$ and the volume of the domain where the field exists. There is an upper limit to the field energy. The maximum strength of the electrical field is limited by the dielectric strength of the material. The maximum electrical field in the air is $E_{max} \approx 30$ kV/cm. Exceeding the maximum field strength leads to dielectric breakdown, wherein the electrical current passes through the dielectric material and creates an electrical arc. The breakdown results in destruction and permanent damage. Therefore, the field strength and the field energy density $w_e$ have to be limited. The volume of the domain is also restricted and defined by the surface of the plates and the distance between them. Therefore, there is a limit $W_{e(max)}$ to the field energy, and it cannot be exceeded. For this reason, it is not possible to withstand a permanent growth of the field energy. Hence, the operation where the source is connected cannot go on indefinitely.

In the case when the source is not connected, mechanical work is obtained on account of the field energy. This energy decreases, and the operation would eventually stop when the field energy is exhausted. Therefore, the operation where the source is disconnected cannot hold indefinitely.

When the need exists for a continuous operation of an electromechanical converter, it is necessary to use concurrently both operating modes. Namely, they should be altered in cycles by switching the source on and off. An interval of operation when the source is connected (on) is followed by another interval when the source is disconnected from the converter (off). In such way, it is possible to provide mechanical work in a continuous manner while keeping the field energy from either reaching $W_{e(max)}$ or dropping to zero. Hence, the process of electromechanical conversion is mostly performed in cycles. Cyclic exchange of the two operating modes is illustrated in Fig. 3.3. In rotating electrical machines, one conversion cycle corresponds to one revolution of the rotor (sometimes, one fraction of the rotor revolution).

**Question (3.2):** Estimate the mechanical work obtained during one cycle with electromechanical converter made of a plate capacitor with one mobile plate. The dimensions of the plates and minimum and maximum distances between the plates are known, while the maximum electrical field strength in the dielectric is $E_{max} = 30$ kV/cm.

**Answer (3.2):** The maximum work obtainable in one cycle is determined by the maximum energy of the coupling field. The surface of the plates $S$, maximum distance between the plates $d$, and maximum energy density of the coupling field $w_e = ½\varepsilon_0 E_m{}^2$ are known. The mechanical work which can be obtained within one cycle is $\Delta W = S\ d\ w_{e(max)}$.

**Question (3.3):** If a converter makes $f$ cycles per second, estimate its average power.

**Answer (3.3):** Average power of the converter making $f$ cycles per second is $P_{a\ v} = f\Delta W = f\ S\ d\ w_{e(max)}$.

### 3.1.5 *Energy Density of Electrical and Magnetic Field*

The power of an electromechanical converter is dependent on the density of energy accumulated within the coupling field. A converter of given dimensions will have higher average power if its coupling field has a higher energy density. Given the converter power, dimensions and mass will be reduced for an increased density of energy. The power-to-size ratio is also called *specific power*. The considerations which follow show that electromechanical converters involving magnetic coupling field possess higher specific power compared to electrostatic machines.

The mechanical work obtained within one cycle of electromechanical converter is dependent on the energy stored in the coupling field. The maximum amount of the field energy is dependent on the energy density and the volume of the converter. If two electrical machines of the same size are considered, the machine with higher density of the field energy will produce higher mechanical work within each conversion cycle. If the repetition rates of conversion cycles are the same for the two machines, the machine having higher energy density will have higher average power.

The energy density of magnetic field exceeds by large the density of energy in electrical field. Permittivity ($D/E$) in vacuum is $\varepsilon_0 = 8.85 \cdot 10^{-12} \approx 10^{-11}$, whereas permeability ($B/H$) amounts $\mu_0 = 4\pi \cdot 10^{-7} \approx 10^{-6}$. Therefore, the energy density of magnetic field $w_m = \mu_0 H^2/2$ is considerably higher than the energy density of electrical field $w_e = \varepsilon_0 E^2/2$. For this reason, electrical machines are mostly operating with magnetic coupling field.

Density of energy accumulated in the coupling field depends on the square of the field strength. In air, electrical field is limited by dielectric strength, $E_{max} \approx 30$ kV/cm $\approx 3$ MV/m. In electrical machines with magnetic field, the field is comprised by magnetic circuit including air gaps and ferromagnetic materials such as iron.

