

# Chapter 5

## Rotating Electrical Machines

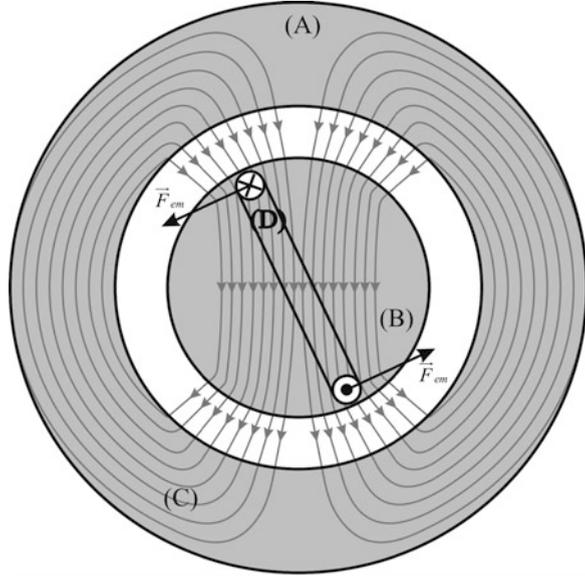
This chapter provides basic information on cylindrical machine. Typical machine windings are introduced and explained, along with the basic forms of magnetic circuits with slots and teeth. This chapter introduces common notation, symbols, and conventions in representing the windings, their magnetic axes, their flux, and magnetomotive force. Typical losses and power balance charts are explained and presented for cylindrical motors and generators. Calculation of the magnetic field energy in the air gap of cylindrical machines is given at the end of this chapter, along with considerations regarding the torque per volume ratio.

Electrical machines are usually rotating devices creating electromagnetic torque due to the magnetic coupling field. The machines perform electromechanical conversion of energy; thus, they are called rotational converters. The stationary part of rotating machines is called stator. The mobile part which rotates is called rotor. The rotating movement of the rotor is accessible via shaft, which serves for rotor mechanical coupling to a work machine. The magnetic coupling field creates torque which acts on the rotor forcing it to rotation. The torque is the result of interaction of electromagnetic forces, and for this reason, it is called electromagnetic torque.

### 5.1 Magnetic Circuit of Rotating Machines

Electrical machines are mainly of cylindrical shape. Stationary part of the machine (stator) is mostly made in the form of a hollow cylinder which accommodates cylindrical rotor capable of rotating in its bearings with negligible friction. Both stator and rotor are made of ferromagnetic material, and between them, there is an air gap. Along the rotor axis, there is a shaft which serves for transferring the electromagnetic torque to a mechanical subsystem. The shaft protrudes out of the machine to facilitate the coupling to work machines. Both stator and rotor contain windings and/or permanent magnets which create the stator and rotor fields. By interaction of

**Fig. 5.1** Cross-section of a cylindrical electrical machine. (A) Magnetic circuit of the stator. (B) Magnetic circuit of the rotor. (C) Lines of magnetic field. (D) Conductors of the rotor current circuit are subject to actions of electromagnetic forces  $F_{em}$

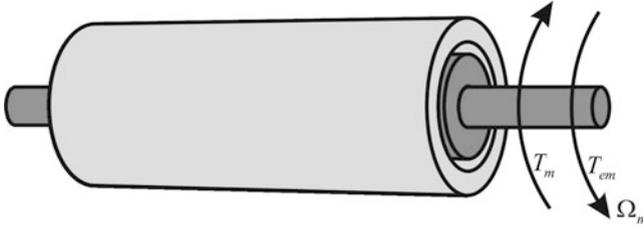


these fields, the electromagnetic torque is created, and it acts on the rotor and creates rotational movement. Figure 5.1 shows cross-section of the magnetic circuit of an electrical machine.

## 5.2 Mechanical Access

Rotating machines are connected via shaft to a load or a work machine. The rotor and shaft are rotating at the mechanical angular speed  $\Omega_m$ . The torque  $T_{em}$  is created by a couple of electromagnetic forces  $F_{em}$  which tend to move the rotor. The product of the torque  $T_{em}$  and speed of rotation  $\Omega_m$  gives the power of electromagnetic conversion  $P_{em} = T_{em}\Omega_m$ . In cases when the torque acts in the direction of rotation, power  $P_{em}$  is positive. Then, electrical energy is being converted to mechanical work; thus, the machine operates in the motor mode. Reference directions for the torque and speed are indicated in Fig. 5.2.

The shaft represents mechanical connection, that is, mechanical access or mechanical output of the machine. It rotates at angular speed  $\Omega_m$  and does the transfer of electromagnetic torque  $T_{em}$ . In the case when the machine operates as a motor, the electromagnetic torque excites movement ( $T_{em} > 0$ ), while mechanical load (load or work machine) resists to motion by an opposing torque, a torque of the opposite sign, denoted by  $T_m$ . In this operating mode, electromagnetic torque  $T_{em} > 0$  tends to accelerate the rotor, while the opposing torque  $T_m > 0$  tends to



