Chapter 20
Steady-State Operation

Mathematical model of electrical machine contains differential and algebraic equations describ,ing the machine operation in given supply conditions and given load. Using the model, it is possible to derive the changes of the rotor speed, electromagnetic torque, the air-gap flux, and phase currents during transients and in steady-state conditions. The model is needed to design the power supply of the machine and to devise control algorithm. At the same time, the model is used to predict performance of the machine in operating conditions of interest and to evaluate whether the machine is suitable for given application.

The study of characteristics of synchronous machines and their performance begins with steady-state analysis. It deals with steady-state operating conditions, where the machine operates with constant speed, torque, and flux amplitude.

In this chapter, mathematical model is used to derive the steady-state equivalent circuit of synchronous machine and to obtain the torque and power expressions in steady-state conditions. Operation at steady state is analyzed and illustrated by means of phasor diagrams. Some basic notions on phasors are reinstated and exercised for isotropic and anisotropic machines. The *power angle* is introduced and defined, and the steady-state torque is expressed in terms of the power angle. This chapter discusses and explains electrical and mechanical properties of synchronous machines supplied from stiff three-phase network. Steady-state operation of synchronous motors and generators is analyzed on the basis of corresponding phasor diagrams, and the expressions for the torque, active power, and reactive power are derived in terms of the power angle.

20.1 Voltage Balance Equations at Steady State

Park transform of the state variables has been carried out with the aim of obtaining the model of a synchronous machine where all the state variables are constant at steady state. Projections of vectors of the stator current, voltage, and flux on *dq* axes

S.N. Vukosavic, *Electrical Machines*, Power Electronics and Power Systems,
DOI 10.1007/978-1-4614-0400-2_20,

of synchronously rotating coordinate frame do not change in steady-state conditions. Therefore, their first-time derivatives are equal to zero. This facilitates obtaining the steady-state equivalent scheme. At steady state, the complex numbers $\underline{u}_{dq}$, $\underline{i}_{dq}$, and $\underline{\Psi}_{dq}$, which represent the vectors of the stator voltage, current, and flux, become complex constants. Their constant, steady-state values are denoted by $\underline{u}_s$, $\underline{i}_s$, and Ψ_s. These complex constants have their amplitude and angle and therefore may be treated as phasors. It is important to notice that complex constants $\underline{u}_s$ and $\underline{i}_s$ represent the steady-state voltages and currents of in three-phase stator winding. Hence, unlike common phasors, the numbers $\underline{u}_s$ and $\underline{i}_s$ represent alternating voltages and currents in a three-phase system.

The absolute value of a common phasor corresponds to the rms value of the AC quantity represented by the phasor. On the other hand, the absolute values of complex numbers $\underline{u}_s$ and $\underline{i}_s$ depend on the leading coefficient of Clarke 3Φ/2Φ transform. Namely, relation between the rms value of the stator phase voltages and the absolute value $|\underline{u}_s| = \mathrm{sqrt}(u_d^2 + u_q^2)$ of the complex constant $\underline{u}_s$ is determined by coefficient K of 3Φ/2Φ transform which is used in deriving the two-phase equivalent of the three-phase machine. With $K = 2/3$, the absolute value $|\underline{u}_s|$ is equal to the peak value of the stator phase voltages. To facilitate the analysis of synchronous machines, it is desirable to represent the voltage balance equations by the steady-state equivalent scheme. At steady state, there is no change of the excitation current and the excitation flux; thus,

$$\begin{aligned} i_R &= \text{const.} = I_R \\ L_m I_R &= \Psi_{Rm}. \end{aligned}$$

In synchronous machines with permanent magnets on the rotor, the rotor flux that encircles the stator winding is constant and equal to Ψ_{Rm}. In steady state, the rotor speed is equal to the synchronous speed; thus, $\Omega_m = \Omega_e$. With angular frequency ω_e of the stator voltages and currents, synchronous speed is equal to $\Omega_e = \omega_e/p$, and it determines the speed of rotation of the stator magnetic field. Therefore, $\Omega_m = \Omega_e = \omega_e/p = \omega_m/p$. Synchronously rotating dq coordinate frame is selected so as to have the d-axis collinear with the excitation flux, that is, with the flux of permanent magnets. Therefore, dq frame revolves at the same speed as the rotor. Relation between the electrical frequency ω and the mechanical speed of rotation Ω in multipole machines is $\omega = p\Omega$. Hence, the frequency which appears in voltage balance equations is equal to $\omega_m = p\Omega_m$. At steady state, equation $\omega_e = \omega_m$ holds.

For isotropic synchronous machine that operates in the steady state, the voltage balance equations of the stator windings are given below:

$$\begin{aligned} u_d &= R_s i_d + L_s \frac{\mathrm{d}i_d}{\mathrm{d}t} - \omega_e L_s i_q = R_s i_d - \omega_e L_s i_q \\ u_q &= R_s i_q + L_s \frac{\mathrm{d}i_q}{\mathrm{d}t} + \omega_e L_s i_d + \omega_e \Psi_{Rm} = R_s i_q + \omega_e L_s i_d + \omega_e \Psi_{Rm}. \end{aligned}$$

On the basis of the two previous equations, it is possible to write

$$u_d = R_s i_d - \omega_e L_s i_q = R_s i_d - p\Omega_m L_s i_q,$$
$$ju_q = \mathrm{j}R_s i_q + \mathrm{j}p\Omega_m L_s i_d + \mathrm{j}p\Omega_m \Psi_{Rm}.$$

By adding the two previous equations, (20.1) is obtained, which represents the steady-state voltage balance in the stator windings and which employs the phasors $\underline{u}_s$ and $\underline{i}_s$:

$$\begin{aligned}\underline{u}_s &= R_S \underline{i}_s + \mathrm{j}\omega_e \underline{\Psi}_s = R_S \underline{i}_s + \mathrm{j}\omega_e L_s \underline{i}_s + \mathrm{j}\omega_e \Psi_{Rm} \\ &= R_S \underline{i}_s + \mathrm{j}p\Omega_m L_s \underline{i}_s + \mathrm{j}p\Omega_m \Psi_{Rm}\end{aligned} \tag{20.1}$$

where

$$\underline{u}_s = u_d + \mathrm{j}u_q, \quad \underline{i}_s = i_d + \mathrm{j}i_q.$$

Flux $\Psi_{Rm} = L_m i_R$ is the part of the excitation flux which encircles the stator windings. With permanent magnet excitation, the flux Ψ_{Rm} represents the part of the flux created by permanent magnets which passes through the air gap and encircles the stator windings. Since direct d-axis is collinear with the rotor flux, projection of the flux $\boldsymbol{\Psi}_{Rm}$ on quadrature q-axis is equal to zero. Therefore, in complex notation, the flux $\underline{\Psi}_{Rm}$ is a real number, that is, $\underline{\Psi}_{Rm} = \Psi_{Rm} + \mathrm{j}0$.

20.2 Equivalent Circuit

At steady state, (20.1) takes the form

$$\underline{U}_s = R_S \underline{I}_s + \mathrm{j}\omega_e \underline{\Psi}_s = R_S \underline{I}_s + \mathrm{j}\omega_e L_s \underline{I}_s + \mathrm{j}\omega_e \Psi_{Rm}. \tag{20.2}$$

where the voltage and current are denoted in uppercase letters which designate the steady-state values. Resistance R_S and inductance L_S are the parameters of the stator phase windings, while the product $\omega_e \Psi_{Rm}$ represents the electromotive force. In cases where $\underline{I}_S = 0$, the stator voltage is equal to $\underline{E}_0 = \mathrm{j}\omega_e \Psi_{Rm}$.

Product $\underline{E}_0 = \mathrm{j}\omega_e \Psi_{Rm}$ represents the electromotive force and also the no load stator voltage. This voltage ($\underline{E}_0$) appears across the stator terminals when the stator current is equal to zero. On the basis of the (20.2), the steady-state equivalent circuit can be represented as a series connection of electromotive force $\underline{E}_0$, reactance $X_S = \omega_e L_S$, and resistance R_S. It should be noted that the equivalent circuit shown in Fig. 20.1 corresponds to an isotropic synchronous machine where $L_d = L_q = L_S$.

