Chapter 7
Single-Fed and Double-Fed Converters

In this chapter, examples of single-fed and double electromechanical converters are analyzed and explained. In both cases, the torque changes are analyzed in cases where the windings have DC currents and AC currents of adjustable frequency. Revolving magnetic field created by AC currents in the windings is introduced and explained. Using the previous considerations, some basic operating principles are given for DC current machines, induction machines, and synchronous machines.

Electrical machines where one or more self-inductances L_{kk} vary as a function of angle θ_m are usually called *single-side supplied converters* or *single-fed machines*. It is shown later that these machines may operate and perform electromechanical conversion in conditions where only the stator windings are fed from electrical source. It is also possible to envisage a single-fed electrical machine where only rotor windings are connected to the source, but this is rarely the case. There exist single-fed electrical machines having windings on the stator only, while the rotor contains no windings and has magnetic circuit with magnetic resistance which depends on the flux direction.

Machines where one or more mutual inductances L_{ij} change with angle θ_m are called *double-side supplied converters* or *double-fed* machines. They have windings on both stator and rotor. Exception to this rule is synchronous machines with permanent magnets on the rotor and DC machines with permanent magnets on the stator, which will be considered later. The effect of permanent magnets on building the flux is equivalent to effects of windings with direct current mounted instead of magnets. Therefore, electrical machines with stator windings fed from electrical source and with permanent magnet on the rotor are classified as double-fed machines. The same holds for permanent magnet DC machines.

In most cases, the windings of double-fed machine are fed from two different electrical sources; thus, there are electrical sources for the stator and the rotor. This two-sided power supply is the reason to call this type of machines *double-fed machines*.

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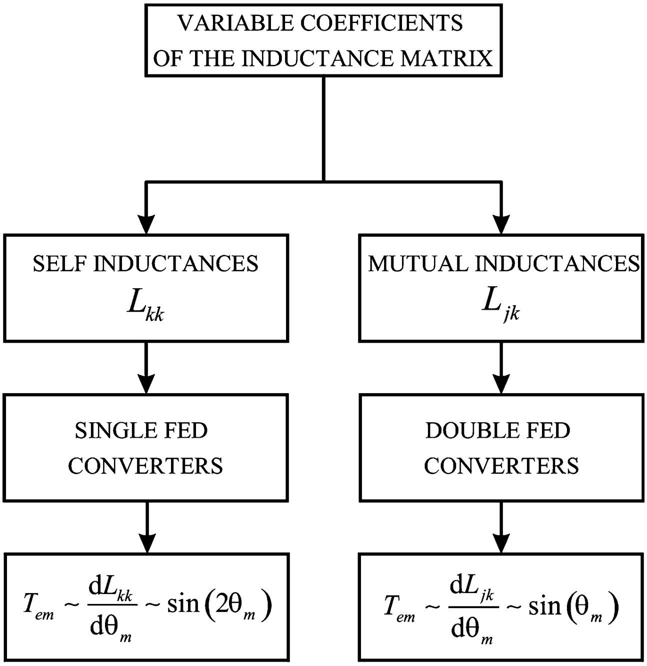


Fig. 7.1 Properties of single-fed and double-fed machines

There are machines whose classification as single- or double-fed is not immediate. An example is induction machine, which is considered later in this book. Induction machine has windings on both stator and rotor. Neglecting the secondary and parasitic effects, it can be stated that self-inductances of the stator and rotor windings of an induction motor are constant, while mutual inductances vary as functions of angle, which may be the basis for classifying induction machines as double-fed machines. Nevertheless, only stator of an induction motor is fed from electrical sources, while rotor winding is short-circuited (*squirrel cage*), and it is not connected to any source. Since an induction motor is fed from the stator side only, it cannot be classified as double-fed machine. On the other hand, there exist induction machines with rotor winding which is separately fed, and these machines truly belong to double-fed machines. Similar dilemma appears when classifying synchronous machine to single-fed or double-fed group. Synchronous machine with permanent magnet on the rotor does not have any rotor windings. Therefore, it is difficult to determine a variable mutual inductance between the stator and rotor winding, as the rotor does not have any windings. On the other hand, a permanent magnet can be represented by a sheet of electrical currents or by a winding with direct current excitation. Thus, there is a basis for classifying permanent magnet synchronous machines as double-fed machines (Fig. 7.1).

7.1 Analysis of Single-Fed Converter

Figure 7.2 shows an elementary single-fed machine. The stator winding has N_1 turns with equivalent resistance R_1, fed from a current source $i(t)$. Depending on variations of the flux and current, there is a voltage $u_1(t)$ across terminals of the winding. Magnetic circuit consists of the immobile stator part with magnetic resistance that is constant. It is considered that induction B_1 in the stator is homogeneous on the magnetic circuit cross-section. Therefore, the flux Φ_1 in one turn can be determined as B_1S_1, where S_1 is the cross-section area of the stator.

Rotor is revolving and its angular displacement from horizontal position is denoted by θ_m. Magnetic circuit of the rotor is made in such way that the magnetic resistance is dependent upon direction of the flux. The rotor is not cylindrical. Instead, it has salient poles. In the case when the rotor is in horizontal position, the stator flux passes through a relatively large air gap of very low permeability (denoted by A in Fig. 7.2). After passing through the air, the flux arrives in the rotor magnetic circuit which is made of high-permeability ferromagnetic material. Then, the flux leaves the rotor magnetic circuit (denoted by B in Fig. 7.2), passing again through the air and entering the stator magnetic circuit. The resulting resistance of the magnetic circuit is then relatively high. For vertically positioned rotor, resulting magnetic resistance is much lower. The field lines pass through a very small air gap, and the magnetic resistance is relatively low. Self-inductance of the stator winding is $L_1 = N_1^2/R_\mu$, where R_μ denotes magnetic resistance across the path of the stator flux. Since the magnetic resistance varies as function of angle, the self-inductance is also variable, fulfilling the requirements of electromechanical conversion.

For the considered machine, the magnetic resistance is variable. Magnetic resistance is also called *reluctance*. For this reason, this type of machine is called reluctant machine, and the torque developed in this machine is called *reluctant torque*.

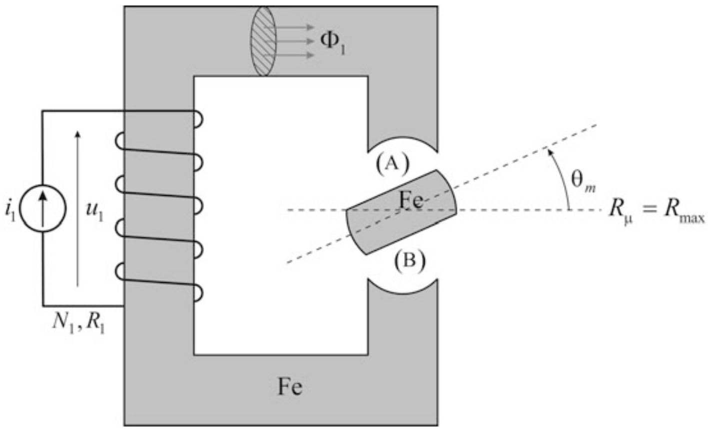


Fig. 7.2 Single-fed converter having variable magnetic resistance

7.2 Variation of Self-inductance

Figure 7.2 shows single-fed converter with variable magnetic resistance. This converter has only one winding; thus, the inductance matrix has only one element, self-inductance of the stator winding $L_1(\theta_m)$. Modeling the process of conversion requires the following function to be known:

$$L_1(\theta_m) = \frac{N_1^2}{R_\mu(\theta_m)}$$

Magnetic resistance R_μ is the ratio of the magnetomotive force $F_1 = N_1 i_1$ and the flux in a single turn $\Phi_1 = B_1 S_1$. Flux of the stator winding is $\Psi_1 = N_1 \Phi_1$. In accordance with the adopted notation, flux of the core is also flux through the contour representing one turn, and it is denoted by Φ. On the other hand, *winding flux* is denoted by Ψ. In a magnetic circuit of cross-section S_{Fe} having an air gap δ and iron core where the intensity H of magnetic field is rather small, magnetic resistance is $R_\mu = \delta/(\mu_0 S_{Fe})$.

