

## Chapter 22

# Variable Frequency Synchronous Machines

This chapter studies the operation and characteristics of three-phase synchronous machines connected to three-phase inverters, static power converters capable of adjusting the stator voltages by means of changing the width of the voltage pulses supplied to the stator terminals. The average voltage of the pulse train is adjusted to suit the machine needs. Variable speed operation of synchronous machine is achieved with variable frequency and variable amplitude of stator voltages. This chapter introduces and explains some basic torque and speed control principles. The need of controlling the stator currents is discussed and explained. Fundamental principles of stator current control are introduced, relying on PWM-controlled three-phase inverter as the voltage actuator. Field-weakening performance of inverter-supplied synchronous machines with buried magnets and surface-mounted magnets is analyzed and explained. The limits of constant power operation in field-weakening mode are determined, explained, and expressed in terms of the stator self-inductance. Based upon the study of operating limits of the machine and operating limits of associated three-phase inverter, steady-state operating area and transient operating area are derived in  $T - \Omega$  plane and studied for inverter-supplied synchronous machines.

### 22.1 Inverter-Supplied Synchronous Machines

Synchronous machines of low and medium power are used in applications such as motion control, vehicle propulsion, industrial robots, or production machines. In these applications, synchronous machines have task to overcome motion resistance and provide the required acceleration and deceleration of moving parts. Synchronous motor is used as an actuator which develops the torque required to control the speed or position and to overcome the motion resistances while driving

the controlled object along predefined trajectories. Synchronous motors are better suited to these tasks than the other electrical motors. Advantages of synchronous machines over the other types of electrical machines include their high specific power, high specific torque, a low inertia, and relatively low losses. Synchronous machines with permanent magnet excitation have no rotor windings and no rotor losses. Therefore, energy efficiency of these motors is considerably improved over other types of motors. With permanent magnets, the rotor flux is obtained without power losses such as  $U_R I_R = R_R I_R^2$ , encountered in machines which have the excitation winding. Moreover, there are no iron losses in the rotor magnetic circuit of synchronous machine. Due to synchronous rotation of the rotor and the stator field, there are no pulsations of magnetic induction  $B$  in magnetic circuit of the rotor.

With no rotor losses and no heat generated within the rotor, the cooling of permanent magnet synchronous machines is greatly simplified, and it is possible to reach larger current densities in the stator winding and larger magnetic induction, which results in higher specific power.<sup>1</sup> Compared to an induction machine of the same rated power, synchronous machine with permanent magnets has 20–30% lower mass and volume. The power balance charts drawn for induction machines and synchronous machines have the same losses in the stator winding, in the stator magnetic circuit, and in the mechanical subsystem. Induction machines have power losses in the rotor which are equal to  $sP_\delta$ , where  $s$  is relative slip while  $P_\delta$  is the air-gap power. Synchronous machines do not have the losses  $sP_\delta$ , and this greatly increases their energy efficiency.

In applications such as motion control, vehicle propulsion, and industry automation, it is required to provide continuous change of the rotor speed. In synchronous machines, the rotor speed corresponds to the synchronous speed  $\Omega_e = p\omega_e$ , which is determined by the supply frequency  $\omega_e$ . For this reason, synchronous motors in motion control applications have to be supplied from separate power sources that produce three-phase voltages of variable frequency and variable amplitude, in accordance with the motor needs. Two synchronous motors within the same industrial robot or electrical vehicle most often rotate at different rotor speeds. Therefore, each motor has its own supply frequency, and it requires a separate power source that provides the stator voltages and currents of desired frequency. Such power sources are usually three-phase inverters with transistor power switches. Commutation of power transistors produces a train of variable width voltage pulses. The pulse width of these pulses affects the average voltage within one commutation period. By sinusoidal change of the pulse width, the average voltage exhibits sinusoidal change with an adjustable amplitude and frequency. Pulse-width-modulation techniques enable generation of pulse trains of an average value that corresponds to the motor needs.