The voltage $\underline{U}_S$ is shown on the left side of the equivalent circuit, and it represents the stator voltage $\underline{U}_S = U_d + \mathrm{j}U_q$. When using the equivalent circuit

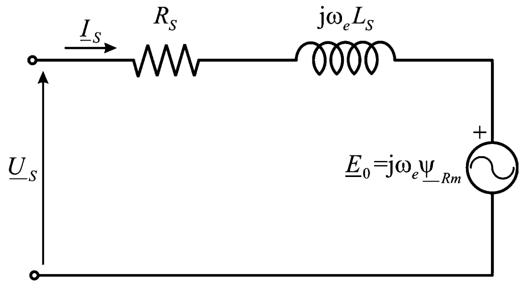


Fig. 20.1 Steady-state equivalent circuit

in Fig. 20.1, where $\underline{U}_S = U_d + jU_q$, it has to be noted that the amplitude of phasors corresponds to peak values of corresponding phase variables. The same equivalent circuit can be used with phasor amplitudes that correspond to rms values.

When synchronous machine operates as motor, electrical power is drawn from the source and brought into machine. The active (real) component of the phasor $\underline{I}_S$ that represents the stator current is directed from the left to the right in Fig. 20.1. When synchronous machine operates as a generator, the phasor $\underline{U}_S$ represents the voltage across the electrical load that receives the electrical energy obtained from the generator. The current supplied to the load is $-\underline{I}_S$. The active component of the stator current in this operating mode is directed from the right to the left, opposite to the reference direction denoted in Fig. 20.1.

20.3 Peak and rms Values of Currents and Voltages

In many cases where the equivalent circuit in Fig. 20.1 is used, it is assumed that the amplitude of relevant phasors corresponds to the peak value of related phase variables. This assumption relies on Clarke transform performed with the leading coefficient $K = 2/3$. Starting with the phase voltages $u_a(t)$, $u_b(t)$, and $u_c(t)$ and applying Clarke and Park transforms, one obtains the stator voltage representation in synchronously rotating coordinate system. The voltage $U_d + jU_q$ has an amplitude $|\underline{U}_S|$ which is equal to the peak value of the phase voltage, U_{eff} ·sqrt(2). Decision to assign the peak values of the phase quantities to phasor amplitudes has to be applied uniquely to all the phasors in the equivalent scheme, namely, to all the voltage, currents, electromotive forces, and flux linkages. Hence, the amplitude $|E_0|$ of the electromotive force corresponds to the peak value of the no load phase voltage. The phasor $\underline{\Psi}_{Rm}$ represents the rotor flux that encircles the stator windings. Its amplitude $|\underline{\Psi}_{Rm}|$ corresponds to the amplitude of the vector $\boldsymbol{\Psi}_{Rm}$, namely, to the peak value the flux reaches in the stator phase windings. The phasor $\underline{\Psi}_S = L_S\underline{I}_S + \Psi_{Rm}$ has the amplitude $|\underline{\Psi}_S|$ which corresponds to the amplitude of the stator flux vector. With the above considerations in mind, the input power to the machine is equal to $P_e = (3/2)\,\mathrm{Re}(\underline{U}_S\underline{I}_S^*)$.[1] The power of losses in the stator windings is calculated as $P_{Cu1} = (3/2)R_S I_S^2$. The power of electromechanical conversion is equal to $P_{em} = P_e - P_{Cu1} = (3/2)\,\mathrm{Re}(\underline{E}_0\,\underline{I}_S^*)$. Since flux vector $\boldsymbol{\Psi}_{Rm}$ coincides with d-axis, which is assigned to be the real axis of the complex $d + \mathrm{j}q$ plane, the electromotive force phasor is equal to $\underline{E}_0 = \mathrm{j}\omega_e\Psi_{Rm}$ and it is collinear with quadrature axis (q), which is at the same time the imaginary axis of the complex $d + \mathrm{j}q$ plane. For this reason, the product $\underline{E}_0\underline{I}_S^*$ assumes the value $\omega_e\Psi_{Rm}I_q$. The electromagnetic torque is equal to the ratio of the power P_{em} and the synchronous speed $\Omega_e = \omega_e/p$:

$$T_{em} = \frac{P_{em}}{\Omega_e} = \frac{3p}{2\omega_e}\omega_e\Psi_{Rm}I_q = \frac{3p}{2}\Psi_{Rm}I_q\,. \tag{20.3}$$

It is also possible to interpret the phasors of the equivalent circuit as complex numbers with amplitudes that represent rms values of relevant phase variables. In this case, the equivalent circuit operates with rms voltages and currents. The phasor $\underline{U}_S$ on the left side of the equivalent circuit represents the stator voltage, and it has an amplitude $|\underline{U}_S|$ that corresponds to the rms value U_{rms} of the stator phase voltages. Now, the electromotive force $|\underline{E}_0|$ corresponds to the rms value of the no load phase voltage. For many, working with rms values of voltages and currents is more handy. When using the phasors that correspond to rms values, the flux phasors have to be treated in the same way. Therefore, the phasor $\underline{\Psi}_{Rm}$, as calculated from the equivalent circuit, obtains an amplitude which is sqrt(2) times smaller than the amplitude of the flux vector $\boldsymbol{\Psi}_{Rm}$. At the same time, the value $|\underline{\Psi}_S| = |L_S\underline{I}_S + \Psi_{Rm}|$ is equal to the amplitude of the stator flux vector divided by sqrt(2). Adopting the phasors that correspond to rms values, the input power to the machine is calculated as $P_e = 3\,\mathrm{Re}(\underline{U}_S\,\underline{I}_S^*)$; the power of losses in the stator winding is obtained as $P_{Cu1} = 3R_S I_S^2$, while the power of the electromechanical conversion is equal to $P_{em} = P_e - P_{Cu1} = 3\,\mathrm{Re}(\underline{E}_0\,\underline{I}_S^*) = 3\omega_e\Psi_{Rm}I_q$. Notice at this point that both Ψ_{Rm} and I_q in the previous expression assume the values that are sqrt(2) times smaller than the values in (20.3). When using phasors that correspond to rms values, the electromagnetic torque is calculated as

$$T_{em} = \frac{P_{em}}{\Omega_e} = \frac{3p}{\omega_e}\omega_e\Psi_{Rm}I_q = 3p\,\Psi_{Rm}I_q\,. \tag{20.4}$$

Question (20.1): A two-pole synchronous machine has the stator self-inductance L_S, while the stator resistance R_S is small and it can be neglected. Machine has permanent magnets on the rotor, and they create the flux Ψ_{Rm} in the stator windings. Machine operates at steady state, connected to a three-phase network of frequency $f_e = 50$ Hz, wherein the rms value of phase voltages is U_{Sn}. The stator voltages

[1] $\mathrm{Re}(\underline{z}) = z_r$ is the real pert of the complex number $\underline{z} = z_r + \mathrm{j}z_i$. The value $\underline{z}^*$ is equal to $\underline{z}^* = z_r - \mathrm{j}z_i$.

have a phase advance δ with respect to corresponding no load electromotive forces. Calculate the power delivered by the network to the machine.

Answer (20.1): The equivalent circuit is analyzed by assuming that the involved phasors represent the rms values. The stator current of the machine is equal to $\underline{I}_S = (\underline{U}_S - \underline{E}_0)/\mathrm{j}X_S$, where $\underline{E}_0 = \mathrm{j}\omega_e \Psi_{Rm}$. The flux $\underline{\Psi}_{Rm} = \Psi_{Rm}$ remains on the real axis. Since the stator voltages lead with respect to electromotive forces by δ, while the electromotive force $\underline{E}_0 = \mathrm{j}\omega_e \Psi_{Rm}$ resides on the imaginary axis, the stator voltages lead with respect to the real axis by $\delta + \pi/2$. Components of the stator voltage in dq frame are $U_d = -U_{Sn}\sin(\delta)$ and $U_q = U_{Sn}\cos(\delta)$. Therefore,

$$\begin{aligned}
\underline{U}_S &= -U_{Sn}\sin\delta + jU_{Sn}\cos\delta, \\
\underline{I}_S &= \frac{\underline{U}_S - \underline{E}_0}{jX_S} = \frac{-U_{Sn}\sin\delta + jU_{Sn}\cos\delta - j\omega_e \Psi_{Rm}}{jX_S} \\
&= \frac{U_{Sn}\cos\delta - \omega_e \Psi_{Rm}}{X_S} + j\frac{U_{Sn}\sin\delta}{X_S}.
\end{aligned}$$

Electrical power received from the network is calculated from the following expression:

$$\underline{S} = 3\underline{U}_S \underline{I}_S^* = 3\frac{U_{Sn}(\omega_e \Psi_{Rm})}{X_S}\sin(\delta) + \mathrm{j}3\frac{U_{Sn}(U_{Sn} - \omega_e \Psi_{Rm}\cos(\delta))}{X_S} = P_e + \mathrm{j}Q_e.$$

The active power received by the machine is $P_e = 3U_{Sn} E_0 \sin(\delta)/X_S$.