Magnetic resistance $R_\mu(\theta_m)$ of the converter given in Fig. 7.2 has its minimum when the rotor is in vertical position. This occurs in the case when $\theta_m = \pi/2$ or $\theta_m = 3\pi/2$. The magnetic resistance is at its maximum when the rotor is in horizontal position. These are the cases with $\theta_m = 0$ or $\theta_m = \pi$, as shown in Fig. 7.3. During the rotor revolution, magnetic circuit in Fig. 7.3 changes in a way that can be modeled assuming that the air gap is variable. It can be concluded that function $R_\mu(\theta_m)$ is periodic with the period of π. For this reason, function $L_1(\theta_m)$ is also periodic and it has the same period. Actual variation of the self-inductance is dependent on the shape of the stator and rotor magnetic circuits.

In order to facilitate the analysis and get to conclusions, function $L_1(\theta_m)$ is approximated by the following trigonometric function:

$$L_1(\theta_m) \approx L_{\min} + \frac{L_{\max} - L_{\min}}{2}(1 - \cos 2\theta_m).$$

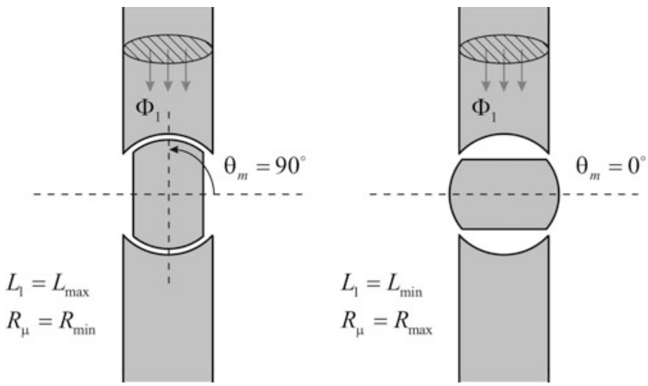


Fig. 7.3 Modeling variations of the magnetic resistance and self-inductance of the winding

which satisfies conditions $L_1(\pi/2) = L_{max}$, $L_1(3\pi/2) = L_{max}$, $L_1(0) = L_{min}$, and $L_1(\pi) = L_{min}$. Therefore, during one turn of the rotor, the inductance has two minima and two maxima. Function $L_1(\theta_m)$ can be represented in the form given by (7.1):

$$L_1(\theta_m) = \frac{1}{2}(L_{min} + L_{max}) - \frac{1}{2}(L_{max} - L_{min})\cos 2\theta_m. \tag{7.1}$$

7.3 The Expressions for Power and Torque

It is of interest to determine the electromagnetic torque and power of electromechanical conversion of the single-fed converter presented in Fig. 7.2. In the case when the stator winding is connected to a current source $i_1(t)$, the energy accumulated in the magnetic coupling field is

$$W_m = \frac{1}{2}L_1(\theta_m)\cdot i_1^2.$$

Since the winding is connected to the current source, the torque can be determined as the first derivative of the field energy W_m. By using expression (7.1) for self-inductance $L_1(\theta_m)$, the electromagnetic torque is determined by (7.2):

$$T_{em} = \frac{dW_m}{d\theta_m} = \frac{1}{2}i_1^2\frac{dL_1}{d\theta_m} = \frac{1}{2}i_1^2(L_{max} - L_{min})\sin 2\theta_m. \tag{7.2}$$

The obtained torque is proportional to the difference between the maximum and minimum inductance, current squared, and sine of doubled angle. Since the flux is proportional to the current, the electromagnetic torque can be also expressed as a function of the flux squared,

$$T_{em} \sim i^2 \sim \Phi^2. \tag{7.3}$$

The highest value of the electromagnetic torque is obtained for $\theta_m = \pi/4$ and $\theta_m = 5\pi/4$, while in positions $\theta_m = \pi/2$, $\theta_m = 3\pi/2$, $\theta_m = 0$, and $\theta_m = \pi$, the torque is equal to zero. In the case when stator current is constant, $i_1(t) = I_1$, and the rotor is moving at a constant speed Ω_m, the torque is proportional to function sin $(2\Omega_m t + \theta_0)$ and its average value is equal to zero. For this reason, the average value of power of electromagnetic conversion is also equal to zero. In other words, the converter shown in Fig. 7.2 with constant (DC) current in the winding cannot perform electromechanical conversion since the average torque and power in one revolution are both equal to zero.

The torque and power with an average values different than zero can be obtained where the winding has an alternating current. For current with angular frequency ω_s, with amplitude I_m, and with initial phase $-\varphi$, squared instantaneous current value is

$$i_1^2 = [I_{\max}\sin(\omega_s t - \varphi)]^2 = \frac{I_{\max}^2}{2}(1 - \cos(2\omega_s t - 2\varphi)).$$

On the basis of (7.2), the electromagnetic torque of single-fed converter with alternating current in its winding is

$$T_{em} = \frac{I_{\max}^2}{4}(1 - \cos(2\omega_s t - 2\varphi)) \cdot (L_{\max} - L_{\min})\sin 2\theta_m.$$

By introducing constant

$$K_m = \frac{I_{\max}^2}{8}(L_{\max} - L_{\min}),$$

expression for the electromagnetic torque obtains the following form:

$$\begin{aligned} T_{em} &= 2K_m \sin 2\theta_m - 2K_m \cos(2\omega_s t - 2\varphi)\sin 2\theta_m \\ &= 2K_m \sin 2\theta_m - K_m \sin(2\theta_m + 2\omega_s t - 2\varphi) \\ &\quad - K_m \sin(2\theta_m - 2\omega_s t + 2\varphi). \end{aligned}$$

Since the rotor revolves at a constant speed, position of the rotor is determined by expression $\theta_m(t) = \Omega_m t + \theta_0$. Taking into account that $\theta_m(0) = \theta_0 = 0$, position of the rotor takes the value $\theta_m(t) = \Omega_m t$; thus, the torque is equal to

$$\begin{aligned} T_{em} = {} & 2K_m \sin 2\Omega_m t - K_m \sin(2\Omega_m t + 2\omega_s t - 2\varphi) \\ & - K_m \sin(2\Omega_m t - 2\omega_s t + 2\varphi)\,. \end{aligned} \tag{7.4}$$

The first member in the above expression is a harmonic function with average value equal to zero. The same conclusion applies to the second member, except in cases where $\Omega_m + \omega_s = 0$. The third member has a nonzero average value if $\Omega_m = \omega_s$. Therefore, a nonzero average value of the torque can be obtained if the angular frequency of stator current is equal to the angular frequency of rotation. The torque is also dependent on the initial phase of the current. By selecting the corresponding phase, one may accomplish either positive or negative average torque. In the case when $\Omega_m = \omega_s$ and $\varphi = 3\pi/4$, average value of the electromagnetic torque is

$$T_{av} = \frac{I_{\max}^2}{8}(L_{\max} - L_{\min}).$$

For initial phase $\varphi = \pi/4$, average value of the electromagnetic torque is

$$T_{av} = -\frac{I_{\max}^2}{8}\left(L_{\max} - L_{\min}\right).$$

It should be noted that operation of the considered machine is based on simultaneous variation of current and rotor position. Namely, the alternating current needs to have the angular frequency ω_s equal to the rotor speed Ω_m. In other words, mechanical and electrical phenomena are to be *synchronous*.

Question (7.1): In the case when current i_1 is constant, prove that in cases with the rotor stopped at position $\theta_m = \pi/2$, the rotor stays in stable equilibrium, while at position $\theta_m = 0$, the rotor is in unstable equilibrium.