**Fig. 5.2** Adopted reference directions for the speed, electromagnetic torque, and load

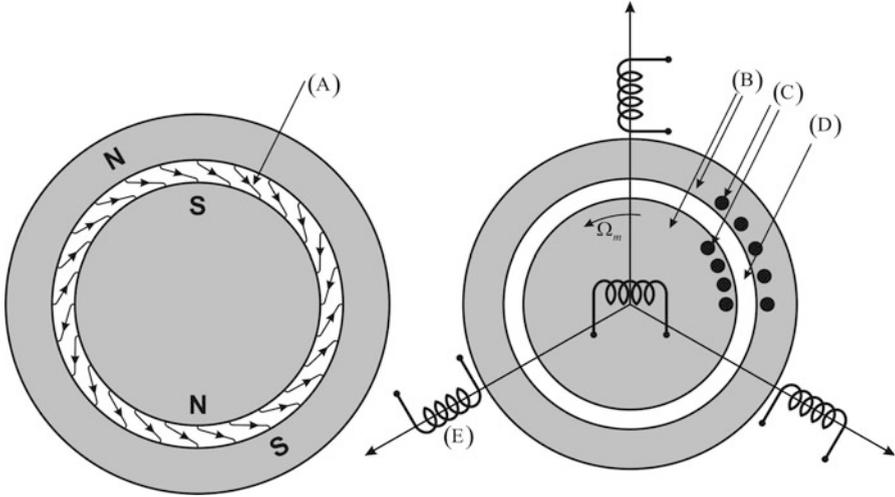
slow it down. The change of speed is determined by Newton law applied to a rotational movement:

$$J \frac{d\Omega_m}{dt} = T_{em} - T_m. \quad (5.1)$$

In general, electrical machine can be operated as a motor or as a generator. A motor performs electromechanical conversion of electrical energy to mechanical work; a generator performs conversion in the reverse direction. For a machine operating in motor mode, torque  $T_{em}$  is of positive sign, whereas in generator mode, the sign of the torque  $T_{em}$  is negative. Electrical generators have their rotor connected to a turbine which turns the rotor, supplying the mechanical power into the machine. Thus, the torque  $T_m$  assumes a negative value with respect to the reference direction shown in Fig. 5.2. Electromagnetic torque of the generator  $T_{em}$  opposes this movement, and it also takes a negative value with respect to the reference direction. Considering adopted reference directions, power of electromagnetic conversion  $P_{em}$  is negative, indicating that the machine converts mechanical work into electrical energy.

### 5.3 The Windings

In addition to magnetic circuits, which direct the magnetic field, electrical machines also have current circuits, also called windings, which conduct electrical current. The windings are made of a number of series connected, insulated copper conductors. In cylindrical machines, the conductors are positioned along the cylinder axis (coaxially). By connecting a number of conductors in series, one obtains a winding. Two conductors connected in series and positioned diametrically constitute one contour or one *turn*. A machine could have a number of windings. They may be placed on both stator and rotor. Each winding has two terminals, which could be short circuited, open, or connected to a power source feeding the machine. By connecting them to a voltage or current source, electrical current is established in the windings. Terminals of the windings are *electrical access* (connection, input) of the machine. Current  $i$  in a winding having  $N$  turns creates magnetomotive force



**Fig. 5.3** Magnetic field in the air gap and windings of an electrical machine. (A) An approximate appearance of the lines of the resultant magnetic field in the air gap. (B) Magnetic circuits of the stator and rotor. (C) Coaxially positioned conductors. (D) Air gap. (E) Notation used for the windings

of  $F = Ni$ . By dividing the magnetomotive force with magnetic resistance  $R_\mu$ , one obtains flux  $\Phi$ . Stator windings create stator flux, whereas rotor windings create rotor flux. There are machines where stator or rotor does not have windings, but the flux is created by permanent magnets.

The resultant flux of the machine is obtained by joint action of the magnetomotive forces of the stator and rotor. Lines of the resulting field run through the magnetic circuit of rotor, air gap, and magnetic circuit of stator. Figure 5.3(A) shows the lines of magnetic field in the air gap. The figure shows the zone where magnetic lines leave magnetic circuit of the rotor and enter the air gap.<sup>1</sup> This zone is called *north*

<sup>1</sup> Figure 5.3 shows an approximate shape of the lines of magnetic field in the air gap, which does not correspond to the air gap field of real machines. Electrical machines have magnetic circuit containing slots and teeth which are described in the following subsections. The presence of slots has an influence on the shape of the air gap field, making it relatively more complicated. In the hypothetical case when the magnetic circuit is of an ideal cylindrical shape and permeability of the ferromagnetic material is considerably higher than that of the air, the lines of magnetic field are perpendicular to the surface separating the air gap and the ferromagnetic material. It is of interest to envisage the surface that separates the air gap from the ferromagnetic material. The boundary conditions relate tangential components of magnetic field  $H$  on either side of this surface with surface electrical currents  $J_S$ . With  $J_S = 0$ , tangential component of magnetic field  $H$  in the air is equal to tangential component of magnetic field in ferromagnetic material. Since  $B_{Fe} < 1.7$  T and  $H_{Fe} = B_{Fe}/\mu_{Fe} \approx 0$ , tangential component of magnetic field in the air is close to zero. Thus, the lines of magnetic field in the air gap are perpendicular to the surface separating the air gap and the ferromagnetic circuit.

*magnetic pole of the rotor.* Diametrically opposed is the south magnetic pole. In the same way, the north and south poles of the stator can be identified. The electromagnetic torque arises from the tendency of rotor poles to take place against the opposite poles of the stator.

The right-hand side of Fig. 5.3 shows cross-section of the machine. Conductors of stator and rotor windings are marked by (C). The stator and rotor could have several windings. For clarity, individual windings are represented by the symbol marked by (E) in Fig. 5.3.

Stator flux can be represented by a vector whose course and direction are determined by positions of its poles, while its algebraic intensity (amplitude) is determined by the flux itself, namely, by the surface integral of magnetic induction  $B$ . Rotor flux can be represented in the same way. In the following subsections, it will be shown that the electromagnetic torque is determined by the vector product of the two fluxes, that is, by the product of the flux amplitudes and the sine of the angle between them.