20.4 Phasor Diagram of Isotropic Machine

Analysis of steady states based on the equivalent circuit relies on complex notation where the vectors of the stator voltages, currents, and flux linkages are represented by corresponding phasors. The voltage balance in the steady-state equivalent circuit can be represented by phasor diagram. The phasor diagram in Fig. 20.2 represents the balance of voltages in isotropic synchronous machine that operates in motoring mode. d-Axis of synchronously rotating coordinate frame is assigned as the real axis of the complex plane.[2] Thus, the q-axis becomes imaginary axis. Park rotational transform is

[2] Steady-state voltages and currents can be represented as phasors in an arbitrary complex plane. The angle between the phasor and the real axis determines the initial phase of considered AC voltages and currents. Apparently, this imposes a constraint on the choice of the position of the real axis. On the other hand, the initial phase is the value of the phase at the instant $t = 0$. Therefore, by choosing the instant $t = 0$, it is possible to select complex planes with different position of their real and imaginary axes. Phasor diagrams of synchronous machines are mostly drawn in the complex plane where the real axis coincides with d-axis of synchronously rotating coordinate system. It is also of interest to notice that other choices are legitimate as well. When solving some problems, the calculations are more simple when the real axis is selected to be aligned with the stator voltage or the stator current.

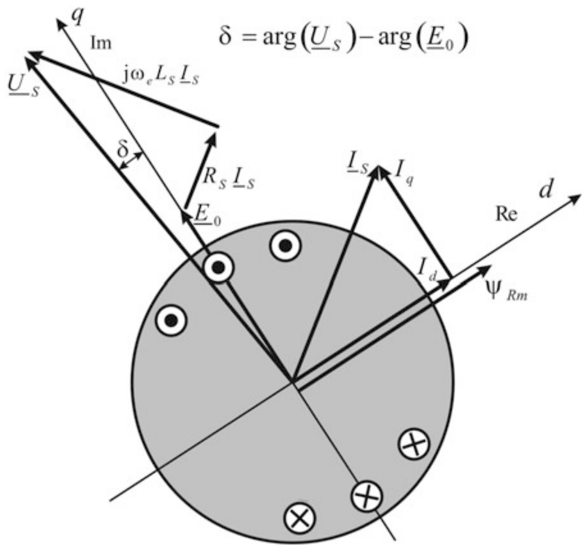


Fig. 20.2 Phasor diagram of an isotropic machine in motoring mode

introduced with d-axis being collinear with the excitation flux. Therefore, the phasor $\underline{\Psi}_{Rm}$ resides on the real axis, and it is equal to $\underline{\Psi}_{Rm} = \Psi_{Rm} + \mathrm{j}0$.

The phasor $\underline{\Psi}_{Rm}$ represents this part of the excitation flux that encircles the stator windings. The no load electromotive force $\underline{E}_0 = \mathrm{j}\omega_e\Psi_{Rm}$ has a phase advance of π/2, and it resides on imaginary axis. The stator voltage $\underline{U}_S$ is obtained by adding the voltage drop across the stator impedance $\underline{Z}_S = R_S + \mathrm{j}\omega_e L_S$ to the no load electromotive force. The voltage drop across the stator resistance is in phase with the stator current $\underline{I}_S$, while the voltage drop across reactance X_S leads by π/2. The angle δ which determines the phase delay of the electromotive force $\underline{E}_0$ behind the voltage $\underline{U}_S$ is called *power angle*. The apparent power of the machine is equal to

$$\begin{aligned}\underline{S} &= 3\underline{U}_S\underline{I}_S^* \\ &= 3\frac{U_{Sn}(\omega_e\Psi_{Rm})}{X_S}\sin(\delta) + \mathrm{j}3\frac{U_{Sn}(U_{Sn} - \omega_e\Psi_{Rm}\cos(\delta))}{X_S} \\ &= P_e + \mathrm{j}Q_e. \end{aligned} \tag{20.5}$$

The active power P_e delivered to the machine by the electrical source is determined by the power angle δ:

$$P_e = 3\frac{U_{Sn}E_0}{X_S}\sin(\delta). \tag{20.6}$$

A positive value of power angle results in a positive power and positive torque. Therefore, in cases where the voltage phasor has a phase advance with respect to the electromotive force, the machine operates as a motor and develops a positive torque.

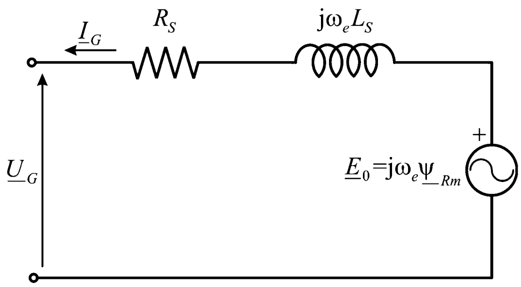


Fig. 20.3 Equivalent circuit suitable for synchronous generators. Reference direction of stator current is altered, $\underline{I}_G = -\underline{I}_S$

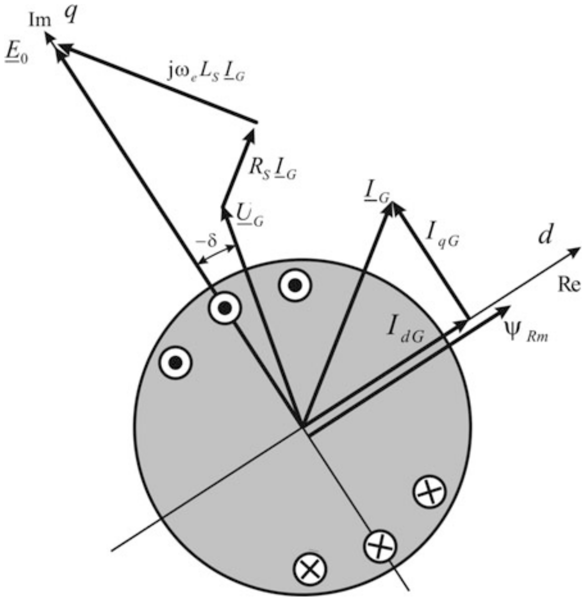


Fig. 20.4 Phasor diagram of an isotropic machine in generating mode

With $\delta < 0$, the voltage lags behind the electromotive force, the power and torque are negative, and thus, the machine operates as a generator.

When an isotropic synchronous machine operates as a generator, the active component of the stator current is negative. Steady-state analysis of synchronous generators is more straightforward if the reference direction of the stator current changes. In Fig. 20.3, the steady-state equivalent circuit is redrawn with altered reference direction for stator current. The new circuit is more intuitive, as it represents the generator which supplies the electrical consumers $\underline{U}_G$ with generator current $\underline{I}_G$. The phasor diagram of an isotropic synchronous generator is given in Fig. 20.4.

At steady state, rotor of synchronous machine revolves at synchronous speed, which is determined by the frequency of the power supply ω_e, this $\omega_e = \omega_m = p\Omega_m$. Whenever a change in the supply voltages or the change in the mechanical subsystem occurs, the machine enters in transient state. An approximate insight of transient behavior of the machine can be obtained from the phasor diagram in Fig. 20.2. In a thought experiment with synchronous motor, a sudden increase in the load torque T_m results in a decrease of the rotor speed. The stator voltage vectors revolve at the speed Ω_e, determined by the supply frequency, while the motion of the electromotive force is determined by the rotor speed. The speed difference $\Omega_e - \Omega_m$ affects the power angle. The electromotive force $\underline{E}_0$ revolves at the same speed as the rotor flux. Therefore, it starts revolving slower than the phasor $\underline{U}_S$, which revolves at constant, synchronous speed $\Omega_e = \omega_e/p$. For that reason, the power angle δ starts increasing. According to (20.6), the input power p_e increases, which results in an increased electromagnetic torque T_{em}. An increased T_{em} counteracts the torque T_m and brings the synchronous machine into a new equilibrium, a new steady-state operating conditions.