Magnetic inductance $B$ in ferromagnetic materials is limited to $B_{max} = 1$–2 T, thus limiting the magnetic inductance achievable in air. Consequently, the maximum field strength $H$ which can be met in electrical machines is close to $H_{max} \approx B_{max}/\mu_0 \approx 1$ MA/m. With $\varepsilon_0 \approx 10^{-11}$ and $\mu_0 \approx 10^{-6}$, the achievable energy density is much higher in the case of magnetic field. Considering two electromechanical converters of the same size, the converter operating with magnetic field could accumulate much higher energy in the coupling field ($10^3$–$10^4$ times) and proportionally higher average power of electromechanical conversion.

### *3.1.6 Coupling Field and Transfer of Energy*

It is of interest to note that both the electromechanical energy conversion with magnetic coupling field and the conversion with electrical field involve both field vectors, electrical field vector $\boldsymbol{E}$ and magnetic field vector $\boldsymbol{H}$. The exchange of energy between electrical and mechanical terminals of electrical machine implies that in the space surrounding the moving part of the machine, there is transfer of energy toward the moving part (motor) or from the moving part (generator). The energy transfer through the surrounding space is measured by *Poynting vector*. Hence, the energy streams through domain if the Poynting vector has a nonzero algebraic intensity. Poynting vector is equal to the vector product of the electrical and magnetic field. It represents the surface density of power, and it is expressed in $W/m^2$. Surface integral of Poynting vector over a surface separating two domains represents the rate of energy transfer from one to the other domain (i.e., the power passed from one domain to another). The course and direction of Poynting vector indicate the course and direction of energy transfer. In the absence of either electrical field $\boldsymbol{E}$ or magnetic field $\boldsymbol{H}$, Poynting vector is equal to zero; thus, no energy transfer is possible. Therefrom, the question arises on how do electrical machines with electrical coupling field acquire magnetic field $\boldsymbol{H}$ required for mandatory Poynting vector.

In an electromechanical converter involving electrical coupling field which is at the state of rest, the magnetic field will be equal to zero and so will be Poynting vector. This is an expected situation since the power of electromechanical conversion is zero in the case when mobile parts of the converter do not move. Namely, the power is equal to the product of the force and speed of motion. At rest, although the electrical force may be present, the speed is equal to zero, thus resulting in zero mechanical power. If the considered converter is in the state of motion, its mobile part moves in the electrical coupling field. This leads to variations in the field strength $\boldsymbol{E}$ and the electrical induction $\boldsymbol{D}$ within the converter. The first time derivative of $\boldsymbol{D}$ contributes to the spatial derivative (i.e., *curl*) of magnetic field $\boldsymbol{H}$. The second Maxwell equation expresses generalized Ampere law, and it reads

$$\text{rot}\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}. \tag{3.10}$$

Since a nonzero spatial derivative of the field ***H*** exists, algebraic intensity of the vector ***H*** cannot be equal to zero at all points of the considered domain. The conclusion is that a certain magnetic field ***H*** exists in electrostatic machines in the state of motion. The field strength $H$ is proportional to the speed of the machine moving parts. In conjunction with the field ***E***, magnetic field ***H*** results in Poynting vector $\boldsymbol{P} = \boldsymbol{E} \times \boldsymbol{H}$.

The same considerations can be derived for an electromechanical converter based on magnetic coupling field. At rest, the magnetic field ***H*** exists in the converter, but the electrical field ***E*** and Poynting vector $P$ are equal to zero. With $P = 0$, there is no flow of energy toward the mobile part of the machine, and the mechanical power is equal to zero. This corresponds to the conclusion that the mechanical power at rest must be zero, as it is the product of the force and the speed. When the considered converter is in the state of motion, its mobile parts move in the magnetic coupling field. This leads to variations in the magnetic field ***H*** and the magnetic induction ***B*** within the converter. The first Maxwell equation expresses the Faraday law in differential form, and it reads

$$\text{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}. \tag{3.11}$$

Hence, the variation of magnetic induction ***B*** results in the spatial derivative (curl) of electrical field, which causes the appearance of the electrical field ***E*** within the converter and leads to nonzero values of the Poynting vector.

## 3.2 Converter Involving Magnetic Coupling Field

Electromechanical conversion in converters involving magnetic coupling field is possible by means of the field acting on the mobile windings and mobile parts made of ferromagnetic materials. In such converters, magnetic field is a precondition for electromechanical conversion of energy. It exists in the space between the stationary and mobile parts of magnetic circuits and current circuits. The mobile parts can perform either linear or rotational movement.