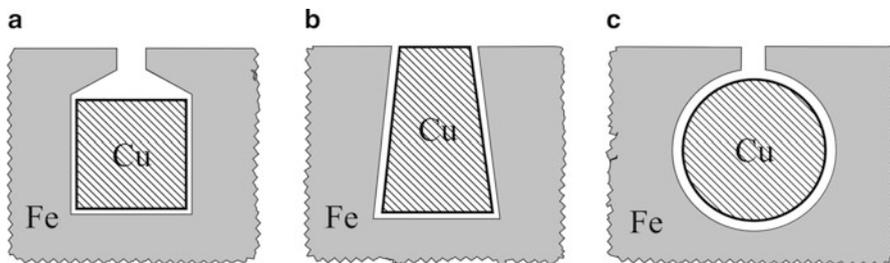
## 5.4 Slots in Magnetic Circuit

Magnetic circuits of the stator and rotor are made of iron sheets in order to reduce power losses. The iron sheets are laid coaxially. Each individual sheet has a cross-section of the form indicated in Fig. 5.4. A number of sheets are assembled and fastened, producing in such way magnetic core. Stator usually assumes the form of a hollow cylinder, whereas rotor is cylindrical, fitting in the stator cavity. Distance between the stator and rotor (air gap) can be from one to several millimeters.

Windings of the machine consist of series connected, mutually insulated copper conductors. Conductors are insulated between each other, as well as from the magnetic circuit and other parts of the machine. The conductors are insulated



**Fig. 5.4** Cylindrical magnetic circuit of a stator containing one turn composed of two conductors laid in the opposite slots



**Fig. 5.5** Shapes of the slots in magnetic circuits of electrical machines. (a) Open slot of rectangular cross-section. (b) Slot of trapezoidal shape. (c) Semi-closed slot of circular cross-section

by lacquer, paper, silicon rubber, or some other insulating material. The insulated copper conductors are placed in *slots* which are positioned coaxially (parallel to machine axis) along the inner side of the stator magnetic circuit or outer side of the rotor magnetic circuit. Examples of some of the slots are shown in Fig. 5.5. Shape of the cross-section of a slot is determined by the need for achieving a smaller or larger leakage flux as well as by the need for mechanical tightening of the conductors placed in these slots. The slots shown in Fig. 5.5 may have one or more conductors. In most cases, the stator slots may have conductors of two different phase windings.

In Fig. 5.6, the cross-section of the stator magnetic circuit shows the axial grooves along the inner surface of the stator. These grooves are called slots. Part of the magnetic circuit between two neighboring slots is called *tooth*. The teeth are formed by cutting the same slot through all the iron sheets which are assembled when forming the magnetic circuit. After the sheets are arranged, one obtains a slot of trapezoidal, or oval, or of some other cross-section. The way the insulated conductors are placed in the slots is illustrated in Fig. 5.6, where the front side of the stator is shown as (F), size view of the stator is shown as (G), whereas the views (H) and (I) show 3D view of the stator with one *section*.

One turn can be obtained by a series connection of conductors placed in different slots (A–B in Fig. 5.6). The two conductors making one turn can be placed in diametrical slots, but there are also turns where this is not the case. The two conductors which belong to one turn pass through the slots and get out of the magnetic circuit at the rear side. At that point, they get connected by the end turns, denoted by A, C, and D in Fig. 5.6. Conductors that are placed in slots, the end turns (D), and front connections (C) between conductors are usually made of a single piece of insulated copper wire.

The slots under consideration may hold more than one copper conductor. Therefore, several turns may reside in the same pair of slots. These turns are connected in series. In such way, one obtains a *coil* or a *section* (C–E in Fig. 5.6). To connect the turns in series, one section has the end turns at the front side of the stator (detail C) as well as at the rear side of stator (detail D).

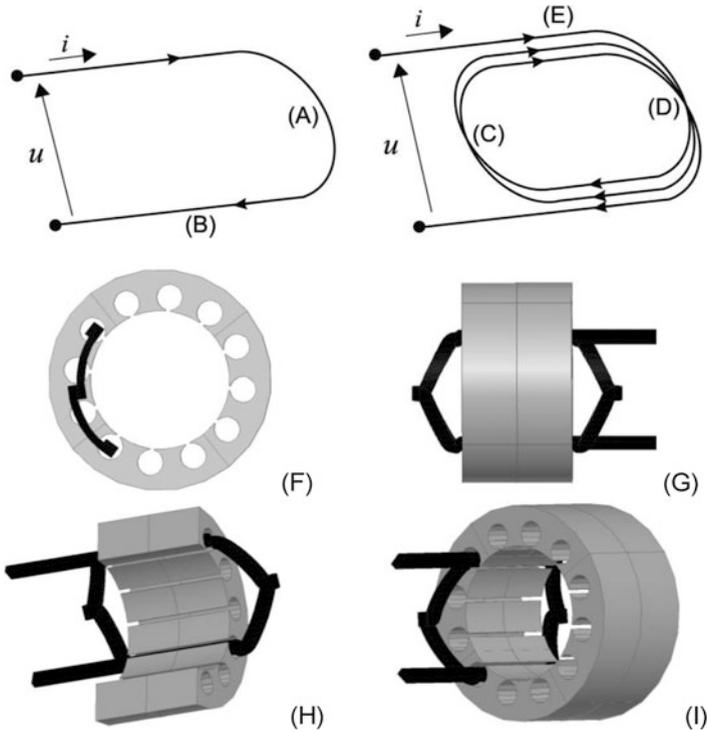


Fig. 5.6 Definitions of one turn and one section

One section can have one or more turns connected in series. One winding can have one or more sections connected in series. Terminals of the winding can be connected to electrical source or electrical load. They represent electrical access to the machine. Electrical machines can have several windings in the stator and/or rotor.