Question (20.2): Make a phasor diagram for synchronous machine starting from the example given in Fig. 21.2 and assuming that stator current $\underline{I}_S$ lags behind the electromotive force by 3π/2. The stator resistance R_S is negligible.

Answer (20.2): The electromotive force $\underline{E}_0$ and the stator voltage $\underline{U}_S$ reside on imaginary axis of the diagram. The voltage amplitude is smaller than the electromotive force by the amount of $X_S I_S$.

Question (20.3): A two-pole synchronous machine operates at steady state with power angle of $\delta = 0$. The stator voltage amplitude is equal to the no load electromotive force. With $\underline{U}_S = \underline{E}_0$, the stator current is equal to zero. At instant $t = 0$, the shaft is loaded by the torque T_m in direction opposite to motion. Discuss the changes in the rotor speed. Assume that the number of pole pairs is $p = 1$, resulting in $\Omega_m = \omega_m$ and $\Omega_e = \omega_e$.

Answer (20.3): At steady state, the rotor revolves at synchronous speed. Therefore, relative position of the stator voltage $\underline{U}_S$ and the electromotive force $\underline{E}_0$ does not change. With $\delta = 0$ and $\underline{U}_S = \underline{E}_0$, there is no stator current and no electromagnetic torque. The change of the rotor speed is determined by $J\, \mathrm{d}\Omega_m/\mathrm{d}t = T_{em} - T_m$. Following the increase of the load torque, the rotor speed decreases. The rotor starts lagging behind the voltage $\underline{U}_S$ and it falls behind the stator magnetic field. The voltage phasor $\underline{U}_S$ leads with respect to $\underline{E}_0$; thus, the angle δ increases. This increase affects the input electrical power P_e and the electromagnetic torque T_{em}. As the torque T_{em} increases with δ, it compensates the increase in the load torque T_m and prevents further decrease of the rotor speed. For this transient to decay, it is necessary to restore the rotor speed $\Omega_m < \Omega_e$ to the synchronous speed Ω_e. Therefore, the torque T_{em} must exceed the motion resistance torque T_m for a brief interval of time, so as to achieve a positive value of acceleration $\mathrm{d}\Omega_m/\mathrm{d}t$. This short interval of acceleration is required to restore the rotor speed to the original value, to the synchronous speed Ω_e. Derivative $\mathrm{d}\delta/\mathrm{d}t$ of power angle is determined by the difference between the synchronous speed and the rotor speed, $\mathrm{d}\delta/\mathrm{d}t = \omega_e – \omega_m = p(\Omega_e - \Omega_m)$. New state of equilibrium is reached when $\omega_e = \omega_m$, resulting in $\mathrm{d}\delta/\mathrm{d}t = 0$.

According to (20.6), the power of synchronous machine depends on no load electromotive force, on stator voltage, and on power angle. Electromotive force E_0 is determined by the excitation current. Different pairs of values (E_0, δ) produce the same power and the same torque, provided that the product $E_0\sin(\delta)$ remains unchanged. Hence, the machine can maintain the same power with different values of the excitation current and different values of the excitation flux Ψ_{Rm}. This degree of freedom can be used for to adjust reactive power Q_e (20.6) exchanged between the machine and the supply network.

Considering the sign of reactive power Q_e, there is convention to consider positive the reactive power taken from the network and delivered to electrical loads of inductive character, such as coils, where the load current lags behind the supply voltage. Reactive power taken from the network by receivers such as capacitors is considered negative. Capacitor current leads with respect to the supply voltage. All the loads of this nature can be considered as *generators* of reactive power. Majority of loads and devices connected to distribution networks are of inductive nature, including electrical motors, transformers, and all the devices that include a series inductance. Parallel capacitors are often connected and used as reactive power compensators that make up for the reactive power generated by other loads.

On the basis of (20.5), reactive power taken from the network by an isotropic synchronous machine is equal to

$$Q_e = 3\,\frac{U_{Sn}(U_{Sn} - E_0\cos(\delta))}{X_S}. \tag{20.7}$$

By reducing the excitation current I_R, no load electromotive force $E_0 = \omega_m L_m I_R$ reduces as well, and it may become smaller than the voltage of the network. With $U_{Sn} > E_0$, reactive power Q_e is positive; therefore, the machine acts as an inductive load. With sufficient increase in excitation current and electromotive force E_0, expression (20.7) becomes negative. Thus, the increase in excitation current results in negative values of reactive power. In such case, machine acts as a capacitive load. Hence, the change in excitation current changes the electromotive force and makes the machine absorb or produce reactive power Q_e.

Synchronous generators in hydroelectric and thermal power plants supply active power consumed by all the electrical loads that are connected to the power grid. Most of electrical loads have inductive nature, and they absorb reactive power. Therefore, besides generating the active power, most generators provide reactive power as well. The amount of reactive power produced by generators is controlled by the excitation current. In a symmetrical three-phase system with sinusoidal voltages and currents, relation $S^2 = P^2 + Q^2$ connects apparent power S, active power P, and reactive power Q. Apparent power S in continuous service is limited by rated voltages and currents. Therefore, an increase in reactive power reduces the available active power that the machine can deliver in continuous service.

An increase in reactive power Q_e increases apparent power. Therefore, it increases the stator current as well. The stator current which is sustainable in continuous service is limited due to the copper losses in stator windings. Excessive current results in overheating. Therefore, the steady-state current cannot exceed the rated current I_n. With rated voltages, the rated current results in the rated apparent power S_n. Starting from the steady-state conditions where $P = P_n$ and $Q = Q_n$, an increase in reactive power increases the apparent power above the rated level S_n. In order to avoid overheating, the active power obtained from the generator has to be reduced.

In cases where the reactive power of power consumers is compensated by using parallel capacitors distributed across transmission and distribution networks, reactive power request imposed on synchronous generators is waived, and their active power does not have to be reduced due to reactive power generation.

20.5 Phasor Diagram of Anisotropic Machine

Anisotropic machine has different inductances in virtual stator phases that reside in d- and q-axes of synchronously rotating dq frame. Therefore, phasor diagram of an anisotropic machine is more complex than phasor diagram of isotropic machine. With $L_d \neq L_q$, the voltage balance in stator winding cannot be expressed by relation $\underline{U}_S = \underline{E}_0 + \underline{Z}_S \underline{I}_S = \underline{E}_0 + (R_S + \mathrm{j}\omega_m L_S)\underline{I}_S$ because the voltage drops $L_d \omega_m I_d$ and $L_q \omega_m I_q$ across the stator inductances have different values of self-inductances. Calculation of real and imaginary components of the stator voltage must be calculated separately as they cannot be expressed by $\mathrm{j}\omega_m L_S \underline{I}_S$. Thus, $U_d = R_S I_d - \omega_m L_q I_q$, while $U_q = E_0 + R I_q + \omega_m L_d I_d$. Reactances $L_d \omega_m$ and $L_q \omega_m$ are denoted by X_d and X_q, respectively.