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<sup>1</sup> Specific power is the quotient of the rated power of the machine and the machine volume or weight. Similar definition holds for specific torque.

## 22.2 Torque Control Principles

In applications where the speed or position of the moving object is controlled and enforced to track some predefined trajectory, synchronous permanent magnet motors are used as *torque actuators*, *executive organs* that provide the driving torque that overcomes the motion resistances and forces the speed or position to maintain desired reference values. Speed regulators and position regulators calculate the difference between the controlled variable  $\Omega_m(\theta_m)$  and its reference value  $\Omega^*(\theta^*)$ , and they calculate the electromagnetic torque  $T_{em}^*$  required to remove the error  $\Delta\Omega = \Omega^* - \Omega_m$  and drive the controlled variable back into the reference track. The speed change is determined by Newton equation of motion,  $J d\Omega_m/dt = T_{em} - T_m$ , where  $T_m$  denotes the load torque disturbance, while  $T_{em}$  denotes the electromagnetic torque provided by the synchronous motor. The motor torque  $T_{em}$  has to overcome the load torque  $T_m$  and to provide the acceleration  $d\Omega_m/dt$  which is needed to achieve desired speed change. While the speed (position) regulator calculates the torque reference signal  $T_{em}^*$ , the synchronous permanent magnet motor has the task of providing the actual shaft torque  $T_{em}$  which corresponds to this reference signal. The speed and accuracy of motion control rely on the torque actuator capability of providing the torque which corresponds to the reference. In other words, the electromagnetic torque has to track the reference  $T_{em}^*$  accurately and with negligible delay, resulting in  $T_{em}^* = T_{em}$ .

In Chap. 19, it has been shown that the electromagnetic torque of synchronous machines depends on the vector product of the stator flux and the stator current. Hence,

$$T_{em} = \frac{3p}{2} (\Psi_d i_q - \Psi_q i_d).$$

In permanent magnet motors, the stator flux components are  $\Psi_d = \Psi_{Rm} + L_d i_d$  and  $\Psi_q = L_q i_q$ , where  $\Psi_{Rm}$  is the flux of permanent magnets which passes through the air gap and encircles the stator windings. Introducing the flux  $\Psi_{Rm}$ , the torque expression becomes

$$T_{em} = \frac{3p}{2} \Psi_{Rm} i_q + \frac{3p}{2} (L_d - L_q) i_d i_q.$$

Most synchronous permanent magnet motors have cylindrical form of the rotor magnetic circuit and a very small difference between magnetic resistances in  $d$ -axis and  $q$ -axis. Therefore, it is justifiable to consider that  $L_d = L_q$ . With that in mind,

$$T_{em} = \frac{3p}{2} \Psi_{Rm} i_q.$$

Hence, the electromagnetic torque of synchronous permanent magnet motor is determined by the  $q$ -component of the stator current. In order to reduce the copper losses in the stator windings and to reduce the rating of the power converter that

supplies the stator winding, it is beneficial to deliver desired torque  $T_{em}$  with the smallest possible stator current. The ratio  $T_{em}/I_S$  between the electromagnetic torque and the stator current  $I_S$  is equal to  $T_{em}/\sqrt{(i_d^2 + i_q^2)}$ , and it has the minimum value when  $i_d = 0$ , that is, when the stator current vector is aligned with  $q$ -axis. In this case, the rotor flux vector and the stator current vector are displaced by  $\pi/2$ .