Flux of one turn is equal to the flux through the contour determined by the conductors of the turn. Flux of a turn is denoted by  $\Phi$ , and it is equal to the surface integral of magnetic induction over the surface leaning on the contour. Flux of a coil having  $N$  turns is equal to  $N\Phi$ .

**Question (5.1):** Is magnetic induction in the teeth of higher or lower intensity compared to the rest of the magnetic circuit? Why?

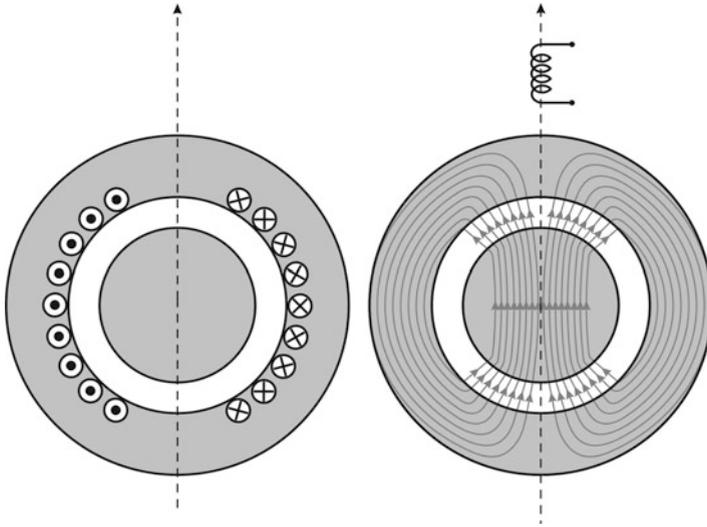
**Answer (5.1):** Flux of the machine passes through magnetic circuit of the stator and through magnetic circuit of the rotor. Within the iron parts of the circuit, there are no air gaps of high magnetic resistance. Passing toward the air gap, lines of magnetic field get through the teeth. The equivalent cross-section is then reduced, since the field is directed toward teeth and not toward slots, where magnetic resistance is much higher. Since the same flux now passes through a smaller equivalent cross-section, magnetic induction in the teeth is higher than the induction in the other parts of magnetic circuit.

## 5.5 The Position and Notation of Winding Axis

Windings can be placed in stator, rotor, or in both parts of the machine, depending on the type of electrical machine. The main types of electrical machines are DC machines, asynchronous machines, and synchronous machines. Introductory remarks on windings of electrical machines are given in Sect. 5.3 “The Windings”. In synchronous and DC machines, the excitation flux can be accomplished by means of permanent magnets. In such cases, the number of windings is smaller since there is no excitation winding. Asynchronous and synchronous machines are also called AC machines, and they usually have three windings in the stator called the three-phase windings. When dealing with machines which have a large number of windings, it is not practical to include a detailed presentation of all these windings. Too many details and unclear presentations do not help drawing conclusions and making decisions. Instead, each of the windings can be denoted by a simple mark which defines its axis, that is, its spatial orientation.

Axis of a winding is determined by direction of the lines of magnetic field created by the currents circulating in the winding conductors. In the preceding subsections, *winding* is defined as a set of several conductors placed in a slot, connected in series, and accessible via winding terminals which are connected to electrical sources or electrical receivers. One *turn* is series connection of two conductors placed in different, mostly diametrical slots. The conductors are connected by end turns at machine ends. Gathered together, they make a contour. Electromotive forces of the two conductors are added to make the electromotive force of the turn/contour. Flux created by current in one turn has direction perpendicular to the surface encircled by the contour. This normal on this surface defines spatial orientation of the turn. The turns making one winding can be distributed along machine perimeter and can be of different spatial orientation.

In cases where all the turns that constitute winding reside in the same pair of slots, the turns share the same magnetic axis. Electrical currents in these turns create magnetomotive force and flux in the same direction. Such winding is called *concentrated winding*. The winding current in concentrated winding creates magnetic field in the air gap. Lines of this field pass through the iron core, where intensity of the field  $H_{Fe}$  is very small due to high permeability of iron. In addition, lines of the field pass through the air gap twice, as shown in Fig. 5.7. Therefore, intensity of the field in the air gap can be determined from  $Ni = 2H_0\delta$ . Magnetic field created by the winding has two distinct zones in the air gap, one where the lines of magnetic field come out of the rotor, pass through the air gap, and enter magnetic circuit of the stator, and the other where the field is in the opposite direction. These zones are called *magnetic poles*. Positions of the poles are determined by the *direction of the field*. This direction extends along the axis of the winding. For a concentrated winding with turns made of series connected diametrically positioned conductors, the axis of the winding corresponds to the axis of each individual turn. This axis is perpendicular to the surface encircled by diametrical conductors.