The phasor diagram is drawn for steady-state operation where the electrical representation of the rotor speed $\omega_m = p\Omega_m$ gets equal to the angular frequency of the supply ω_e. The voltage balance equations along the d- and q-axes are

$$
\begin{aligned}
U_d &= -U_s \sin\delta = R_s I_d - \omega_e L_q I_q, \\
U_q &= +U_s \cos\delta = R_s I_q + \omega_e L_d I_d + \omega_e \Psi_{Rm}.
\end{aligned} \tag{20.8}
$$

In these equations, variable $\Psi_{Rm} = L_m I_R$ represents the part of the rotor flux which passes through the air gap and encircles the stator windings. This definition is also used in permanent magnet machine, where variable Ψ_{Rm} represents the part of the flux of permanent magnets which encircles the stator windings. By solving (20.8), one obtains the stator currents I_d and I_q (20.9). Relation between phasors of voltages, currents, electromotive force, and flux in an anisotropic machine are presented in Fig. 20.5:

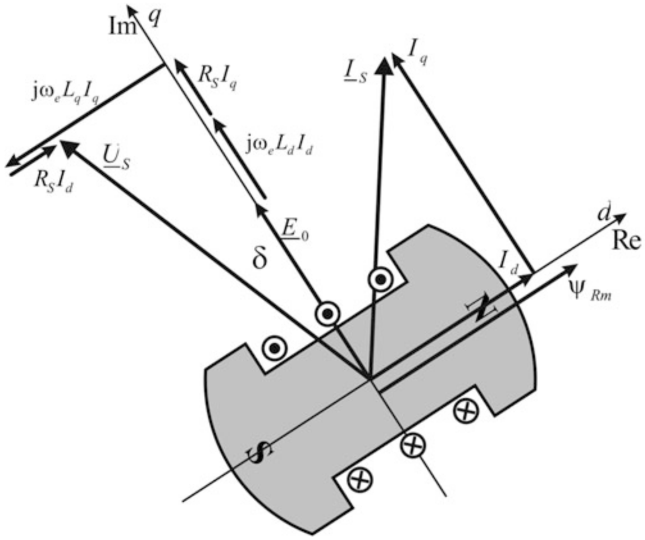


Fig. 20.5 Phasor diagram of an anisotropic machine ($\omega_m = \omega_e$)

$$
\begin{aligned}
I_q &= +\frac{U_s \sin\delta}{\omega_e L_q} = \frac{U_s \sin\delta}{X_q}, \\
I_d &= \frac{U_s \cos\delta - \omega_e \Psi_{Rm}}{\omega_e L_d} = \frac{U_s \cos\delta - E_0}{X_d}.
\end{aligned} \tag{20.9}
$$

20.6 Torque in Anisotropic Machine

By selecting the complex plane with the real axis collinear with the rotor flux, as shown in Fig. 20.5, the stator voltage phasor $\underline{U}_S = U_d + jU_q$ is equal to $-U_S$ sin (δ) + jU_S cos(δ), where $U_S = |\underline{U}_S|$, while δ represents the power angle, the angle between $\underline{U}_S$ and $\underline{E}_0$. On the basis of (20.8), which gives voltage balance in the windings, one can calculate currents I_d and I_q. In large synchronous machines, the voltage drop across the stator resistance R_S can be neglected. From (20.9),

$$
I_q = \frac{U_s \sin\delta}{X_q}, \quad I_d = \frac{U_s \cos\delta - E_0}{X_d}.
$$

Since the notation used above denotes the rms values of voltages and currents, the power absorbed by the machine from the supply network is equal to $P_e = \text{Re}\,(3\underline{U}_S\underline{I}_S^{\,*}) = 3(U_d I_d + U_q I_q)$. With $R_S \approx 0$, there are no significant copper losses in the winding. Moreover, the iron losses in the stator magnetic circuit have been neglected as well. Therefore, the input power P_e gets passed through the air gap to the rotor and converter into mechanical power. Hence, P_e is equal to the power of electromechanical conversion. The torque can be determined by dividing the power P_e by the synchronous speed $\Omega_e = \omega_e/p$:

$$\begin{aligned}T_{em} &= \frac{P_e}{\Omega_e}\\ &= \frac{3p}{\omega_e}\left[-U_S\sin(\delta)\left(\frac{U_S\cos(\delta)-E_0}{X_d}\right)+U_S\cos(\delta)\left(\frac{U_S\sin(\delta)}{X_q}\right)\right]\\ &= \frac{3p}{\omega_e}\frac{U_SE_0\sin(\delta)}{X_d}+\frac{3p}{\omega_e}\frac{U_S^2}{2}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\sin(2\delta) = T_{EXC}+T_{REL}.\end{aligned} \tag{20.10}$$

The torque T_{em} has component T_{EXC} which is the product of the excitation flux and the stator currents, and it depends on the electromotive force and the stator voltage. It is created by electromagnetic forces that result from interaction of the rotor field and the stator currents. This torque component is equal to

$$T_{EXC} = \frac{3p}{\omega_e}\frac{U_SE_0\sin(\delta)}{X_d}. \tag{20.11}$$

The torque component T_{REL} is called *reluctant torque*. It does not depend upon excitation flux Ψ_{Rm}, and it exists even in machines where the excitation flux is equal to zero. Reluctant torque is thrusting the rotor toward position of minimum magnetic resistance. Namely, the rotor is driven in position where the magnetic resistance along the path of the stator flux assumes the smallest magnetic resistance. In cases where $L_d > L_q$, reluctance torque acts toward moving the d-axis in position aligned with the stator flux. In terms of the phasor diagram, it acts toward aligning the q-axis and the stator flux $\underline{U}_S$. The torque component T_{REL} depends on the square of the stator voltage:

$$T_{REL} = \frac{3p}{\omega_e}\frac{U_S^2}{2}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\sin(2\delta). \tag{20.12}$$

20.7 Torque Change with Power Angle

Diagram in Fig. 20.6. shows the torque change of an anisotropic machine in terms of the power angle. The maximum value of the torque is reached for an angle smaller than π/2. The maximum torque is limited by the machine reactances. Likewise the maximum torque of induction machines, the maximum torque of synchronous machines is larger with smaller reactances (inductances).

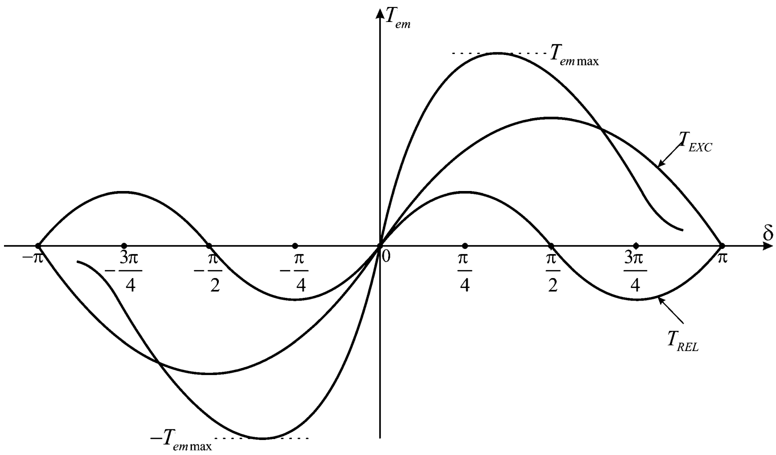


Fig. 20.6 Torque change in anisotropic machine in terms of power angle δ

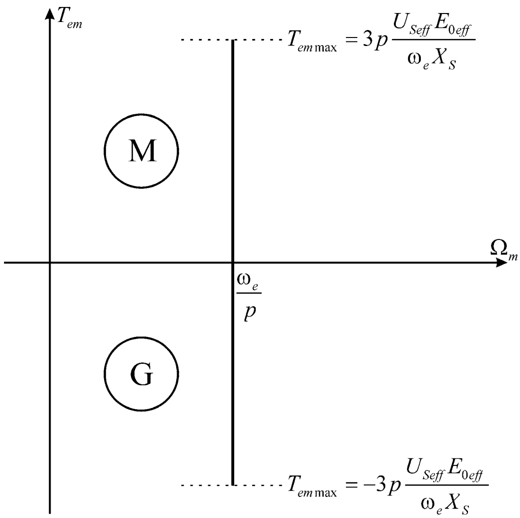
In the case of an isotropic machine, $X_d = X_q = X_S$. The maximum value of the torque which is available from an isotropic machine is obtained with the power angle $\delta = \pi/2$, and it is equal to

$$T_{em\ \max} = \frac{3p}{\omega_e} \frac{U_S E_0}{X_d}. \tag{20.13}$$

20.8 Mechanical Characteristic

Mechanical characteristic $T_{em}(\Omega_m)$ of an electrical machine is the steady-state dependence of electromagnetic torque and the rotor speed. It depends on the power supply conditions, namely, on the amplitude and frequency of supply voltages. Mechanical characteristic obtained with the rated power supply conditions is called *natural characteristic*. Mechanical characteristic of a synchronous machine is shown in Fig. 20.7. Previous diagram (Fig. 20.6) is not a mechanical characteristic. Instead, it shows dependence of the electromagnetic torque and the power angle, $T_{em}(\delta)$, while mechanical characteristic $T_{em}(\Omega_m)$ gives dependence of the electromagnetic torque and the rotor speed. Both $T_{em}(\delta)$ and $T_{em}(\Omega_m)$ dependences are defined in steady-state conditions. Synchronous machine operates in steady state only in cases where the rotor speed Ω_m corresponds to the speed of rotation of the stator magnetic field, denoted by Ω_e and called synchronous speed. Therefore, mechanical characteristic $T_{em}(\Omega_m)$ of synchronous machine is a straight line defined by $\Omega_m = \Omega_e$. The peak values of the torque are reached at the ends of the straight line $\Omega_m = \Omega_e$ given in Fig. 20.7. The peak torque is limited by the machine reactances, and it is equal to $T_{em\ max} = 3pU_SE_0/(\omega_eX_S)$.