Forces acting on mobile parts are dependent on the magnetic induction and current in conductors. Mechanical work can be obtained on account of the field energy or work of the source which is connected to the current carrying conductors.

### *3.2.1 Linear Converter*

Figure 3.4 shows a simple electromechanical converter involving homogeneous magnetic field and a straight part of the conductor performing linear motion. The subsequent analysis is focused on motoring operation of the converter, wherein

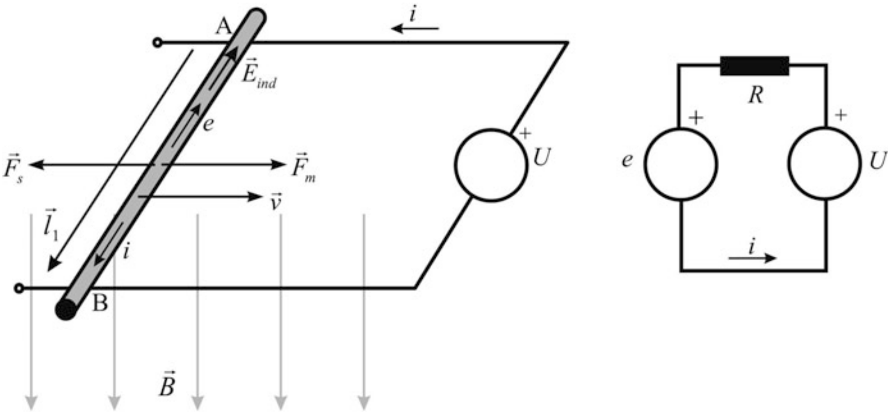


**Fig. 3.4** A linear electromechanical converter with magnetic coupling field

the electrical energy, obtained from a constant voltage source $U$, is converted to mechanical work. Mobile conductor AB of length $l_1$ touches fixed parallel conductors connected to the source $U$. The mobile conductor AB, fixed parallel conductors, and source $U$ make current circuit shown on the right-hand side of Fig. 3.4. The resistance of conductor AB can be neglected, whereas the sum of resistances of all remaining conductors in the current circuit is denoted by $R$.

The source $U$ causes the current $i$ in the circuit. Direction of the current corresponds to the direction of vector $l_1$ shown in Fig. 3.4 along conductor AB. The conductor is placed in an *external*[1] homogeneous magnetic field of induction $\boldsymbol{B}$. The electromagnetic force $\boldsymbol{F}_m$ acting on the conductor is determined by (3.12):

$$\vec{F}_m = i\left(\vec{l}_1 \times \vec{B}\right). \tag{3.12}$$

Since vector $l_1$ is orthogonal to the vector of magnetic induction, algebraic intensity of the force is equal to $F_m = l_1 i B$. The electromagnetic force in Fig. 3.4 is directed from left to right. It is assumed that the force makes the conductor move in the same direction at a speed $v$. The conductor is subjected to an external force $\boldsymbol{F}_{ex}$, which opposes this movement. In the state of dynamic equilibrium, acceleration of the conductor is zero, the speed of motion $v$ is constant, and the sum of the forces acting on the conductor is equal to zero. Therefore, the algebraic intensities of the external and electromagnetic forces are equal:

$$\vec{F}_{ex} + \vec{F}_m = 0; \quad \vec{F}_{ex} = -\vec{F}_m; \quad \left|\vec{F}_{ex}\right| = \left|\vec{F}_m\right| = i l_1 B. \tag{3.13}$$

[1] Magnetic field caused by external phenomena is called *external field*. External phenomena do not make part of the system under consideration, and they are not related or caused by the considered system. External magnetic field can be created by external conductors carrying electrical current, external permanent magnets, the Earth magnetic poles, and other sources.

While the conductor moves in magnetic field, the electromotive force $e_{AB}$ is induced between its ends. Electrical field $E_{ind}$ induced in the conductor is determined by the vector product of the speed $v$ and magnetic induction $B$. Since the vector of the induced field does not vary along the conductor, the electromotive force $e = e_{\mathrm{AB}}$ can be calculated from (3.14):

$$e = \left(-\vec{l}_1\right) \cdot \vec{E}_{ind} = \left(-\vec{l}_1\right) \cdot \left(\vec{v} \times \vec{B}\right). \tag{3.14}$$

Vector of the induced electrical field is collinear with the conductor. Therefore, the electromotive force is equal to $e = l_1 vB$. The sign of the induced electromotive force $e = e_{\mathrm{AB}}$ is related to the adopted reference direction, shown in Fig. 3.4. Positive value of the electromotive force, $e = e_{\mathrm{AB}} > 0$, acts toward increasing the potential at the conductor end A with respect to the potential at the end B.