Answer (7.1): Electromagnetic torque given by (7.2) is proportional to $\sin(2\theta_m)$. If the rotor is stopped at position $\theta_m = \pi/2$, electromagnetic torque is equal to zero. A hypothetically small displacement $\Delta\theta$ in positive direction places the rotor in a new position where $2\theta_m = \pi + 2\Delta\theta$, where $\sin(2\theta_m) < 0$. A negative torque arises, tending to drive the rotor back to the previous position. In the case when the rotor makes a small move $\Delta\theta$ to negative direction, a positive torque arises, tending to return the rotor to the previous position. In the case when the rotor is stopped at position $\theta_m = 0$, the equilibrium is unstable. A hypothetically small movement $\Delta\theta$ in positive direction leads to creation of a positive torque, proportional to factor sin ($\Delta\theta_m$). Positive torque tends to increase the initial displacement and drive the rotor away from the initial position. The deviation is also cumulatively increased if a hypothetically small movement $\Delta\theta$ is made in the negative direction.

Question (7.2): Is it possible to accomplish a nonzero average value of the torque using an alternating current of angular frequency $\omega_s \neq \Omega_m$?

Answer (7.2): On the basis of (7.4), nonzero average value of the torque can be obtained also in the case when angular frequency of the current is $\omega_s = -\Omega_m$.

7.4 Analysis of Double-Fed Converter

A double-fed machine shown in Fig. 7.4 has windings on both moving and still parts. Stator has the magnetic circuit and winding with N_1 turns having resistance R_1. The current in the stator winding is $i_1(t)$, while the voltage $u_1(t)$ across terminals of the winding depends on variations of the flux and current. The rotor has cylindrical magnetic circuit and built-in rotor winding with resistance R_2 and with electrical current $i_2(t)$. Depending on variations of the flux and rotor current, the voltage across terminals of the rotor winding is $u_2(t)$. Electromagnetic coupling between the stator and rotor is accomplished by variable mutual inductance.

One part of the field lines representing the flux in the stator winding passes through magnetic circuit of the rotor and through rotor winding, and this part is

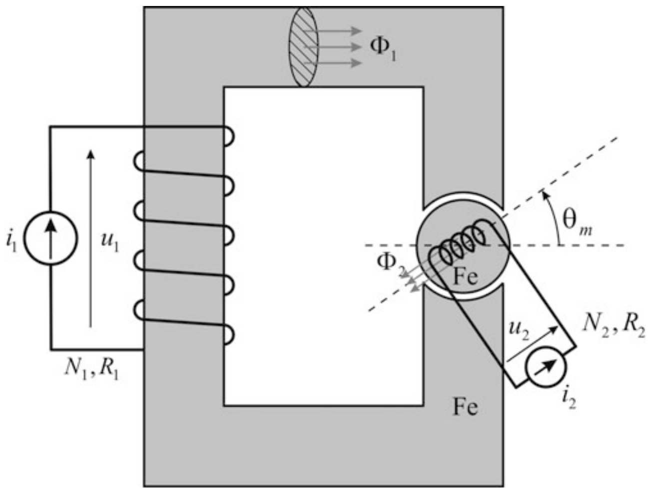


Fig. 7.4 Double-fed electromechanical converter with magnetic coupling field

called mutual flux. Since the rotor is cylindrical, it gives the same magnetic resistance in all directions. It is therefore called *isotropic*. The air gap is constant; thus, rotation of the rotor does not cause any variation of magnetic resistance along the stator flux path. Therefore, the self-inductance of the stator winding is constant.

For the particular form of the stator magnetic circuit shown in Fig. 7.4, it appears that the rotor self-inductance L_2 would change in the course of rotor revolution. The variation of self-inductance is not the key property of double-fed converters. Nevertheless, the variation of L_2 will be briefly explained for clarity. Direction of the rotor flux is determined by the position of the magnetic axis of the rotor winding, that is, by the angle θ_m. This angle determines the displacement between the rotor magnetic axis and the horizontal axis. As the rotor turns, the rotor flux is facing magnetic resistance which is dependent on the rotor position θ_m. Namely, for $\theta_m = \pi/2$, the rotor flux is passing through a relatively small air gap and it enters the magnetic circuit of the stator. When $\theta_m = 0$, the rotor flux passes from the rotor magnetic circuit into the surrounding airspace with permeability and high magnetic resistance. With $\theta_m = \pi/2$, the path of the rotor flux through the air is shorter compared to the rotor flux path through the air for $\theta_m = 0$. Therefore, the magnetic resistance and self-inductance of the rotor are both dependent on position θ_m. Variation of L_2 is dependent upon the shape of magnetic circuit. Assuming that stator magnetic circuit is modified in such way that it firmly embraces the rotor cylinder, variation of inductance L_2 would be smaller. In cases where both stator and rotor magnetic circuits are cylindrical (see Fig. 5.10.), the rotor self-inductance L_2 remains constant and does not depend on the rotor position.

In the subsequent analysis of the operation of a double-fed converter, variation of the rotor self-inductance as function of the shift θ_m is neglected, and it is assumed that $L_2 = \text{const}$.

7.5 Variation of Mutual Inductance

Mutual inductance between the stator and rotor windings is dependent on the rotor position θ_m. When the rotor is in position where the rotor magnetic axis is horizontal, magnetic axes of stator and rotor windings are perpendicular. The lines of the stator flux do not affect the flux through the rotor turns, nor do the lines of the rotor flux contribute to the flux in the stator turns. Therefore, with the rotor axis in horizontal position, the mutual inductance L_{12} is equal to zero. On the other hand, in positions $\theta_m = \pi/2$ and $\theta_m = 3\pi/2$, magnetic coupling between the windings is strong, magnetic axes of the two windings reside on the same line, and mutual inductance L_{12} reaches its maximum absolute value L_m. The sign of the mutual inductance depends on relative position between magnetic axes of the two windings. Physically, the question is whether the fluxes add or subtract. When magnetic axes are oriented in the same direction, a positive current in one winding tends to increase the flux in the other winding. Therefore, the mutual inductance is positive. In cases where magnetic axes of stator and rotor are in opposite directions, a positive current in one winding tends to decrease the flux in the other winding and the mutual inductance is negative. Variation of the mutual inductance with the rotor angle θ_m depends on the shape of magnetic circuit and also on the distribution of conductors making up the windings. In majority of cases, this inductance can be approximated by

$$L_{12}(\theta_m) = L_m \sin \theta_m.$$

The inductance matrix expresses the total flux of the stator Ψ_1 and total flux of the rotor Ψ_2 in terms of the winding currents i_1 and i_2. Self-inductances $L_{11} = L_1$ and $L_{22} = L_2$ are positioned along the main diagonal of the matrix, while the remaining matrix elements are equal to the mutual inductance between the two windings $L_{12} = L_{21} = L_m \sin \theta_m$, as illustrated in Fig. 7.5.

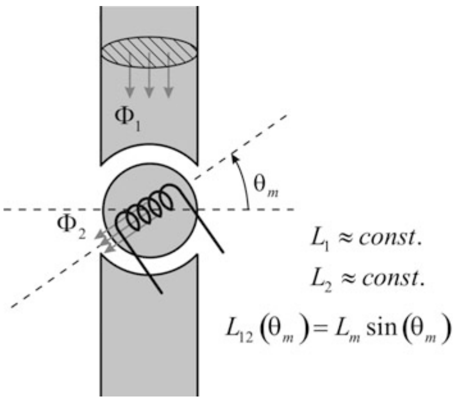


Fig. 7.5 Calculation of the self-inductances and mutual inductance of a double-fed converter with magnetic coupling field

$$\underline{\Psi} = \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} L_1 & L_{12} \\ L_{12} & L_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \underline{L}\ \underline{i}. \tag{7.5}$$

The energy of magnetic field can be expressed as function of currents and elements in the inductance matrix. Expression for the energy can be given in the form of a sum, in the matrix form, or as a scalar expression

$$\begin{aligned} W_m &= \frac{1}{2} \sum_{j=1}^{2} \sum_{k=1}^{2} L_{jk} i_j i_k \\ &= \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + L_{12} i_1 i_2 = \frac{1}{2}\, \underline{i}^T \underline{L}\, \underline{i}. \end{aligned} \tag{7.6}$$