Fig. 20.7 Mechanical characteristic



20.9 Synchronous Machine Supplied from Stiff Network

Most synchronous machines are connected to three-phase network with AC voltages having the line frequency of $f_e = 50$ or 60 Hz. Other machines are supplied from static power converters which provide a three-phase system of voltages of variable frequency and amplitude. In the former case, the stator frequency ω_e and the voltage amplitude are determined by external factors and cannot be changed. The network where the voltage frequency and amplitude do not change and remain constant is called *stiff network*. In synchronous machines supplied from a stiff network, the synchronous speed cannot be changed, and the steady-state value of the rotor speed remains constant. In the latter case, the machine is supplied from a separate source, from static power converter which produces a three-phase system of voltages of adjustable amplitude and frequency. In most such cases, static power converter supplies only one machine. Therefore, the frequency and amplitude of the stator voltages can be varied and adjusted to achieve desired flux and desired variation of the rotor speed.

In hydropower plants and thermal power plants, large power synchronous machines are used as generators, and they are connected to three-phase network of constant frequency. Mechanical power is obtained via shaft from steam turbines or waterwheels and turbines. This power is converted into electrical and delivered to the network. The rated voltage of large synchronous generators ranges from 6 up to 25 kV. They are connected to transmission networks with rated voltages from 110 up to 700 kV. Each synchronous generator has a dedicated transformer that connects the stator terminal to high-voltage transmission network.

Synchronous machines of lower power are used in motion control applications, where each machine has a dedicated static power converter that provides the supply voltages of variable amplitude and frequency. Motion control applications include propulsion of vehicles, control of industrial robots, motion tasks in production machines, and other similar tasks where it is necessary to provide a continuous change of the rotor position, speed, and torque. In motion control applications, synchronous machines operate mostly as motors which overcome the motion resistances within work machines. In speed control loops, synchronous motors are used as *torque actuators*, and their task is to provide the torque which corresponds to the *torque command*. The torque command is calculated within the speed controller, and it corresponds to the torque required to overcome the motion resistances and to achieve desired variation of the speed. Providing variable torque at variable rotor speed requires the stator voltage frequency and amplitude to be changed in continuous manner. This is achieved by supplying the stator winding from three-phase inverters which comprise semiconductor power switches and operate on pulse width modulation principles.

It is of interest to study operation of synchronous machines in both cases that are mentioned above. The first to consider is the operation of synchronous machines supplied from a stiff network. The subsequent analysis considers three-phase synchronous machine connected to three-phase network with symmetrical AC voltages, wherein the voltages have constant line frequency and constant amplitude. Without the lack of generality, it is assumed that the shaft of synchronous machine is connected to driving turbine which provides the source of mechanical work and that the synchronous machine operates as a generator which converts mechanical work into electrical energy. The steady-state performance of synchronous machine is considered, described by the steady-state equivalent circuit in Fig. 20.1, phasor diagram in Fig. 20.2. In further discussion, it is of interest to investigate the impact of changes in the power angle δ on the electromagnetic torque (20.3), active and reactive power (20.5). In steady-state conditions, gradual changes in the torque of the driving turbine result in changes of the power angle and the active power, while the changes in excitation current affect the reactive power.

20.10 Operation of Synchronous Generators

Synchronous generators in thermal and hydropower plants have the rated voltages that range from 6 to 25 kV and rated power that ranges from several tens of MW up to several hundreds of MW. Generator shaft is coupled to a steam turbine or a water turbine which provides the driving torque $T_T > 0$. This torque supports the motion of the rotor; thus, Newton equation of motion has the form $J\mathrm{d}\Omega_m/\mathrm{d}t = T_{em} + T_T$. At steady state, $\mathrm{d}\Omega_m/\mathrm{d}t = 0$; thus, the electromagnetic torque of the generator is negative and equal to $T_{em} = -T_T$. The power of electromechanical conversion $P_e = T_m\Omega_m$ is also negative, illustrating the fact that the electromechanical energy conversion within the machine has opposite direction, since the mechanical work gets converter into electrical energy. In the phase diagram of generator (Fig. 20.4), the electromotive force $\underline{E}_0$ leads with respect to the voltage $\underline{U}_S$; thus, the power angle δ is negative. Power P_e delivered to the machine from the network is negative in generator mode, and it is equal to

$$P_e = 3\frac{U_{Sn}E_0}{X_S}\sin(\delta) = -P_G, \tag{20.14}$$

where P_G denotes the active power delivered from the generator to the network. Assuming that the network has a constant line frequency, the change in power angle is described by $\mathrm{d}\delta/\mathrm{d}t = \omega_e - p\Omega_m$. With constant ω_e, an increase in the rotor speed causes the power angle to reduce. An equilibrium point is reached when $\Omega_m = \omega_e/p$.

It is of interest to study behavior of synchronous generators in cases where the driving turbine torque T_T exhibits small changes and also in cases where the line frequencies ω_e changes by a very small amount.

20.10.1 Increase of Turbine Power

The power of steam turbines or water turbines can be increased or decreased according to requirements. Variation of the steam turbine power is achieved by means of opening or closing the valves that feed the steam from the boiler to the steam turbine and also by operating the valves that change the air and coal dust supply to the boiler. The power of a water turbine changes in a similar manner. An increased turbine power results in a larger turbine torque T_T which is passed to the synchronous generator, where it tends to increase the rotor speed. Starting from the state of equilibrium where $T_{em0} = -T_{T0}$, an increase of the turbine torque to a new value of $T_{T1} = T_{T0} + \Delta T$ leads to an increase of the rotor speed according to equation $J\mathrm{d}\Omega_m/\mathrm{d}t = T_{em} + T_T = +\Delta T$.

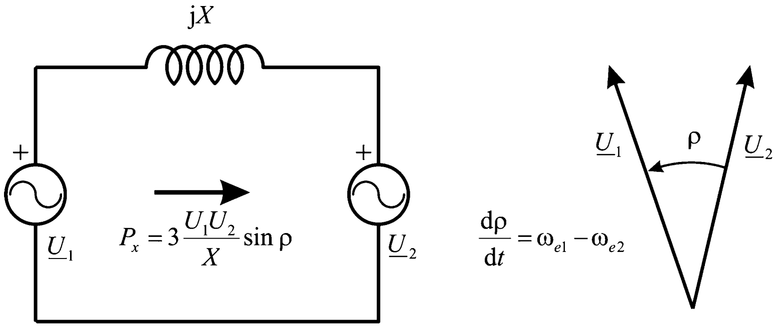
An increase of the rotor speed changes the power angle δ according to $\mathrm{d}\delta/\mathrm{d}t = \omega_e - p\Omega_m$. The motion of the electromotive force $\underline{E}_0 = \mathrm{j}p\Omega_m\Psi_{Rm}$ is determined by the rotor speed, while the voltage $\underline{U}_S$ rotates at synchronous speed $\Omega_e = \omega_e/p$, determined by the network frequency. With an increase in the rotor speed, the power angle delta decreases, and the phase lead $-\delta$ of electromotive force with respect to the stator voltage increases. The power angle assumes negative values in generator mode, and it goes further in negative direction. The generator power $P_G = -P_e = -3(U_{Sn}E_0/X_S)\sin(\delta)$ is increased. Along with that, the electromagnetic torque $T_{em} = P_e/\Omega_e < 0$ increases in magnitude. With $J\mathrm{d}\Omega_m/\mathrm{d}t = T_{em} + T_T$, a new equilibrium with $J\mathrm{d}\Omega_m/\mathrm{d}t = 0$ can be reached when the electromagnetic torque reaches the value of $T_{em} = -T_{T1}$. The nature of transient phenomena that take place while reaching the new equilibrium is discussed later. In this new steady state, electrical power delivered by the generator to the transmission network is increased. It is assumed that the excess power in the network gets counteracted by an increase in electrical power consumption of electrical loads that are connected to the network. If the assumption that the network is stiff holds, the end of the excess power does not put in question the above considerations. In an actual network, however large, the line frequency may exhibit small changes in consequence to variation in electrical power consumption or variation in mechanical power delivered by steam and water turbines.