**Fig. 5.7** Notation of a winding and its axis

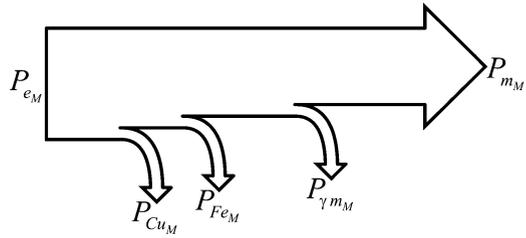
While analyzing electrical machines, consideration of all conductors of all windings would be too complex and of little use. Therefore, the windings are represented by special marks, similar to those of the coils. Orientations of each mark should be such that it extends along the axis of the winding. Namely, direction of the mark should be aligned with the lines of magnetic field established by electrical current in the winding. The way of marking the axis of a winding is shown in Fig. 5.7.

Windings of electrical machines can be made in such way that one slot contains more than one conductor. Conductors placed in one slot do not have to belong to the same winding. In a three-phase machine, there are three separate stator windings, having a total of six terminals. One slot may contain conductors belonging to two or even three separate windings. Three parts of one stator winding are often called *phases* (three-phase windings).

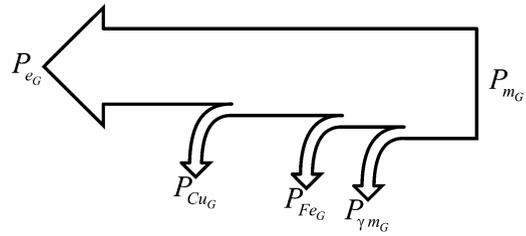
## 5.6 Conversion Losses

Conversion process in electrical machines involves power losses in magnetic circuits, in windings, and in mechanical subsystem. Losses in magnetic circuits are a consequence of alternating magnetic induction in ferromagnetic materials, and it is divided in hysteresis losses and eddy currents losses. Losses due to eddy currents can be reduced by lamination. Laminated magnetic circuit is made of iron sheets separated by thin layers of electrical insulation. In such way, eddy currents are suppressed along with eddy current losses. Winding losses are proportional to the winding resistance and square of electrical current. Mechanical losses are

**Fig. 5.8** Balance of power of electrical machine in motoring mode



**Fig. 5.9** Balance of power of electrical machine in generator mode



consequence of resistance to rotor motion. They are caused mainly by friction in the bearings and air resistance in the air gap. When electrical machine operates in the steady state motoring mode, it takes the power  $P_{eM} = \sum ui_M$  from the electrical power source. In motoring mode, it is convenient to assume the reference direction for the power  $P_{eM}$  and current  $i_M$  from the source toward the machine. During the process of electromechanical conversion, certain amount of energy is lost in magnetic circuit at the rate of  $P_{FeM}$ , also called power losses in iron. In windings, energy is lost at the rate of  $P_{CuM}$ , also called power losses in copper. Internal mechanical power which is transferred to the rotor is the product of electromagnetic torque and speed of rotation,  $T_{em}\Omega_m$ . The motion resistance caused by friction in the bearings and friction in the air results in mechanical losses  $P_{\gamma mM}$ . Power  $P_{mM} = T_{em}\Omega_m - P_{\gamma mM} = T_m\Omega_m$  is transferred via shaft to a work machine. In the motor mode, the source power is the machine input, power  $P_{mM}$  is the output, whereas the sum  $P_{FeM} + P_{CuM} + P_{\gamma mM}$  determines the power of losses. Ratio  $\eta = P_{mM}/P_{eM}$  is the coefficient of efficiency, and it is always less than one. The balance of power for an electrical machine operating in motoring mode is shown in Fig. 5.8.

In the case when machine operates in generator mode, it converts mechanical work to electrical energy. The balance of power for the generator mode is shown in Fig. 5.9. Generator receives mechanical power  $P_{mG}$ , obtained from a hydroturbine, an endothermic motor, or some other similar device.

In Fig. 5.8, the mechanical power is considered positive if it is directed from the turbine toward electrical machine. In this case, the turbine is the source of mechanical power. With the reference directions for power and current adopted for motoring mode (Fig. 5.8), where the power is considered positive when being supplied from the electrical machine and being delivered to the work machine, then the mechanical power  $P_m$  in generator mode has a negative value. Therefore, the reference direction in generator mode is often changed and determined so as to obtain positive values of

power  $P_{mG}$ , received from the turbine, and a positive value of power  $P_{eG}$ , supplied by the generator to electrical circuits and receivers, connected to the stator winding. The same reference direction is usually taken for the electrical current  $i_G$ .

Power  $P_{mG}$  represents input to the generator, and it comes from the turbine or other source of mechanical power. Within the machine, one part of the input power is lost on overcoming motion resistances encountered by the rotor. By subtracting power  $P_{\gamma mG}$  from input power  $P_{mG}$ , one obtains internal mechanical power which is converted to electrical power. One part of the obtained electrical power is lost in windings, where the copper losses are  $P_{CuG}$ , and in magnetic circuit, where the iron losses are  $P_{FeG}$ . The remaining power is at disposal to electrical consumers supplied by the generator. At the ends of stator winding, one obtains currents and voltages which determine the generated electrical power  $P_{eG} = \sum ui_G$ , which can be transferred to electrical consumers.

The coefficient of efficiency can be increased by designing the machine to have reduced losses windings (copper losses) and magnetic circuit (iron losses). By increasing the cross-section of conductors, the resistance of copper conductors is decreased which leads to reduced copper losses. By increasing the cross-section of magnetic circuit, the magnetic induction decreases for the same flux. Consequently, the iron losses are smaller. On the other hand, this approach to reducing the current density and magnetic induction leads to an increased volume and mass of the machine. The specific power, determined by the ratio of the power and mass of the machine, becomes smaller as well. Therefore, for an electrical machine of predefined power, decreased current and flux densities lead to an increase in quantities of copper and iron used to make the machine. At the same time, dimensions of the machine are increased as well.