7.6 Torque Expression

Since the mutual inductance is variable and it changes with the rotor position, one of the three members in the expression for the field energy (7.6) varies with angle θ_m in the following manner:

$$L_{12} i_1 i_2 = i_1 i_2\ L_m \sin\ \theta_m. \tag{7.7}$$

The electromagnetic torque can be determined as the first derivative of the field energy W_m. Expression for the electromagnetic torque, given by (7.8), shows that the torque is proportional to the product of the currents in the stator and rotor windings and that it is dependent on the mutual inductance L_m as the angle θ_m. Namely, it changes with $\cos\theta_m$.

$$T_{em} = \frac{\mathrm{d}W_m}{\mathrm{d}\theta_m} = \frac{\mathrm{d}}{\mathrm{d}\theta_m}\left[\frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + L_{12} i_1 i_2\right] = i_1 i_2 L_m \cos\theta_m. \tag{7.8}$$

The electromagnetic torque of a double-fed machine can be expressed as a product of two currents but can also be written as a product of two fluxes. In order to prove this statement, it is necessary to express the currents in terms of the fluxes, which is accomplished by inverting the inductance matrix,

$$\underline{\Psi} = \underline{L}\ \underline{i} \quad \Rightarrow \quad \underline{i} = \underline{L}^{-1}\ \underline{\Psi}\ ,$$

resulting in the expressions for the currents

$$\begin{aligned} i_1 &= \frac{L_2}{L_1 L_2 - L_{12}^2}\ \Psi_1 - \frac{L_{12}}{L_1 L_2 - L_{12}^2}\ \Psi_2, \\ i_2 &= \frac{-L_{12}}{L_1 L_2 - L_{12}^2}\ \Psi_1 + \frac{L_1}{L_1 L_2 - L_{12}^2}\ \Psi_2. \end{aligned}$$

By multiplying the above expressions for i_1 and i_2, one obtains the torque expression which comprises the factor $\Psi_1\Psi_2$. On the other hand, since $\Psi_1 = N_1\Phi_1$ and $\Psi_2 = N_2\Phi_2$, the torque can be expressed as function of the product of the fluxes in individual turns,

$$T_{em} \sim i_1 i_2 \sim \Phi_1 i_2 \sim \Phi_2 i_1 \sim \Phi_1\Phi_2.$$

On the basis of the obtained expression, it is possible to conclude:

- In single-fed machine, the electromagnetic torque is proportional to the current squared, ${i_1}^2$. It can be expressed in terms of the total flux squared, ${\Psi_1}^2$, or the flux in one turn squared, ${\Phi_1}^2$.
- In double-fed machine, the electromagnetic torque is proportional to the product of the two winding currents, $i_1 i_2$, or to the product of the two winding fluxes, $\Psi_1\Psi_2$, or to the product of the fluxes in one stator and rotor turn, $\Phi_1\Phi_2$.

7.6.1 Average Torque

Electromagnetic torque in a double-fed machine is given in (7.8). When the rotor revolves at a constant angular speed Ω_m, position of the rotor is $\theta_m = \Omega_m t + \theta_0$. It can be assumed that at the instant $t = 0$, angle $\theta_m(0) = \theta_0$ gets equal to zero; thus, position of the rotor is $\theta_m = \Omega_m t$. If electrical currents in the windings are constant ($i_1 = I_1$ $i_2 = I_2$), it can be concluded that the torque will changed according to cos $\Omega_m t$, with the average value equal to zero. Therefore, a double-fed machine with constant (DC) currents in the stator and rotor windings produces electromagnetic torque with average value equal to zero. Therefore, the average value of the conversion power is equal to zero as well. If one of the currents is variable, it is possible to synchronize its changes with the rotor revolution and obtain a nonzero average torque and power.

Question (7.3): Assuming that the current of the other winding is constant, $i_2 = I_2$, and that $\theta_m = \Omega_m t$, determine the variation of current i_1 which will give the torque with nonzero average value.

Answer (7.3): According to expression (7.8), the electromagnetic torque is determined by the product of functions cos $\Omega_m t$ and $i_1(t)$. Product $i_1(t)$ cos $\Omega_m t$ can have a nonzero average if i_1 is an alternating current with the ω_1 equal to the rotor angular speed Ω_m.

7.6.2 Conditions for Generating Nonzero Torque

The subsequent analysis proves that the electromagnetic torque of a double-fed machine with alternating currents in the stator and rotor windings may assume a nonzero average value, provided that the frequencies of currents and the rotor speed meet certain conditions. The currents can be expressed in terms of their amplitude I, angular frequency ω, and initial phase, $i = I\cos(\omega t - \varphi)$. When both the stator and rotor currents have nonzero angular frequencies, they change periodically, as well as the stator and rotor fluxes. It is also possible to distinguish the case where one of the currents has its angular frequency equal to zero, $\omega = 0$. This actually means that such current does not change, maintaining the value of $i = I\cos(\varphi)$. As a matter of fact, $\omega = 0$ results in a DC current. The currents of the stator and rotor may have different amplitudes, frequencies, and initial phases. Let the angular frequency of the stator current be ω_1, the angular frequency of the rotor current ω_2, the relevant amplitudes I_{1m} and I_{2m}, and the initial phases $-\varphi_1$ and $-\varphi_2$. Instantaneous values of the winding currents are given by (7.9):

$$\begin{aligned} i_1 &= I_{1m}\cos(\omega_1 t - \varphi_1), \\ i_2 &= I_{2m}\cos(\omega_2 t - \varphi_2). \end{aligned} \tag{7.9}$$

By introducing these expressions into (7.8), one obtains the electromagnetic torque as

$$\begin{aligned} T_{em} &= i_1 i_2 L_m \cos\theta_m \\ &= I_{1m}\cos(\omega_1 t - \varphi_1)\cdot I_{2m}\cos(\omega_2 t - \varphi_2)\cdot L_m\cos\theta_m\,. \end{aligned}$$

With $\theta_m = \Omega_m t$, the torque T_{em} is a product of three periodic functions. By introducing coefficient $K_n = L_m I_{1m} I_{2m}/4$, this equation assumes the form

$$\begin{aligned} T_{em} = {} & K_n\cos(\omega_1 t - \varphi_1 + \omega_2 t - \varphi_2 + \Omega_m t) \\ & + K_n\cos(\omega_1 t - \varphi_1 + \omega_2 t - \varphi_2 - \Omega_m t) \\ & + K_n\cos(\omega_1 t - \varphi_1 - \omega_2 t + \varphi_2 + \Omega_m t) \\ & + K_n\cos(\omega_1 t - \varphi_1 - \omega_2 t + \varphi_2 - \Omega_m t). \end{aligned} \tag{7.10}$$

The electromagnetic torque has the amplitude proportional to the mutual inductance and to the product of the amplitudes of the winding currents ($L_m I_{1m} I_{2m} = 4K_n$). Variation of the torque is determined by four cosine functions having different frequencies. Their frequencies can be expressed by $\omega_1 \pm \omega_2 \pm \Omega_m$. For the function $\cos(\omega t - \varphi)$ to assume a nonzero average value, it is necessary that the angular frequency ω is equal to zero. Hence, for the expression (7.10) to have a nonzero average value, one of frequencies $\omega_1 \pm \omega_2 \pm \Omega_m$ has to be equal to zero. Therefore, conclusion is reached that a nonzero average value of the torque T_{em} is obtained in cases where the angular frequencies (ω_1 and ω_2) of electrical currents in the windings and the rotor speed Ω_m meet one out of four conditions given in expression (7.11):

$$\begin{aligned} \Omega_m &= \omega_1 + \omega_2, \\ \Omega_m &= \omega_1 - \omega_2, \\ \Omega_m &= -\omega_1 + \omega_2, \\ \Omega_m &= -\omega_1 - \omega_2. \end{aligned} \tag{7.11}$$

7.7 Magnetic Poles

Double-fed electrical machine has magnetic circuit where it is possible to observe two magnetic poles of the stator and two magnetic poles of the rotor. Position of the *north* magnetic pole of the rotor can be determined as a zone where the lines of magnetic field, created by electrical currents in the rotor windings, come out of the rotor magnetic circuit and enter the air gap. Similarly, one can define the *south* magnetic pole of the rotor, as well as the magnetic poles of the stator. Double-fed machine under the scope has two stator poles and two rotor poles. Since $L_{12} = L_m \sin\theta_m$, it can be concluded that one cycle of variation of the mutual inductance corresponds to one full mechanical rotation of the rotor.