In a thought experiment with an electrical network that has several generators and a number of constant power electrical consumers, it is of interest to consider the events that take place when all the steam and water turbines increase their mechanical power at the same time. If all the electrical consumers retain the same power, then the sum of power generated by all the generators in the network must remain constant as well. Namely, unless otherwise stated, the network does not have means to accumulate or store the excess of electrical energy. Therefore, the sum of power generated by all the synchronous generators has to remain constant. With $P_G = -P_e =$ const., the electromagnetic torque T_{em} has to remain constant as well. According to Newton equation of motion $J\mathrm{d}\Omega_m/\mathrm{d}t = T_{em} + T_T$, an increase in T_T in conditions where T_{em} remains constant results in an increase of the rotor speed. Hence, in the considered thought experiment, the rotor speed in all generators will increase, increasing in such way the line frequency of the considered network. Excess energy that comes from the steam and water turbines is not converted into electrical energy. Instead, it is stored as kinetic energy of revolving masses, $W_{kin} = \sum \frac{1}{2} J \Omega_m{}^2$. If the situation with excess turbine power persists, the line frequency of the network is continuously increased due to continuous increase in the rotor speed of synchronous machines.

In practical power networks, an increase in the line frequency indicates the excess power received from the steam and water turbines, while a decrease in the line frequency indicates the lack of mechanical power delivered to generators and/or an excessive consumption.

Question (20.4): The problem statement relies on the above thought experiment, where the power of steam and water turbines increases, where the power of electrical consumers remains constant, and where the rotor speed and the line frequency increase. Assume that the power system considered above has one high-voltage transmission line connected to a much larger power system. This second, larger power system can be treated as a stiff network. Series reactance X_S of the transmission line is known, as well as the voltage amplitudes U_1 and U_2 of the two power systems. Both systems have symmetrical systems of three-phase voltages. Consider the effects of such connection on the behavior of the system with the excess power.

Answer (20.4): The two electrical power systems with voltages U_1 and U_2, connected by means of the transmission line with the series reactance X, can be represented as two voltage sources connected by a series impedance jX. This representation is similar to the equivalent circuit in Fig. 20.1, where the voltage $\underline{U}_S$ gets connected to the voltage $\underline{E}_0$ across the series impedance.



Phasors $\underline{U}_1$ and $\underline{U}_2$ have the phase difference ρ. This phase difference changes according to equation $d\rho/dt = \omega_{e1} - \omega_{e2}$, where ω_{e1} is the line frequency of the first power system while ω_{e2} is the angular frequency of the second power system. According to (20.6), the power exchanged between the two power systems can be expressed as $P_x = 3(U_1U_2/X)\sin\rho$. As the mechanical power of turbines in the first power system increases, the line frequency ω_{e1} will increase as well due to an increase in rotor speed of synchronous generators within the first power system. At the same time, the line frequency ω_{e2} in the second, larger power system remains the same. The angle ρ will increase due to $d\rho/dt > 0$. Therefore, the phasor $\underline{U}_1$ increases the phase advance with respect to the phasor $\underline{U}_2$, and the power $P_x \sim \sin\rho$ increases. Hence, the excess power of the first power system gets passed to the second power system. On the long run, a new steady-state condition appears with $\omega_{e1} = \omega_{e2}$.

Notice at this point that the energy and power exchange between the two power systems takes place automatically, without any need for the system operator to commutate any switches or to issue any commands. At the same time, connection between the two power systems helps in resolving problems of temporary excess of power in one of the systems as well as problems of temporary increase in power consumption.

20.10.2 Increase in Line Frequency

Line frequency in power network determines the phase angle of the stator voltage $\underline{U}_S$. When the network frequency increases, the power angle δ changes. Considering synchronous generator with $\delta < 0$ and with the power angle time derivative $d\delta/dt = \omega_e - p\Omega_m$, negative value of the power angle becomes closer to $\delta = 0$. Generator power $P_G = -3(U_{Sn}E_0/X_S)\sin(\delta)$ decreases, as well as the magnitude of the electromagnetic torque. These considerations can be applied to any and all synchronous generators connected to the electrical power network. Therefore, whenever the line frequency $\omega_e = 2\pi f_e$ of the power system increases, the power received from synchronous generators reduces. At the same time, a sudden increase of power consumed by electrical loads reduces the speed of revolving rotors and causes a decrease in the line frequency.

On the basis of the above examples, it can be concluded that electrical power systems with synchronous generators have a strong coupling between the power and the line frequency. This *P-f* relation is one of basic power system properties, and it provides the grounds for the power regulation and the frequency regulation.

20.10.3 Reactive Power and Voltage Changes

A synchronous generator delivers the power $P_G = -3(U_{Sn}E_0/X_S)\sin(\delta)$ with different values of the electromotive force and different values of power angle. In order to maintain a constant power, it is of interest to keep the product $E_0\sin(\delta)$ constant. The electromotive force E_0 can be varied by changing the excitation current of the generator. It is possible to change the excitation current, to change the flux Ψ_{Rm} and the electromotive force $E_0 = \omega_m \Psi_{Rm}$, and yet to maintain the same power, provided that the product $E_0\sin(\delta)$ remains the same. This degree of freedom can be used to change the reactive power Q_e absorbed by the machine from the three-phase network. Reactive power of an isotropic synchronous machine is proportional to the voltage difference between the stator voltage and the electromotive force E_0:

$$Q_e = 3\frac{U_{Sn}(U_{Sn} - E_0\cos(\delta))}{X_S}. \tag{20.15}$$

By increasing the excitation current I_R, electromotive force $E_0 = \omega_m L_m I_R$ becomes larger than the stator voltage; thus, the reactive power Q_e becomes negative. In this way, synchronous generator becomes a source of reactive power, and its equivalent impedance has capacitive nature. Majority of electrical consumers has an inductive power factor and consumes reactive power, behaving as a coil. Therefore, electrical power system must comprise adequate sources of reactive power.

Transmission of reactive power across transmission and distribution line results in significant voltage drops. Most transmission lines have their equivalent series reactance *X* considerably larger than equivalent series resistance *R*. In cases where a three-phase electrical load gets connected at the end of the transmission line, where it absorbs reactive power *Q* and has the voltage $\underline{U}_{END} = U_{END}$, the current of the transmission line is equal to $\underline{I} = -\mathrm{j}Q/(3U_{END})$. The voltage at the beginning of the transmission line is equal to $\underline{U}_{BEG} = \underline{U}_{END} + \mathrm{j}X\underline{I} = U_{END} + XQ/(3U_{END})$. The voltage drop $\mathrm{j}X\underline{I}$ is collinear (in phase) with the voltages. Hence, the value *XI* is directly subtracted from U_{BEG} in order to obtain U_{END}.

Notice at this point that replacing reactive power consumer by resistive load which absorbs active power $P = Q$ results in a significant reduction of the voltage drop. With resistive load, the current of the same amplitude results in much smaller difference in amplitudes of $\underline{U}_{BEG}$ and $\underline{U}_{END}$. With $\underline{I} = P/(3U_{END})$, $\underline{U}_{BEG} = \underline{U}_{END} + \mathrm{j}XP/(3U_{END})$. The voltage drop $\mathrm{j}X\underline{I}$ is perpendicular (phase shifted by $\pi/2$) with respect to U_{END}, and this circumstance results in $|\underline{U}_{BEG}| - |\underline{U}_{END}| << XI$.