Current $i = (U - e)/R$ exists in the circuit shown in the Fig. 3.4. At steady state, time varying electrical current $i(t)$ assumes a constant value $I = (U - l_1 vB)/R$. Power of the source $P_i = Ui = ei + Ri^2$ contains the component $P_{\mathrm{AB}} = ei = l_1 vB$ as well as the losses $P_\gamma = Ri^2$. The losses in conductors are caused by Joule effect, and they depend on the equivalent resistance and square of the current. The remaining power $P_{\mathrm{AB}}$ is transferred to the moving conductor. By maintaining the movement, electromagnetic force $\boldsymbol{F}_m$ performs the work against external force $\boldsymbol{F}_{ex}$ which is opposite to motion. Vectors of the force and speed of motion are collinear. Therefore, the mechanical power is equal to $P_{meh} = F_m v = l_1 ivB$. Power $P_{meh}$ is the output power of the electromechanical converter which converts electrical energy obtained from the source $U$ to mechanical work. Since $P_{meh} = F_m v = P_{\mathrm{AB}} = ei = l_1 vBi$, distribution of the source power $P_i$ can be described by expression

$$P_i = Ui = ei + Ri^2 = F_m v + Ri^2 = P_{meh} + Ri^2. \tag{3.15}$$

Therefore, power from the source is divided in the thermal losses and mechanical power, the latter being the result of electromechanical conversion. The power delivered by the induced electromotive force $e$ is equal to $P_e = e(-i) = -ei < 0$. Consequently, the electromotive force $e$ behaves as a receiver, taking over the electrical power $ei = l_1 vBi$ which is then converted to mechanical power $P_{meh} = F_m v = ei$. In the presented example, the mechanical power of the electromechanical converter is equal to the product of the electromotive force and current. Equation 3.16 in certain form is present in all electrical machines:

$$ei = F_m v. \tag{3.16}$$

Joule losses are determined by the power $P_\gamma = Ri^2$, and they are turned into heat. Conductors and other parts of the converter are heated. Compared to ambient temperature, their temperatures are increased. Due to elevated temperatures, these parts of the converter transfer their heat to the ambient by convection, conduction, or radiation. When the power of losses $P_\gamma$ becomes equal to the heat power transferred to the ambient, the temperature increase stops and the system enters the thermal equilibrium. Since the electromechanical converters are used for