In due course, *multipole* machines will be defined and explained. The matter concerns electrical machines made to have more than one pair of magnetic poles. In most cases, the number of poles on the stator is equal to the number of rotor poles. A four pole machine has two north and two south poles on the stator and the same number of poles on the rotor. Such machine is said to have $p = 2$ pairs of poles. In a four pole double-fed machine, mutual inductance varies as $L_{12} = L_\mathrm{m} \sin(p\theta_\mathrm{m}) = L_\mathrm{m} \sin(2\theta_\mathrm{m})$, thus making two cycles during one revolution of the rotor. A nonzero average value of the torque is obtained in the case when $\pm\,\omega_1 \pm \omega_2 \pm p\Omega_m = 0$, where Ω_m denotes the *mechanical* angular frequency of the rotor motion.

In this book, letter ω denotes the angular frequencies of voltages and currents, while letter Ω_m denotes the speed of rotor motion, also called mechanical angular speed. The former is often referred to as the *electrical* frequency ω, while the later is called *mechanical* speed Ω. Therefrom, mechanical speed Ω_m may have its electrical counterpart $\omega_m = p\Omega_m$.

Later on, revolving vectors are defined representing the spatial distribution of the magnetic induction B, magnetic field H, but also the voltages and currents in multiphase winding. The speed of rotation of such vectors in space is also denoted by letters Ω.

The expressions *electrical* frequency ω and *mechanical* speed Ω will be better defined in the course of presentation, as well as the relation $\omega = p\Omega$. For the time being, it is understood that two-pole machines are considered, resulting in $p = 1$ and $\omega = \Omega$, unless otherwise stated.

7.8 Direct Current and Alternating Current Machines

The analysis of double-fed machines can be used for demonstration of the basic operating principles of DC machines, induction machines, and synchronous machines. The latter two are also called AC machines. These machines will be studied in the remaining part of the book. All three types of machines have windings on both stator and rotor. Rotation of the rotor changes mutual inductance between the stator and rotor windings.

It has been shown that development of electromagnetic torque with nonzero average value requires the electrical stator frequency ω_1, electrical rotor frequency ω_2, and the rotor speed Ω_m[1] to meet the condition $\omega_1 \pm \omega_2 \pm \omega_m = 0$.

DC machines have a DC current in the stator windings ($\omega_1 = 0$), while in the rotor windings they have an AC current. The angular frequency of the rotor currents is determined by the speed of rotation, $\omega_2 = p\Omega_m$[2].

Induction machines have alternating currents in stator windings and alternating currents in rotor windings. According to (7.11), the sum of $p\Omega_m$[3] and rotor frequency ω_2 has to be equal to the stator frequency ω_1. Therefore, $p\Omega_m = \omega_1 - \omega_2$. The rotor mechanical speed Ω_m lags behind ω_1/p by ω_2/p. The rotor frequency $\omega_2 = \omega_1 - p\Omega_m$ of induction machines is also called *slip* frequency, as it defines the slip of the rotor speed behind the value of ω_1/p, determined by the stator frequency and called *synchronous speed*.

Synchronous machines have alternating currents in stator windings, while the rotor conductors carry DC current. Since $\omega_2 = 0$, condition (7.11) reduces to $\omega_1 = p\Omega_m$. Therefore, the rotor speed Ω_m is uniquely determined by the stator electrical frequency, $\Omega_m = \omega_1/p$. Hence, all the two-pole ($p = 1$) synchronous machines connected to the three-phase grid with the line frequency of $f_s = 50$ Hz make 50 turns per second, or 50·60 = 3,000 revolutions per minute (rpm). A four pole ($p = 2$) synchronous machine supplied by $f_s = 60$ Hz runs at $60 \cdot 60/p = 1{,}800$ rpm. Hence, these machines run *synchronously* with the supply frequency and therefore their name.

7.9 Torque as a Vector Product

The principles of operation of DC machines, induction, and synchronous machines as well as the main differences between them are more obvious when the stator and rotor fluxes are represented by corresponding vectors. Electromagnetic torque can

[1] In cases where machine has p pairs of poles, the condition for torque development is $\omega_1 \pm \omega_2 \pm p\Omega_m = 0$. Notation Ω_m is angular speed of rotor motion, hence mechanical speed. Angular frequency $\omega_m = p\Omega_m$ is *electrical* representation of the rotor speed. It defines the period $T_{\omega m} = 2\pi/\omega_m$ which marks passing of north magnetic poles of the rotor against north magnetic poles of the stator. With $p > 1$, this happens more than once per each mechanical revolution. In a machine with $p > 1$ pole pairs, angular distance between the two neighboring north poles is $\Omega_m T_{\omega m} = 2\pi/p$. A four pole machine ($p = 2$) has two north and two south poles. Two north poles are at angular distance of $\Omega_m T_{\omega m} = 2\pi/2 = \pi$. Therefore, any north magnetic pole of the rotor passes against stator north pole twice per turn. In a two-pole machine ($p = 1$), starting from the north magnetic pole, one should pass angular distance of $\Omega_m T_{\omega m} = 2\pi/1 = 2\pi$ in order to arrive at the next north pole, the very same pole from where one started. Namely, a two-pole machine has only one north magnetic pole and one south magnetic pole.

[2] In a two-pole DC machine, the number of pole pairs is $p = 1$. Therefore, $\omega_2 = p\Omega_m = \Omega_m$. With $p > 1$, the condition reads $\omega_2 = \omega_m = p\Omega_m$.

[3] In a two-pole induction motor, $p = 1$.

be expressed as a vector product of the stator and rotor flux vectors. In other words, the torque is obtained by multiplying the amplitude of the stator flux vector, the amplitude of the rotor flux vector, and the sine of the angle between the two vectors. A proof of this statement will be presented later on for all the machines studied in this book. Moreover, the electromagnetic torque developed by an electrical machine can be determined by calculating the vector product of:

- Stator flux and rotor flux vectors
- The stator and rotor magnetomotive force vectors (current vectors)
- The stator flux vector and the rotor magnetomotive force vector (current vector)
- The rotor flux vector and the stator magnetomotive force vector (current vector)

Obtaining electromagnetic torque as vector product of the flux and current can be demonstrated by taking the example of a contour placed in an external, homogeneous magnetic field, as shown in Fig. 7.6. The contour is made of a conductor carrying electrical current I. The conductor is shaped in the form of a flat rectangle of width D and length L, encircling the surface $S = DL$. In the considered position, angle between the normal n_1 on the surface plane and vector of magnetic induction is α. Angle α determines the electromagnetic torque acting on the contour.