Since the flux  $\Psi_{Rm}$  of permanent magnets does not change, the torque of permanent magnet synchronous motor is controlled by changing the stator current  $i_q$ . Given the torque reference  $T_{em}^*$ , it is necessary to establish the stator currents with their  $dq$  components  $i_d = 0$  and  $i_q = (2/(3p)) \cdot T_{em}^*/\Psi_{Rm} = K \cdot T_{em}^*$ . The torque control of permanent magnet synchronous motor is accomplished by regulating the stator current and delivering the stator current component  $i_q$  so that it corresponds to the desired torque. The speed and accuracy in delivering desired torque are uniquely defined by the speed and accuracy in regulation of the stator current. Current regulation relies on supplying the stator winding from the three-phase inverter, static power converter which employs switching power transistors in order to adjust the stator voltage and achieve desired phase currents.

Figure 22.1 shows a three-phase inverter with transistor power switches. The inverter supplies the stator winding of a synchronous motor. The mains voltages  $u_R$ ,  $u_S$ , and  $u_T$  are rectified within the diode rectifier that makes use of six power diodes. The rectifier provides the DC voltage  $E$  which exists in intermediate circuit also called DC link circuit. The elements  $L_{DC}$  and  $C_{DC}$  serve as the filter that removes AC components from the rectifier output. Braking unit with transistor  $Q_K$  and resistor  $R_K$  serves to reduce the DC voltage  $E$  in braking intervals, when the motor operates as generator which passes the braking power back into the DC link, charging the DC link capacitor and increasing the DC link voltage. The braking energy cannot be returned to the mains due to the nature of the three-phase diode rectifier. Therefore, the braking power gets dissipated in the resistor  $R_K$ . Six power transistors  $Q_1$ – $Q_6$  are commutated in order to obtain pulse-shaped phase voltages. The state of the power transistor switches is determined so as to drive the three-phase currents toward their reference values  $i_a^*(t)$ ,  $i_b^*(t)$ , and  $i_c^*(t)$ .

The problem of current control in three-phase motors requires more detailed study, and it is beyond the scope of this chapter. For this reason, only some basic information and principles are presented in further text.

It is of interest to determine the reference values for the phase currents of a synchronous permanent magnet motor with flux  $\Psi_{Rm}$ , with the rotor position equal to  $\theta_m$ , with  $p = 1$  pole pairs, and with the torque reference  $T_{em}^*$ . Assuming that the current regulator manages to maintain the phase currents on their reference values, the phase current references

$$\begin{aligned} i_a^*(t) &= \left(\frac{2}{3} \frac{T_{em}^*}{\Psi_{Rm}}\right) \cos\left(\theta_m + \frac{\pi}{2}\right), \\ i_b^*(t) &= \left(\frac{2}{3} \frac{T_{em}^*}{\Psi_{Rm}}\right) \cos\left(\theta_m + \frac{\pi}{2} - \frac{2\pi}{3}\right), \\ i_c^*(t) &= \left(\frac{2}{3} \frac{T_{em}^*}{\Psi_{Rm}}\right) \cos\left(\theta_m + \frac{\pi}{2} - \frac{4\pi}{3}\right), \end{aligned} \quad (22.1)$$