Design policy of reducing the flux and current densities decreases the overall energy losses in copper and iron in the course of electrical machine service. Nevertheless, the overall effects of this design policy may eventually be negative. Namely, the increase in efficiency is obtained on account of an increased consumption of iron and copper. At this point, along with the energy spent during the operating lifetime of electrical machines, it is of interest to take into account the energy spent in their manufacturing. Production of the electrolytic copper, used to make the windings, requires considerable amounts of energy. The same way, production of insulated iron sheets for making magnetic circuits requires energy. Therefore, the energy savings due to reduced copper and iron losses are counteracted by increased energy expenditure in machine manufacturing.

Choice of the flux and current density in an electrical machine is made in the design phase, and it represents a compromise. For machines to be used in short time intervals, followed by prolonged periods of rest, it is beneficial to use increased flux and current density. Increased copper and iron losses are of lesser importance, as the machines are mostly at rest. On the other hand, savings in copper and iron contribute to significant reduction in energy used in machine manufacturing.

Contemporary electrical motors are fed from power converters which can adjust voltages and currents of the primary source to the requirements of machines. Among other things, the possibility of varying conditions of supply is used for the purpose of bringing a machine to the operating regime where power losses are reduced.

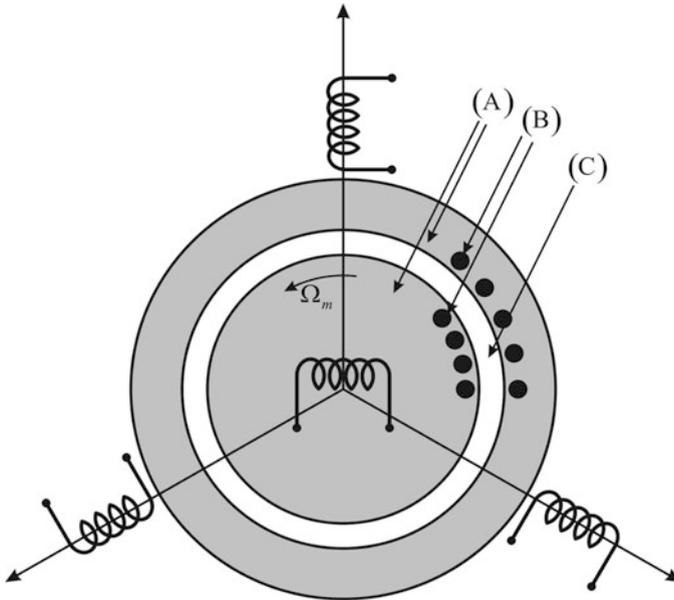
## 5.7 Magnetic Field in Air Gap

Between stator and rotor, there is a clearance, often called *air gap*. In Fig. 5.10, the air gap is denoted by (C). The clearance  $\delta$  is considerably smaller than diameter of the machine and ranges from a fraction of millimeter for very small machines up to 10 mm for large machines.

The permeability of ferromagnetic materials (iron) is very large. Since the flux through the air gap is equal to the flux through magnetic circuit, similar values of magnetic induction are encountered in both the air gap and iron. Since  $\mu_{Fe} \gg \mu_0$ , the magnetic field  $\mathbf{H}$  in iron is negligible. It can be assumed that the magnetic field  $\mathbf{H}$  has a significant, nonzero value  $H_0$  only in the air gap (Fig. 5.11). The contour integral of the field  $\mathbf{H}$  is reduced to the value given by (5.2) which relates the magnetomotive force  $F$  to the line integral of magnetic field along the closed contour:

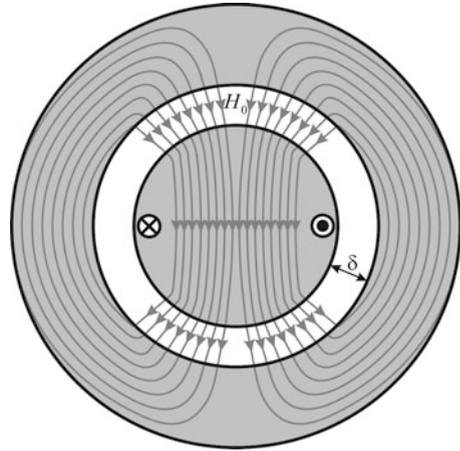
$$F = \sum Ni = \oint_C \vec{H} \cdot d\vec{l} \cong 2H_0\delta. \quad (5.2)$$

Doubled value of the product  $H_0\delta$  in (5.2) exists since the lines of magnetic field pass through the air gap twice, as shown in Fig. 5.11.



**Fig. 5.10** Cross-section of an electrical machine. (A) Magnetic circuits of the stator and rotor. (B) Conductors of the stator and rotor windings. (C) Air gap

**Fig. 5.11** The magnetic field lines over the cross-section of an electrical machine

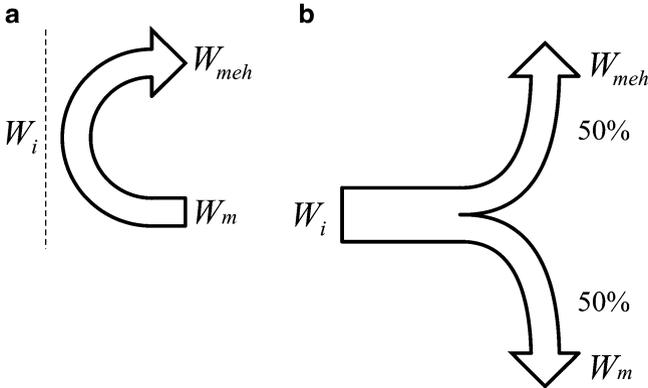


Since  $Ni = 2H_0\delta$ , it is of significance to have smaller gap  $\delta$ . In this way, the required field  $H_0$  can be accomplished with a smaller current in the windings and lower losses. However, there are limits to the minimum applicable air gap. The air gap must be sufficient to ensure that the stator and rotor do not touch under any circumstances. A finite precision in manufacturing mechanical parts and a finite eccentricity of the rotor as well as the existence of elastic radial deformation of the shaft in the course of operation prevent the use of air gaps inferior to 0.5–1 mm. Otherwise, there is a risk that the rotor could scratch the stator in certain operating conditions.