Magnetic momentum of the contour is a vector collinear with the normal n_1 on the surface S surrounded by the contour. The orientation of the normal is determined by the direction of electrical current in the contour and the right-hand rule. The amplitude of the magnetic momentum m is determined by the product of the contour current I and the surface S,

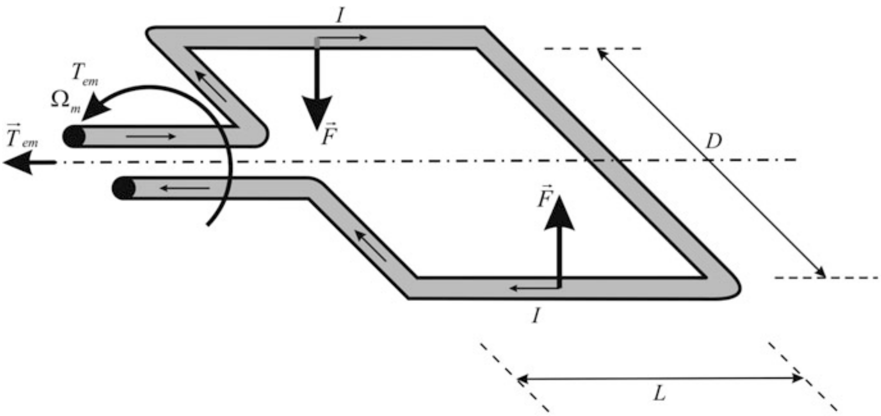
$$\vec{m} = I \cdot S \cdot \vec{n}_1 \tag{7.12}$$

The electromagnetic torque acting on the contour is equal to the vector product of the magnetic momentum m and the magnetic induction B. The torque can be determined from (7.13). In Fig. 7.6, the torque vector extends in the axis of rotation of the contour, and its direction is determined from the coupled forces by the right-hand rule. Maximum value of the torque $T_m = D \cdot L \cdot I \cdot B$ is obtained at position $\alpha = \pi/2$.

$$\vec{T}_{em} = \vec{m} \times \vec{B}, \quad \left|\vec{T}_{em}\right| = S \cdot I \cdot B \cdot \sin\alpha = D \cdot L \cdot I \cdot B \cdot \sin\alpha. \tag{7.13}$$

Result (7.13) can be checked by analyzing the forces acting on parts of the rectangular contour. For contour parts of the length L, orthogonal to the lines of magnetic field, the electromagnetic force is determined by expression $F = L \cdot I \cdot B$. On parts of the contour of the length D, the forces are acting in the direction of rotation, but they are collinear and of opposite directions. Therefore, their opposing actions are canceled. Force arm K is equal to

$$K = \frac{D}{2} \sin\alpha,$$



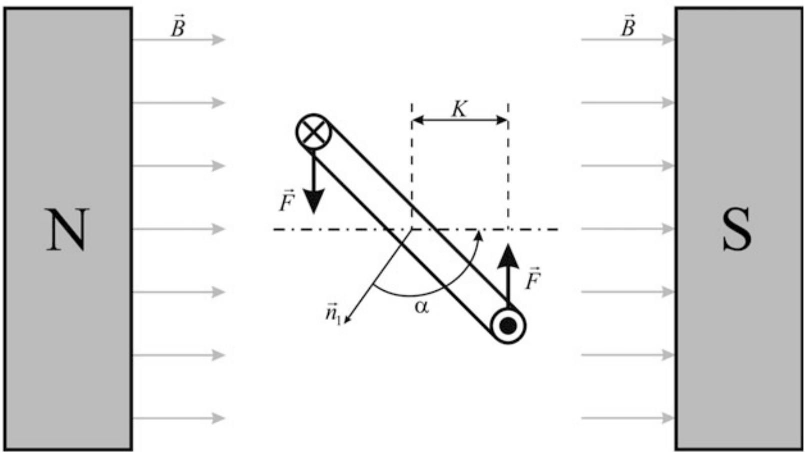


Fig. 7.6 Torque acting on a contour in homogenous, external magnetic field is equal to the vector product of the vector of magnetic induction ***B*** and the vector of magnetic momentum of the contour. Algebraic intensity of the torque is equal to the product of the contour current I, surface $S = L{\cdot}D$, intensity of magnetic induction B, and sin(α). Its course and direction are determined by the normal n_1 oriented in accordance with the reference direction of the current and the right-hand rule

thus, the electromagnetic torque acting on the contour of Fig. 7.6 is

$$T_{em} = 2 \cdot F \cdot K = 2(L \cdot I \cdot B)\frac{D}{2}\sin\alpha = D \cdot L \cdot I \cdot B \cdot \sin\alpha.$$

The preceding expression obtained for the torque can be represented as function of the flux and magnetomotive force. Maximum value of the flux through the contour is $\Phi_m = SB = DLB$, and it is obtained in position $\alpha = 0$. Since the contour has one turn ($N = 1$), current I in the contour is equal to the magnetomotive force $F_m = NI = I$. Starting from expression (7.13), the electromagnetic torque can be expressed as

$$T_{em} = F_m \cdot \Phi_m \cdot \sin\alpha \tag{7.14}$$

Flux through the contour is a scalar quantity. By associating the course and direction of magnetic induction $\boldsymbol{B}$ to the flux Φ, it is possible to conceive the flux vector. Magnetomotive force of the contour is a vector whose orientation is determined by the normal n_1, which is collinear with the vector of magnetic momentum of the contour. Therefore, the value of expression (7.14) is determined by the vector product of the magnetomotive force vector and the flux vector. In a like manner, it can be shown that the electromagnetic torque of a cylindrical rotating machine is determined by the vector product of the magnetomotive force of the stator and the rotor flux. By rearranging the expressions, it is possible to express the torque as the vector product of the stator and rotor fluxes. It is also possible to express the torque in terms of stator and rotor magnetomotive forces or in terms of the stator flux and the rotor magnetomotive force.

7.10 Position of the Flux Vector in Rotating Machines

The stator flux vector and the rotor flux vector of an electrical machine have the spatial orientation which depends on electrical currents they originate from. A DC current in stator windings creates stator flux which does not move relative to the stator. A DC current in rotor windings creates rotor flux which does not move with respect to the rotor. In cases where rotor turns, such rotor flux revolves with respect to the stator at the rotor speed. It will be shown later that a set of stator windings with AC currents may produce stator flux vector which revolves with respect to the stator at a speed determined by the angular frequency of AC currents. More detailed definition of the flux per turn, flux per winding, and the method of representing flux as a vector are given in Chap. 4.

The analysis which shows that the electromagnetic torque of a machine can be determined from the vector product of fluxes and magnetomotive forces is a part of the chapters dealing with DC and AC machines. Induction, synchronous, and DC machines differ inasmuch as they have DC or AC currents in stator and rotor windings.

The electromagnetic torque of DC and AC machines can be determined on the basis of the vector product between the stator and rotor flux vectors. Provided with the stator flux per turn (Φ_S), rotor flux per turn (Φ_R), and with the angle $\Delta\theta$ between the stator and rotor flux vectors, the electromagnetic torque can be calculated from the expression $|\Phi_S \times \Phi_R| = \Phi_S \Phi_R \sin(\Delta\theta)$.

In cases when relative position of the two flux vectors varies according to the law $\Delta\theta = \omega t$, the electromagnetic torque will, according to (7.13), exhibit oscillations

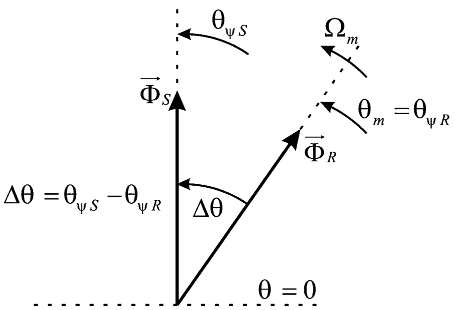


Fig. 7.7 Change of angular displacement between stator and rotor flux vectors in the case when the stator and rotor windings carry DC currents

and change as ($\sin\omega t$). Average value of such torque is equal to zero. In order to create an electromagnetic torque with nonzero average, it is necessary that relative position between the stator and rotor flux vectors does not change. A constant displacement $\Delta\theta$ is obtained in cases where both flux vectors are stationary with respect to the stator but also in cases where the two vectors rotate at the same speed and in the same direction, keeping their relative displacement $\Delta\theta$ constant.

A constant displacement $\Delta\theta$ cannot be achieved in electrical machines that have DC currents in both stator and rotor windings. Namely, windings carrying DC current create a magnetomotive force and flux along the winding axis. Therefore, the flux caused by DC currents cannot move relative to the winding. Therefore, DC currents in stator windings create a stationary stator flux. DC currents in rotor windings create a rotor flux that does not move with respect to the rotor. With the rotor in motion, the rotor flux revolves at the rotor speed, moving in such a way relative to the stator flux. In these conditions, the angle $\Delta\theta$ changes while the electromagnetic torque oscillates and has the average value equal to zero.