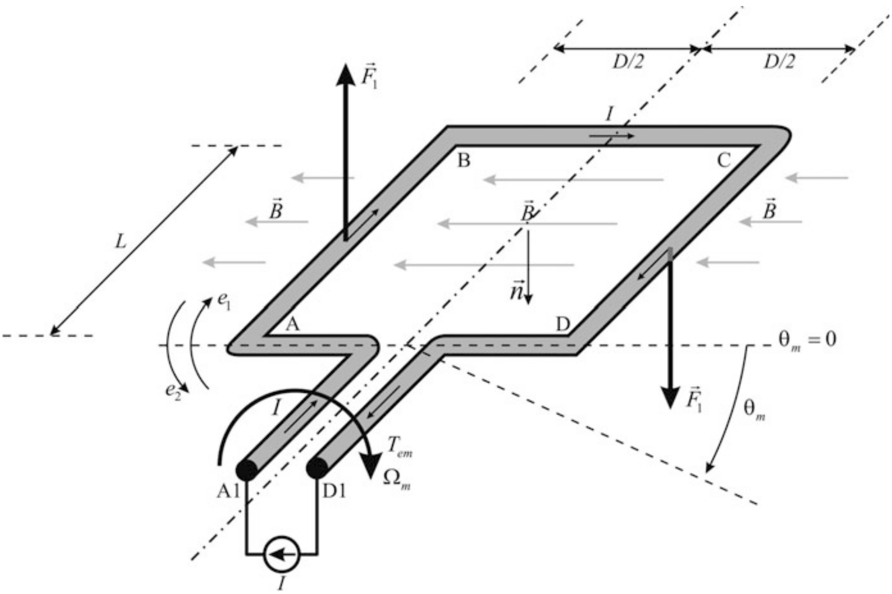


**Fig. 3.5** A rotational electromechanical converter involving magnetic coupling field

converting electrical energy to mechanical work, it is necessary to keep the conversion losses as small as possible. Due to thermal losses, the coefficient of efficiency $\eta$ of power converters is reduced. In addition, generated heat has to be removed so that the converter does not become overheated. It is required, therefore, to have a corresponding solution for heat transfer and cooling. The losses can be reduced by decreasing the equivalent resistance $R$. However, reducing resistance by increasing the cross section of conductors leads to an increased consumption of copper, increasing in this way the cost, weight, and size of converters.

The power converter shown in Fig. 3.4 can also run in generator mode. Direction of the current will be reversed and also direction of the electromagnetic force. In order to support the motion, direction of the external force $\boldsymbol{F}_{ex}$ has to be changed as well. In generator mode, mechanical power is converted to electrical energy. Generator operation is analyzed in more detail in Sect. 2.4.

### 3.2.2 Rotational Converter

Electromechanical conversion is most frequently performed by using rotational machines, which convert electrical energy to mechanical work of rotational movement. An example of simple rotational converter is shown in Fig. 3.5. Contour ABCD is made out of copper conductors. It has dimensions $D \times L$, and it rotates in homogeneous external magnetic field $B$. The contour rotates clockwise around horizontal axis, shown in Fig. 3.5. The position of the contour is determined by angle $\theta_m$, and it varies at the rate $\Omega_m = \mathrm{d}\theta_m/\mathrm{d}t$, where $\Omega_m$ represents the angular speed in rad/s. At certain instant, the contour is in position $\theta_m = 0$, when lines of the magnetic field are parallel to surface $S = D \times L$, surrounded by the contour. Terminals of the contour are connected to power supply which provides the current $I$ in the conductor.

Electromagnetic force $\boldsymbol{F}_1$ acts on parts AB and CD of the conductive contour. These parts are of length $L$ and are orthogonal to the magnetic field; thus, the force is determined by expression $F_1 = LIB$. The electromagnetic force does not act on the transversal parts BC and DA of length $D$, because the current in these parts is collinear with the magnetic field. At position $\theta_m = \pi/2$, the transversal parts BC and DA are subjected to the actions of forces in the direction of rotation, but the forces are collinear and of opposite directions; therefore, their actions are mutually canceled.

The couple of electromagnetic forces in Fig. 3.5 creates the torque $T_{em} = DF_1$. Assuming that the contour rotates with angular frequency $\Omega_m = \mathrm{d}\theta_m/\mathrm{d}t$, the developed mechanical power at the considered instant ($t = 0$, $\theta_m = 0$) is equal to $P_{meh} = T_{em}\Omega_m = DLIB\Omega_m$. Power $P_{meh}$ is the output power of the electromechanical converter which converts the electrical energy obtained from the supply $I$ to mechanical work.

It is of interest to compare the obtained mechanical power with the electrical power taken from the source. Between terminals A1 and D1 of the constant current source $I$, there is voltage $u = v_{A1} - v_{D1}$. The source is connected to the contour ABCD, and the voltage is $u = RI + \mathrm{d}\Phi/\mathrm{d}t$, where $\Phi$ denotes the flux through surface $S$ encircled by the contour, while $R$ denotes the equivalent resistance of the conductors making the contour. Reference direction of the flux is the direction of the positive normal $n$ to surface $S$. This normal is in accordance with the direction of circulation along the contour ABCD, that is, the current in designated direction of circulation along the contour creates a magnetic field which is aligned with the normal $n$. In Figs. 3.5 and 3.6, the normal is denoted by vector $n$. The contour can rotate around horizontal axis; thus, the flux through the surface $S$ depends upon the angle $\theta_m$ between the vectors of magnetic induction and the plane in which the surface $S$ reclines.