## 5.8 Field Energy, Size, and Torque

Cylindrical electrical machines based on magnetic coupling field develop the electromagnetic torque through an interaction of magnetic field with winding currents. The available electromagnetic torque can be related to the machine size. In addition, the available torque can be estimated from the energy of the magnetic field in the electrical machine.

Cylindrical electrical machines based on magnetic coupling field have an immobile stator and a revolving rotor. The rotor is turning around the axis of cylindrical machine. The axis is perpendicular to the cross-section of the machine presented in Figs. 5.10 and 5.11. The measure of mechanical interaction of the stator and rotor is the torque. When the torque is obtained by action of the magnetic coupling field, it is called the *electromagnetic torque*. The torque is created due to the interaction of the stator and rotor fields. Magnetic fields of stator and rotor can be obtained either by inserting permanent magnets into magnetic circuit or by electrical currents in the windings. Left part in Fig. 5.3 illustrates the torque which tends to align different magnetic poles of the stator and rotor. In order to determine the relation between the



**Fig. 5.12** Energy exchange between the source, field, and mechanical subsystem within one cycle of conversion. (a) Interval when the source is off,  $\Phi = \text{const}$ . (b) Interval when the source is on,  $I = \text{const}$

available torque and the energy accumulated in the coupling magnetic field, it is of use to summarize the process of electromechanical conversion, taking into account the cyclic nature of the process as well as the two phases in one conversion cycle. During the first phase, electromechanical converter is connected to the electrical source. The source supplies the energy which is split in two parts. One part increases the energy accumulated in the coupling field, while the other part feeds the process of electromechanical conversion. During the second phase of the conversion cycles, the electrical source is disconnected from the electromechanical converter, and the mechanical energy is obtained from the energy stored in the coupling field. In Fig. 5.12, marks  $W_i$ ,  $W_{meh}$ , and  $W_m$  denote energy of the source, mechanical energy obtained from the converter, and energy accumulated in the magnetic coupling field, respectively. The cycle of electromechanical conversion in converters with magnetic coupling field is analogous to the cycle of converters based on electrical coupling field, the later being described in Sect. 3.1.4, *Conversion Cycle*.

In the case when the source is separated from machine windings (Fig. 5.12a), the voltage across terminals of the winding is  $u \approx Nd\Phi/dt = 0$ . Neglecting the voltage drop  $Ri$ , the flux in a short-circuited winding is constant. In the absence of electrical source, mechanical work can be obtained only on account of the energy of the coupling field. Therefore, in such conditions,  $dW_{meh} = -dW_m$ . This assertion can be illustrated by example where a mobile iron piece is brought into magnetic field of a coil. A shift  $dx$  of the iron piece produces mechanical work  $dW_{meh} = Fdx$ , where  $F$  is the force acting on the piece. Self-inductance of the coil  $L(x)$  is dependent on the position of the piece of iron, as the piece changes the magnetic resistance to the coil flux. In the case when this coil is separated from the source and short circuited, and resistance  $R$  of the coil is negligible, the first derivative of the

flux is zero, and the flux  $\Psi$  is constant. Variation of the field energy due to a shift  $dx$  is given by (5.3), where  $i$  is the coil current:

$$dW_m = d\left(\frac{1}{2}Li^2\right) = d\left(\frac{\Psi^2}{2L}\right) = -\frac{\Psi^2}{2L^2}dL = -\frac{1}{2}i^2dL. \quad (5.3)$$

Since  $dW_{meh} = -dW_m$ , the force acting on mobile piece of iron in magnetic field of a short-circuited coil can be determined from (5.4):

$$\begin{aligned} dW_{meh} &= F dx = -dW_m \\ \Rightarrow F &= \frac{1}{2}i^2 \frac{dL}{dx}. \end{aligned} \quad (5.4)$$

If the electrical source is connected (Fig. 5.12b), electrical current in the winding is determined by the source current  $I$ . The current is constant, while the flux changes. Upon shift  $dx$ , the source delivers energy  $dW_i = u I dt$ , where  $u = d\Psi/dt$  is the voltage across the coil terminals. Work of the source is given by (5.5):

$$dW_i = u I dt = I d\Psi. \quad (5.5)$$

Since the coil current is constant, the corresponding increase of energy of the magnetic field can be obtained by applying (5.6):

$$dW_m = d\left(\frac{1}{2}LI^2\right) = \frac{1}{2}I^2dL = \frac{1}{2}I d\Psi = \frac{1}{2}dW_i. \quad (5.6)$$

From the previous equation, it follows that work of the source is split to equal parts<sup>2</sup>  $dW_m = dW_{meh} = dW_i/2$ . Expression for the force acting on the mobile piece of iron in cases where the source is connected is  $F = \frac{1}{2}I^2dL/dx$ , which corresponds to expression (5.4), developed for the case when the source is disconnected and the coil is short circuited.