Whenever electrical loads absorb reactive power, there are considerable voltage drops across the transmission lines. As a consequence, it is necessary to increase the stator voltage of synchronous generators in order to maintain constant voltage at electrical loads. For this to achieve, it is necessary to increase the excitation current I_R of synchronous generators, which leads to increased flux Ψ_{Rm} and increased electromotive force E_0. At the same time, an increase in E_0 contributes to increased reactive power delivered from generators to the network.

From the above considerations, it is concluded that the voltage across the network and the reactive power flow are strongly related. This *U*-*Q* relation constitutes the bases for the voltage control in electric power systems.

20.10.4 Changes in Power Angle

Changes in mechanical power received from steam or water turbines result in transient response of synchronous machine which ends in a new steady-state condition with a different value of power angle. A sudden increase in electrical load of the generator produces the same effect. It is of interest to investigate the transient response of the machine torque, power, and power angle during such transients.

With constant power supply frequency, the stator voltage vector rotates at constant synchronous speed $\Omega_e = \omega_e/p$, determined by the supply frequency ω_e. The rotor flux is created by the current I_R in the excitation winding, which revolves with the rotor. Therefore, the vector of the rotor flux revolves at the rotor speed Ω_m. The excitation flux $\underline{\Psi}_{Rm}$ creates the electromotive force $\underline{E}_0 = \mathrm{j}\underline{\Psi}_{Rm}\Omega_m$ in the stator windings. In steady state, the stator voltage and electromotive force are represented by phasors $\underline{U}_S$ and $\underline{E}_0$. Power angle δ represents the difference in initial phases of the stator voltage and the electromotive force. It changes according to the law[3] $\mathrm{d}\delta/\mathrm{d}t = \omega_e - \omega_m = \omega_e - p\Omega_m$. The phase of the stator voltage depends on the supply frequency ω_e, while the phase of the electromotive force depends on the rotor speed $p\Omega_m$. Therefore, changes in power angle δ are defined by the speed difference between the synchronous speed and the rotor speed. At the same time, the electromagnetic torque of synchronous machine operating in the steady state depends on the product of U_S, E_0, and sine of the power angle.

[3] In phasor diagrams, phasors $\underline{U}_S$ and $\underline{E}_0$ represent an AC stator voltage and an AC electromotive force. The power angle δ represents the phase difference or the *electrical* shift between the stator voltage and the electromotive force. Therefore, the first-time derivative of power angle is equal to $\mathrm{d}\delta/\mathrm{d}t = p(\Omega_e - \Omega_m) = \omega_e - p\Omega_m = \omega_e - \omega_m$, where p is the number of pole pairs.

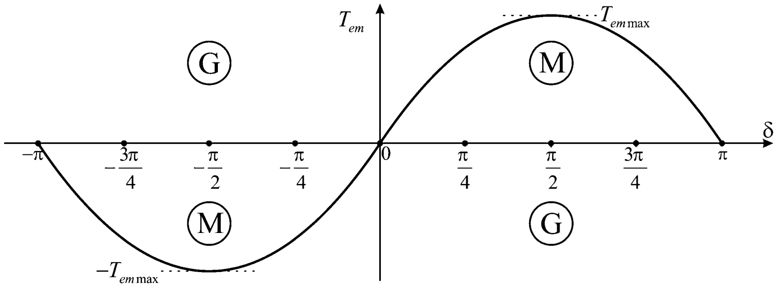


Fig. 20.8 Torque change in isotropic machine in terms of power angle

Variation of the torque in terms of the power angle δ is presented in Fig. 20.8. If the machine operates with no load, the power angle is equal to zero, as well as the electromagnetic torque. If the motion resistance torque T_m appears, directed toward reducing the rotor speed, the power angle increases because its derivative $\mathrm{d}\delta/\mathrm{d}t = \omega_e - p\Omega_m$ becomes positive. The increase in the power angle increases the electromagnetic torque. A new equilibrium is reached when $T_{em} = T_m$.

Assuming that the machine is connected to steam or water turbine which provides the torque T_T, directed toward increasing the rotor speed, the rotor speed Ω_m exceeds the synchronous speed Ω_e, and the power angle $\mathrm{d}\delta/\mathrm{d}t = \omega_e - p\Omega_m$ obtains a negative first derivative and a negative value. A negative value of δ means that the electromotive force phasor $\underline{E}_0$ leads with respect to the voltage $\underline{U}_S$. Machine develops electromagnetic torque $T_{em} < 0$ which is opposite to the rotor motion; thus, the machine operates as a generator, converting mechanical work into electrical energy.

In cases where the power angle is relatively small, it is justified to adopt the approximation $\sin(\delta) \approx \delta$ and to represent the torque T_{em} by an approximate expression $T_{em} \approx k\delta$. At steady state, the electromagnetic torque is in equilibrium with the torque components resulting from the mechanical subsystem. When the machine operates as motor, electromagnetic torque is equal to motion resistances T_m. When the machine operates as generator, electromagnetic torque is equal to the torque provided by steam or water turbines. In both cases, the rotor speed does not change since $J\mathrm{d}\Omega_m/\mathrm{d}t = T_{em} - T_m = 0$ in motoring mode and $J\mathrm{d}\Omega_m/\mathrm{d}t = T_{em} + T_T = 0$ in generator mode. At steady state, the rotor revolves at synchronous speed $\Omega_m = \Omega_e$. It is of interest to determine the character of transients which appear due to disturbances such as the load torque changes or changes in the line frequency ω_e. In the subsequent considerations, it is assumed that the number of pairs of magnetic poles is $p = 1$; thus, the angular frequencies of electrical variables ω_m and ω_e are equal to corresponding mechanical speeds Ω_m and Ω_e. Therefore, Newton equation of motion is written as $J\ \mathrm{d}\omega_m/\mathrm{d}t = T_{em} - T_m$, where T_m represents the motion resistances. Alternatively, $-T_m = T_T$ represents the driving torque of the steam or water turbine. With that in mind, the change in the power angle is determined by

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = \omega_e - p\Omega_m = \omega_e - \omega_m. \tag{20.16}$$

Newton equation for a two-pole machine takes the form

$$J \frac{d\Omega_m}{dt} = J \frac{d\omega_m}{dt} = T_{em} - T_m, \tag{20.17}$$

while the change in the power angle depends on differential equation

$$J \frac{d^2\delta}{dt^2} = -T_{em}(\delta) + T_m. \tag{20.18}$$

Staring from no load conditions where $\delta = 0$, $T_{em} = 0$, and $\omega_m = \omega_e$, and introducing the step change in the load torque T_m (or the turbine torque T_T), a transient interval follows where the rotor speed, electromagnetic torque, and power angle change and fluctuate before entering another steady-state condition. During these transients, the rotor speed is not equal to the synchronous speed. Taking into account that position of the *dq* coordinate system is determined by the rotor position, while the vector of the stator voltage depends on the power supply frequency, it is concluded that the stator voltage vector will move with respect to selected *dq* coordinate frame. For this reason, projections U_d and U_q of the stator voltage vector $\boldsymbol{U}_S$ on the axes of *dq* frame will change during transients. Electrical subsystem of synchronous machine is represented by voltage balance equations in virtual stator phases that are placed in *d*-axis and *q*-axis of *dq* frame. Changes in corresponding voltages U_d and U_q introduce nonzero derivatives of flux linkages Ψ_d and Ψ_q, thus bringing the electrical subsystem in transient state. Hence, during transients in mechanical subsystem, where the electromagnetic torque, the rotor speed, and the power angle exhibit transient changes, the electrical subsystem of synchronous machine does not remain in steady-state condition, and it enters transient state of its own.

While in transient state, electrical subsystem of synchronous machines cannot be represented by the steady-state equivalent circuit. At the same time, the transient torque cannot be represented by simplified expression such as $T_{em} \approx k\delta$, which is derived from the steady-state equivalent circuit.

Within the next chapter, it will be shown that the time constants of mechanical subsystems are considerably larger than the time constants of electrical subsystem. This circumstance will be used to simplify transient analysis of synchronous machines.