In accordance with the notation in Fig. 3.6, the angle $\theta_m$ is equal to zero at the position where the magnetic field is parallel to the surface $S$. In zero position, flux $\Phi$ is equal to zero. When the contour makes an angular shift of $\theta_m$, the normal $n$ to surface $S$ is shifted to position $n_1$. Assuming that the external field is homogeneous, $\Phi$ can be represented by the expression $\Phi(\theta_m) = \Phi_m \sin(\theta_m) = \Phi_m \sin(\Omega_m t)$, where $\Phi_m = BS$ is the maximum value of flux which is attained at position $\theta_m = \pi/2$. By using the obtained expression for the flux, the voltage across the terminals of the source is calculated as $u = RI + \Omega_m \Phi_m \cos(\Omega_m t)$. At position $\theta_m = 0$, the power delivered by the source $I$ to the converter is given in (3.17):

$$\begin{aligned} P_i = uI = RI^2 + I\Omega_m\Phi_m = RI^2 + I\Omega_m BS \\ = RI^2 + DLIB\Omega_m = P_{meh} + RI^2. \end{aligned} \tag{3.17}$$

Therefore, the power of the source $I$ is partially converted to mechanical power, whereas the remaining part accounts for conversion losses that are turned into heat

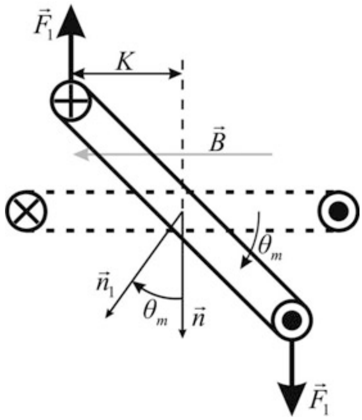


**Fig. 3.6** Variations of the flux and electromotive force in a rotating contour

due to Joule effect. At position $\theta_m = 0$, the electromagnetic torque acting on the contour is equal to $T_{em} = \Phi_m I = P_{meh}/\Omega_m$. After the angle is shifted to $\theta_m$, the arm $K$ of force $F_1$ is shown in Fig. 3.6, and it is equal to $K = (D/2)\cos\theta_m$. Therefore, torque $T_{em}$ varies as function of angle $\theta_m$ in accordance with (3.18):

$$T_{em} = \Phi_m I \cos\theta_m. \tag{3.18}$$

Equations similar to (3.18) determine the electromagnetic torque of all rotating electrical machines. The analysis of operation of the converter shown in Fig. 3.5 leads to the conclusion that the average value of the torque during one full revolution is zero. This can be changed by insertion of additional contours or by changing the supply current, as will be elaborated in due course.

By changing the direction of the current or direction of rotation, the electromechanical converter shown in Fig. 3.5 will operate in the generator mode of operation, converting mechanical work to electrical energy. Voltage and current of the current source $I$ will have opposite signs, while the source $I$ will act as a receiver of electrical energy.

### *3.2.3 Back Electromotive Force*[2]

The arrows denoted by $e_1$ and $e_2$ in Fig. 3.5 indicate two possible reference directions for the induced electromotive force. The choice of reference direction

[2] Back electromotive force (abbreviated BEMF) is also called counter-electromotive force (abbreviated CEMF), and it refers to the induced voltage that acts in opposition to the electrical current which induces it. BEMF is caused by changes in magnetic field, and it is described by Lenz law. The only difference between the electromotive force (EMF) and BEMF is the reference direction and, hence, the sign.

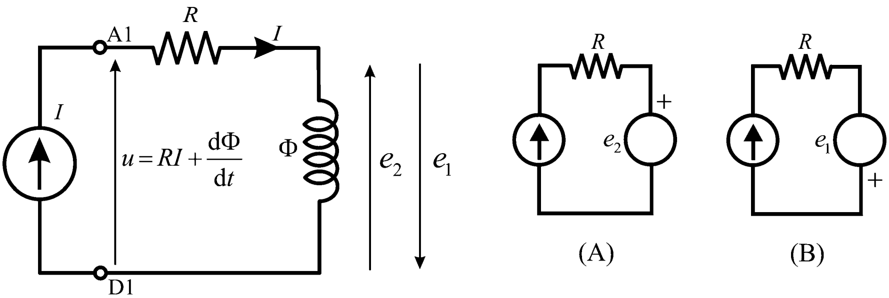


**Fig. 3.7** Definition of reference direction for electromotive and back electromotive forces

determines the sign of the induced electromotive force, as well as its connection in the equivalent scheme of the electric circuit; thus, it is useful to give the corresponding explanation and expressions for the induced electromotive force in both cases.