In the considered case, the flux vectors are shown in Fig. 7.7. Stator flux Φ_S does not move, while rotor flux Φ_R revolves at rotor speed Ω_m. With $\theta_{\Psi S} = 0$, the angle $\Delta\theta$ between the two vectors is function of the speed of rotor rotation $\Delta\theta = -\Omega_m \cdot t$, while variation of the torque is determined by function $\sin(-\Omega_m \cdot t)$; thus, its average value is zero. In order to accomplish a constant value of the angle between stator and rotor fluxes, both vectors have to be still or moving at the same speed. In any case, one of the windings, stator or rotor, has to create a magnetic field that revolves with respect of the originating winding. Although the principles of operation of the DC machines and induction and synchronous machines are yet to be explained and analyzed in detail, it is of interest to indicate the position of the stator and rotor flux vectors in these machines.

A **DC machine** is shown in the part A of Fig. 7.8. Stator flux is represented by vector $\boldsymbol{\Phi}_S$. Flux Φ_S is immobile, created by DC currents in the stator windings. Rotor flux is represented by vector $\boldsymbol{\Phi}_R$. Flux Φ_R is created by alternating currents in the rotor conductors. Usually, rotor winding has a large number of turns, but in Fig. 7.8a, it is represented by conductors P1 and P2. In these conductors, there is an alternating current with angular frequency of $\omega_2 = \Omega_m$. During one turn of the rotor, currents in conductors P1 and P2 make one full cycle of their periodical change, being positive during one half period and negative during another half period.

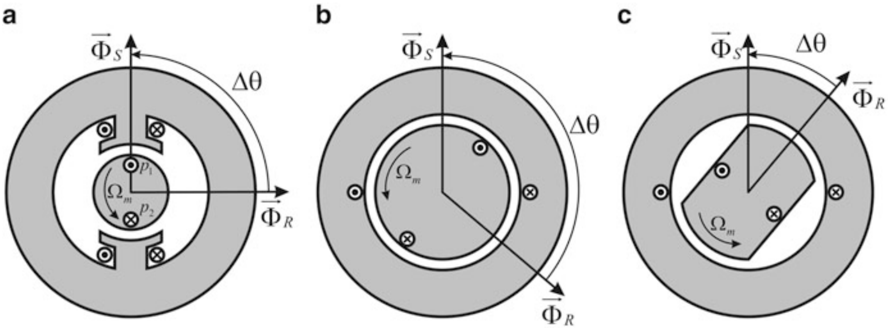


Fig. 7.8 Position of stator and rotor flux vectors in DC machines (**a**), induction machines (**b**), and synchronous machines (**c**)

It is assumed that the rotor revolves at the speed Ω_m. Since the current in rotor conductors changes sign synchronously with rotor revolutions, the current in rotor conductor passing by the south magnetic pole of the stator will always be directed toward the spectator (⊙). In Fig. 7.8a, the rotor is in position where the conductor P1 passes under the south magnetic pole of the stator.

The preceding statement can be supported by the following discussion. In position of the rotor shown in Fig. 7.8a, conductor P2 is below the north magnetic pole of the stator and carries the current directed away from the spectator (⊗). Having passed one half of the rotor turn, conductor P2 comes in place of the conductor P1, below the south magnetic pole of the stator. At the same time, direction of rotor current changes. Hence, in conductor P2, direction (⊗) changes into (⊙). Therefore, direction of the current in the rotor conductor below the south magnetic pole of the stator remains toward the spectator. It can be shown in a like manner that the rotor conductor passing by the north stator pole keeps the direction away from the reader (⊗).

Distribution of rotor currents described above does not move with respect to the stator. Rotor currents create magnetomotive force and flux which are immobile with respect to the stator. What remains unclear at this point is the way of supplying the rotor winding with alternating currents having an angular frequency equal to the rotor speed. This will be explained in more detail in the chapter dealing with DC machines.

Under considerations, the AC currents in rotor conductors create rotor flux vector Φ_R which revolves with respect to the rotor itself. The magnetic field which rotates with respect to the originating windings is called *rotating* or *revolving magnetic field.* Conditions to be met for AC currents to create rotating magnetic field will be explained in more detail in the chapter dealing with induction machines.

In the course of rotation of the rotor in Fig. 7.8a, the rotor flux vector is still with respect to the stator and orthogonal to the stator flux vector, regardless of the speed and direction of rotation. For this reason, rotor magnetic field in a DC machine can be called *halted rotating field.*

Induction machines have AC currents of angular frequency ω_1 in stator conductors. In rotor conductors, there are AC currents with different angular frequency (ω_2). A simplified representation of an induction machine having one pair of magnetic poles ($p = 1$) is shown in Fig. 7.8b. By the previous example of a DC machine, it is shown that AC currents in rotor conductors create rotor magnetic field and rotor flux vector which rotate with respect to the rotor. The speed of rotation of the flux vector with respect to the originating winding is determined by the angular frequency of the winding currents.

In a like manner, stator flux $\boldsymbol{\Phi}_S$ in an induction machine rotates at a speed $\Omega_1 = \omega_1$ with respect to the stator, while rotor flux $\boldsymbol{\Phi}_R$ rotates at a speed $\Omega_2 = \omega_2$ with respect to the rotor. Since the rotor revolves at the speed[4] $\Omega_m = \omega_m$, the speed of rotation of rotor flux is $\Omega_m + \omega_2$. Vector of the stator flux rotates at the speed of ω_1; thus, the speed difference between stator flux vector and rotor flux vector is $\omega_1 - \Omega_m - \omega_2$. According to (7.11) which gives the condition for delivering the power and torque with nonzero average values, the sum $\omega_1 - \Omega_m - \omega_2$ must be equal to zero.

On the basis of previous considerations regarding the operation of induction machines, the following conclusions can be drawn:

- The stator and rotor flux vectors rotate at the same speed. The speed of rotation of the magnetic field in induction machine is Ω_1, and it is determined by the angular frequency ω_1 of the stator currents. In a two-pole machine, this speed is $\Omega_1 = \omega_1$.
- Angle $\Delta\theta$ between the stator and rotor flux vectors is constant in a steady state. Machine provides electromagnetic torque proportional to $\sin(\Delta\theta)$, and it is constant in a steady state.
- Rotor of a two-pole machine revolves at the speed which is different than the speed $\Omega_1 = \omega_1$ of the magnetic field. The speed difference $\omega_2 = \omega_1 - \Omega_m$ is called *slip*. Slip of a two-pole induction machine ($p = 1$) is equal to the angular frequency of the rotor currents (ω_2).

Synchronous machines have AC currents of frequency ω_1 in the stator windings and a DC current in the rotor windings.[5] The stator flux vector $\boldsymbol{\Phi}_S$ of a two-pole ($p = 1$) machine rotates at the speed $\Omega_1 = \omega_1$ with respect to the stator, while the rotor flux vector $\boldsymbol{\Phi}_R$ rotates at the same speed as the rotor, $\Omega_m = \omega_m$. A simplified representation of a two-pole synchronous machine is shown in Fig. 7.8c. Generation of the electromagnetic torque with a nonzero average value requires that relative position of the two flux vectors does not change. In other words, the angle $\Delta\theta$ has to remain constant. For this reason, the rotor speed and the speed of

[4] Example in Fig. 7.8b considers a two-pole machine having $p = 1$ pair of magnetic poles. Due to $\omega = p\Omega$ and $p = 1$, mechanical speed (angular frequency) Ω corresponds to electrical speed (angular frequency) ω.

[5] There exist synchronous machines that have permanent magnets in place of DC excited rotor windings.

revolving stator flux vector have to be the same. Therefore, the stator and rotor flux vectors of a synchronous machine rotate synchronously with the rotor. In a two-pole synchronous machine, angular frequency of stator currents has to be equal to the rotor speed. In machines having several pole pairs ($p > 1$), this condition takes the form $\omega_1 = p\Omega_m$.

7.11 Rotating Field

The analysis carried out in the preceding subsection shows that the condition for developing an electromagnetic torque with a nonzero average value is that relative position $\Delta\theta$ between the stator and rotor flux vectors remains constant. In DC machines, both fluxes are still with respect to the stator, while in AC machines, induction and synchronous, the two fluxes revolve at the same speed.