Neither of the two described processes could last for a long time. If the source is disconnected permanently, the energy of the coupling field  $W_m$  is converted to

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<sup>2</sup> Distribution of the work delivered by the source corresponds to expressions  $dW_m = dW_{meh} = dW_i/2$  if the medium is linear, that is, if permeability  $\mu$  of the medium does not depend on the field strength. Then, the coefficient of self-inductance  $L(x)$  depends exclusively on the position  $x$  of the iron piece. Thus, ratio  $\Psi/I$  does not depend neither on flux nor on the electrical current. Under the described conditions, the statements (5.5) and (5.6) are correct. Consequently, relations  $dW_m = dW_{meh} = dW_i/2$  hold as well. In cases when magnetic induction in iron reaches the level of magnetic saturation, the characteristic of magnetization of iron  $B(H)$  becomes nonlinear. The saturation is followed by a drop in permeability  $\mu$  of the medium (iron). In such cases, inductance of the coil is a function of both position and flux,  $L = \Psi/i = f(x, \Psi)$ . Consequently, the expression for increase of the field energy would take another form. Subsequent analysis leads to the conclusion that with nonlinear medium, the work of the source is not to be split in two equal parts.

mechanical work until it is completely exhausted. On the other hand, should the source be permanently connected, one part of the source energy is converted to mechanical work, while the other part increases energy of the field. An increase in the field energy is followed by a raise in the magnetic field  $H$  and magnetic induction  $B$ . Magnetic induction in a magnetic circuit comprising iron parts cannot increase indefinitely. Accumulation of energy  $W_m$  is limited by magnetic saturation in iron sheets, which limits magnetic induction to  $B_{max} < (1.7 \div 2)T$ . Since neither of the two phases in one conversion cycle cannot persist indefinitely, they have to be altered in order to keep the field energy within limits. Therefore, the electromechanical conversion is performed in cycles which include interval when the source is disconnected (left side of Fig. 5.12) and interval when the source is connected (right side of the figure). An interval when the source is connected must be followed by another interval when the source is disconnected in order to prevent an excessive increase or decrease of the energy accumulated in the coupling field.

The expressions for electrical force given by (3.7) and (3.9) can be applied in the cases when the medium is linear, that is, when permeability  $\mu$  of the medium does not depend on the field strength.

All electromechanical converters are operating in cycles. In the first phase, mechanical work is obtained from the source, while in the second phase, it comes from the energy accumulated in the coupling field. The cyclic connection and disconnection of the source does not have to be made by a switch. Instead, the electrical source can be made in such way to provide a pulsating or alternating voltage which periodically changes direction or stays zero for a certain amount of time within each cycle. *In the case of an AC voltage supply, the cycle of electromechanical conversion is determined by the cycle of the supply voltage.* The mechanical work which can be obtained within one cycle is comparable to the energy of the coupling field. In the example presented in Fig. 5.12, energy of the coupling field assumes its maximum value  $W_{m(max)}$  at the instant when the source is switched off. If the field energy is reduced to zero at the end of the cycle, then the mechanical work obtained during one full cycle is twice the peak energy of the field,  $W_{meh(1)} = 2W_{m(max)}$ . For rotating machines, one cycle is usually determined by one turn of the rotor.<sup>3</sup> Mechanical work obtained by action of the electromagnetic torque  $T_{em}$  during one turn is equal to the product of the torque and angular path ( $2\pi T_{em}$ ). Therefore, the electromagnetic torque of an electrical machine can be estimated on the basis of the peak energy accumulated in the coupling field.

The relevant values of magnetic field  $H$  are exclusively those in the air gap. In ferromagnetic materials, the field  $H$  is negligible due to a rather large permeability. Therefore, energy of the coupling field is located mainly in the air gap.

Product of the torque  $T_{em}$  and angular speed of rotation  $\Omega_m$  is the mechanical power which is transferred to work machine via shaft. In the case of a generator, the energy is converted in the opposite direction. Namely, mechanical work is

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<sup>3</sup> Exceptions are electrical machines with more than one pair of magnetic poles, explained later on.

converted into electrical energy. Given the reference directions, the product of generator torque  $T_{em}$  and speed  $\Omega_m$  is negative, indicating that the mechanical power is transferred to electrical machine via shaft.

**Question (5.2):** If dimensions of a machine and the peak value of magnetic induction are known, estimate the electromagnetic torque which can be developed by this machine.

**Answer (5.2):** Energy accumulated in the electromagnetic field is located mainly in the air gap. Volume of the air gap is  $V = \pi LD\delta$ , where  $D$  is diameter of the machine,  $L$  is axial length, and  $\delta$  is the air gap. Magnetic induction  $B$  in the air gap depends on electrical current in the winding. Lines of the magnetic field which pass through the air gap enter the ferromagnetic material. Usually, the ferromagnetic parts are made of iron sheets which make up the stator and rotor magnetic circuits. Therefore, induction in the air gap cannot exceed value  $B_{max} \approx 1.7$  T. Excessive values of  $B$  would cause magnetic saturation in the iron sheets. Therefore, the density of energy accumulated in magnetic field in the air gap cannot exceed  $\frac{1}{2} B_{max}^2 / \mu_0$ . The maximum energy of the coupling field can be estimated as  $W_{m(max)} \approx \frac{1}{2} \pi LD\delta B_{max}^2 / \mu_0$ , while the electromagnetic torque of the machine can be estimated by dividing the obtained energy by  $2\pi$ .