Figure 3.7 shows the equivalent scheme of the mobile contour, fed from a constant current source via terminals A1 and D1. Character $\Phi$ denotes flux through the surface $S$ encircled by the contour. Reference direction for the flux is determined by the normal on the surface $S$, denoted by $n$ in Figs. 3.5 and 3.6.

The normal $n$ is aligned with the magnetic field created by the current $I$ which circulates along the contour (ABCD) in designated direction. Flux $\Phi$ depends on the angle $\theta_m$. Rotation of the contour in the direction indicated in Fig. 3.5 leads to a growth of the flux $\Phi$ that the external magnetic field makes through the surface $S$.

The total magnetic flux through the surface $S$ depends on the external magnetic field $B$, but it also changes with the current that circulates within the contour. Namely, the contour current creates a magnetic field of its own, and this field contributes to the total magnetic flux. The total flux can be expressed by $\Phi = LI + BS\sin(\theta_m)$. The coefficient $L$ defines the ratio between the flux $\Phi$ and the current $I$ in cases where the external magnetic field does not exist. The ratio $L = \Phi(I)/I$ is called *the self-inductance* of the contour.

Each change of the flux $\Phi$ induces an electromotive force in the contour. This electromotive force depends on the first time derivative of the flux. Under the action of electromotive force, a current appears in the contour. The intensity of this current depends upon the equivalent resistance of the circuit. In the case shown in Fig. 3.7, the contour is fed from a constant current source. The equivalent resistance of a constant current source is $R_{eq} = \infty$; thus, presence of an electromotive force $e_1$ does not cause any change of current. In the case when the contour is galvanically closed, that is, when terminals A1 and D1 are short-circuited or connected to a voltage source or a receiver of finite equivalent resistance, the presence of electromotive force $e_1$ will cause a change of current and a change of flux.

According to Lenz rule, electromotive forces are induced in coils due to changes in magnetic flux. Electrical currents appear as a consequence of induced electromotive forces. Induced currents oppose to the flux change and tend to maintain the initial flux value. Electrical current in a coil creates magnetic field and the flux which is proportional to the self-inductance of the coil. Direction of this *self-flux* is opposite to the original flux change. Hence, the induced electromotive force produces the current and the self-flux in direction that tends to cancel the original flux change. For that reason, induced electromotive forces are also called *counter-electromotive forces* or *back electromotive forces*.

Considering the setup in Fig. 3.5, during rotation of the contour in the direction indicated in the figure, the flux due to external magnetic field rises. Electromotive force $e_1$, given by (3.19), appears in the contour:

$$e_1 = -\frac{d\Phi}{dt}. \tag{3.19}$$

Since an increase of the flux results in $e_1 < 0$, a current appears opposite to the direction of circulation along the galvanically closed contour ABCD. Therefore, the induced current creates its own magnetic field and the self-flux of the contour in the direction opposite to the indicated normal $n$. Total flux is equal to the sum of fluxes due to external field, which is growing, and the self-flux which is of negative sign.

Electromagnetic induction opposes to changes of the flux to the degree which depends on the circuit parameters. When the equivalent resistance of the circuit is $R_{eq} = \infty$, the induced electromotive force does not cause any change in electrical current which would have opposed to changes in the flux. In cases when the equivalent resistance of the contour is zero ($R_{eq} = 0$, the case of a superconductive contour with short-circuited terminals), the phenomenon of electromagnetic induction prevents any changes of flux. Since the voltage balance equation is given in (3.20)

$$u = Ri - e_1 = Ri + \frac{d\Phi}{dt}, \tag{3.20}$$

in conditions with $u = 0$ and $R = 0$, the flux cannot change due to $d\Phi/dt = 0$. Therefore, notwithstanding eventual changes in the external magnetic field, the total flux through a short-circuited superconductive contour is constant.

For a contour fed from a constant current source, shown in Fig. 3.5, the equivalent schemes of the electrical circuit are shown in Fig. 3.7. For the reference direction of the induced electromotive force, it is possible to use $e_1$ or $e_2$, as indicated in Figs. 3.5 and 3.7. If the reference direction $e_1$ is chosen, the equivalent scheme (B) of Fig. 3.7 applies, and algebraic intensity of the electromotive force is determined by (3.19). Alternatively, the equivalent scheme (A) and (3.21) apply. Quantity $e_2 = +d\Phi/dt$ is called *back electromotive force* or *counter-electromotive force*:

$$e_2 = -e_1 = \frac{d\Phi}{dt}. \tag{3.21}$$