With the rotor revolving at a speed Ω_m, the angle $\Delta\theta$ can remain constant provided that at least one of the two fluxes (Φ_S or Φ_R) revolves with respect to the winding whose magnetomotive force originates the flux. The magnetic field which rotates with respect to the originating winding is called *rotating magnetic field*. It will be shown later that creation of a rotating field in induction and synchronous machines requires at least two separate windings on the stator, also called *phases* or *phase windings*. With two-phase windings on the stator, the spatial displacement between the winding axes has to be $\pi/2$. The alternating currents in two-phase windings have to be of the same amplitude and the same angular frequency. The difference of their initial phases has to be $\pi/2$, the same as the spatial displacement between the phase windings. In this case, stator currents result in a rotating magnetic field. The amplitude of the stator flux and its speed of rotation can be changed by varying the amplitude and frequency of the stator currents. In this chapter, an introductory example is given, illustrating the generation of a rotating field by the stator with two-phase windings.

Figure 7.9 shows two stator windings with their magnetic axes spatially displaced by $\pi/2$. Axes of the windings are denoted by α and β. The winding in axis α has the same number of turns as the winding in axis β. Both windings carry alternating currents of the same amplitude I_m and frequency ω_S,

$$i_\alpha(t) = I_m \cos(\omega_S t),$$
$$i_\beta(t) = I_m \cos\left(\omega_S t - \frac{\pi}{2}\right) = I_m \sin(\omega_S t),$$

but their initial phases differ by $\pi/2$. Each winding creates a magnetomotive force along its own axis. Magnetomotive force amplitude depends on the current and the number of turns. The winding flux is proportional to the magnetomotive force and inversely proportional to magnetic resistance. If magnetic circuits of the stator and rotor are of cylindrical shape, magnetic resistance R_μ incurred along the flux path

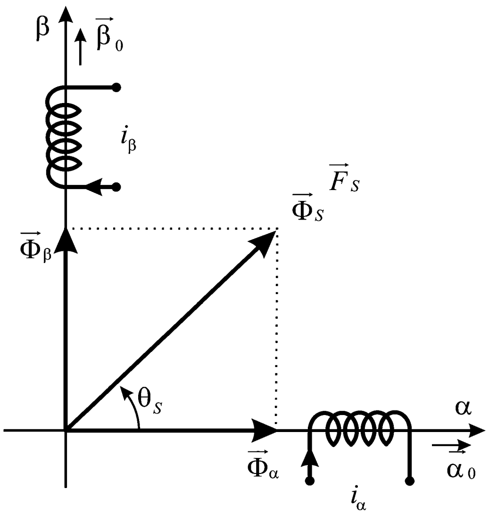


Fig. 7.9 Two stator phase windings with mutually orthogonal axes and alternating currents with the same amplitude and frequency create rotating magnetic field, described by a revolving flux vector of constant amplitude. It is required that initial phases of the currents differ by π/2

does not depend on the flux spatial orientation. For this reason, the magnetic resistance to the flux $\boldsymbol{\Phi}_\alpha$ is equal to the magnetic resistance to the flux $\boldsymbol{\Phi}_\beta$. With both windings having the same number of turns and the same magnetic resistances, the fluxes $\boldsymbol{\Phi}_\alpha$ and $\boldsymbol{\Phi}_\beta$ are obtained by multiplying the number of turns N by electrical currents i_α and i_β, respectively, and dividing the product by the magnetic resistance R_μ. Maximum values of the fluxes Φ_α and Φ_β are

$$\Phi_{\alpha\max} = \frac{N_\alpha I_m}{R_\mu} = \Phi_{\beta\max} = \frac{N_\beta I_m}{R_\mu} = \Phi_m$$

The instantaneous values of the fluxes are

$$\Phi_\alpha(t) = \Phi_m \cos(\omega_S t),$$
$$\Phi_\beta(t) = \Phi_m \sin(\omega_S t).$$

The two fluxes contribute to the resulting flux Φ in the electrical machine, which can be represented by a vector in $\alpha - \beta$ coordinate frame. Functions $\Phi_\alpha(t)$ and $\Phi_\beta(t)$ represent projections of such flux vector on α-axis and β-axis. The amplitude of the resulting flux is Φ_m. With the assumed electrical currents, the resultant magnetic field created by the pair of windings in Fig. 7.9 rotates at the speed $\Omega_S = \omega_S$. During rotation, algebraic intensity of the flux vector does not change and it remains Φ_m. This example demonstrates the possibility for a system of two windings to create magnetic field which rotates with respect to the windings. It is important to notice that the windings must carry alternating currents and that the angular frequency of electrical currents ω_S determines the speed of magnetic field rotation Ω_S.

Rotating magnetic field is a prerequisite for DC, induction, and synchronous machines, analyzed within this book. In each of the three machine types, windings exist with AC currents creating magnetic field that revolves with respect to the winding itself, also called rotating magnetic field.

7.12 Types of Electrical Machines

7.12.1 Direct Current Machines

Electrical machines where the stator winding carries a DC current, while the rotor winding carries AC currents, and where the stator flux vector and the rotor flux vector do not move with respect to the stator are called *DC current machines*. Stator windings of DC machines are fed by DC, *direct current*. Rotor conductors in such machines carry AC currents with the frequency determined by the speed of rotation. The power source feeding a DC machine does not provide AC currents and voltages, but instead it gives DC currents and voltages. The method of directing DC current from the power source into the rotor conductors involves *commutator*, mechanical device explained further on. The action of commutator is such it receives DC source current and feeds the rotor winding with AC currents, the frequency of which is determined by the rotor speed.

Induction and synchronous machines have AC currents in their stator windings. The angular frequency ω_1 of these currents provides a rotating magnetic field. Therefore, these machines belong to the group of *AC machines*. The speed of rotation of the magnetic field is determined by the angular frequency ω_1. It is shown by the analysis of the structure in Fig. 7.9 that a system of two orthogonal stator windings could create magnetic field that revolves at the speed determined by the angular frequency of AC currents. Practical AC machines usually have a system of stator windings consisting of three parts, *three phases*, that is, three-*phase windings*. Magnetic axes of three-phase windings are spatially shifted by $2\pi/3$. The initial phases of AC currents carried by the windings should be displaced by $2\pi/3$ in order to provide rotating field. Amplitude I_m of AC currents determines the algebraic intensity of the flux vector, while the angular frequency $\omega_1 = \omega_S$ determines the speed of rotation Ω_S of the magnetic field.

7.12.2 Induction Machines

In addition to AC currents carried by the stator windings, induction machines also have AC currents in the rotor conductors. Magnetic field created by the stator currents rotates at the speed $\Omega_1 = \omega_1$, while the rotor field revolves at the speed $\Omega_2 = \omega_2$ with respect to the rotor. The speeds of rotation of the stator and rotor flux vectors have been discussed in the previous section, where it is shown that the angular frequency of rotor currents, also called the slip frequency, corresponds to the difference between the angular frequency of stator currents and the rotor speed.

7.12.3 Synchronous Machines

Like induction machines, synchronous machines have a system of stator windings with AC currents creating magnetic field which revolves at the speed determined by the angular frequency of stator currents. Currents of the rotor winding of a synchronous machine are constant. They are supplied from a separate DC current source. Rotor current creates the rotor flux which does not move with respect to the rotor. Therefore, the rotor flux rotates together with the rotor and has the same speed Ω_m. There are synchronous machines which do not have the rotor winding. Instead, the rotor flux is obtained by placing permanent magnets within the rotor magnetic circuit. It has been shown before that the torque generation within an electrical machine requires the angle between the stator and rotor flux vectors to be constant. Therefore, the stator flux vector of a synchronous machine has to rotate at the same speed as the rotor. In other words, the stator flux has to move *synchronously* with the rotor.

Among these machines, each type has its merits, limitations, and specific field of application.

Further analysis of electrical machines requires some basic knowledge on the machine windings, skills in analyzing the magnetic field in the air gap, and understanding the principles of rotating magnetic field.