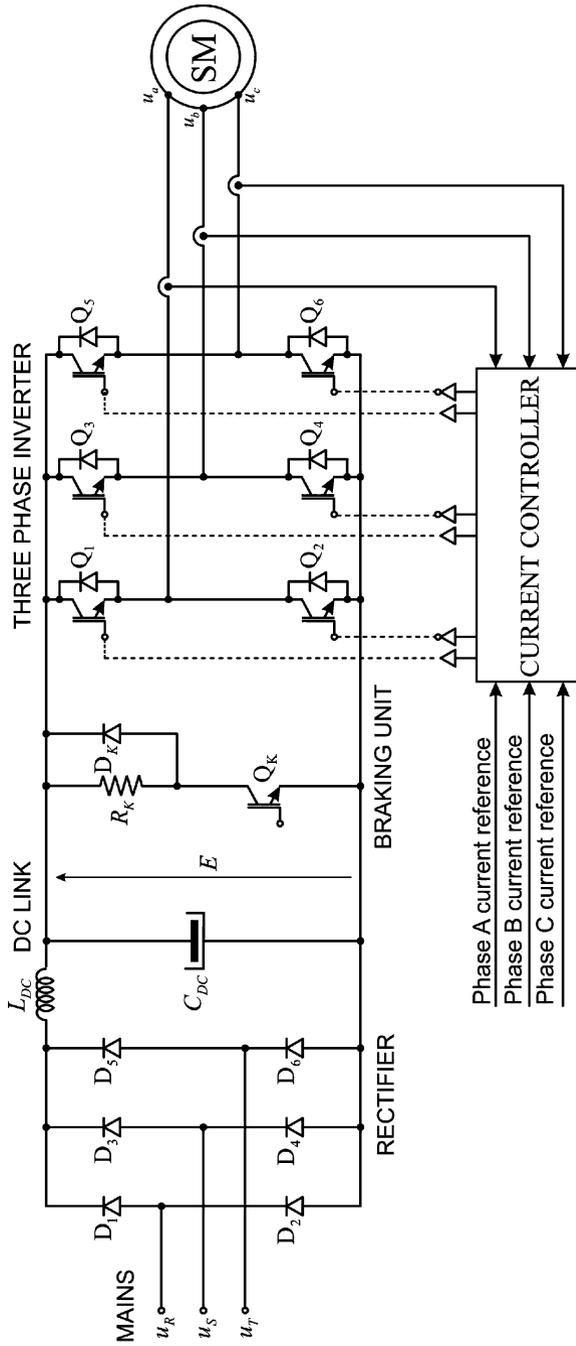


Fig. 22.1 Power converter topology intended to supply synchronous permanent magnet motor and the associated current controller

result in the stator phase currents  $i_a(t) = i_a^*(t)$ ,  $i_b(t) = i_b^*(t)$ , and  $i_c(t) = i_c^*(t)$ . They produce the stator current vector with projections  $i_d = 0$  and  $i_q = (2/3) \cdot T_{em}^* / \Psi_{Rm}$  on the axes of synchronous  $d$ - $q$  coordinate frame. In turn, the motor delivers the electromagnetic torque  $T_{em} = T_{em}^*$ .

### 22.3 Current Control Principles

Basic topology of static power converter intended to supply three-phase motor is given in Fig. 22.1. By switching action of power transistors  $Q_1$ – $Q_6$ , it is possible to change the phase voltage supplied to the stator winding. The phase voltages  $u_a$ ,  $u_b$ , and  $u_c$  affect corresponding phase currents, thus providing the grounds for the current control. The aim of the current control is obtaining the current components  $i_d$  and  $i_q$  that correspond to desired values. Current component  $i_d$  does not contribute to the torque, and it is mostly set to zero. Desired torque is obtained by establishing current component  $i_q$  in proportion to the desired torque. With rotor flux vector collinear with  $d$ -axis, and with  $i_d = 0$ , the vector of the stator current is orthogonal to the flux. Further discussion outlines the basic principles of setting current components  $i_d$  and  $i_q$  to desired values.

The current components in  $d$ - $q$  coordinate frame are uniquely defined by the phase currents  $i_a$ ,  $i_b$ , and  $i_c$ . The problem of obtaining desired current components  $i_d$  and  $i_q$  is therefore equivalent to the problem of controlling the phase currents. The reference values of the phase currents  $i_a^*$ ,  $i_b^*$ , and  $i_c^*$  are calculated in (22.1), and they depend on the torque reference  $T_{em}^*$  and the rotor position  $\theta_m$ . The torque reference comes from the speed controller or the position controller, while the rotor position is usually measured by the sensor attached to the rotor shaft. By setting the phase currents according to (22.1), resulting vector of the stator current is perpendicular to the vector of the rotor flux.

Within the current controller in Fig. 22.1, measured phase currents  $i_a$ ,  $i_b$ , and  $i_c$  are compared to corresponding reference values in order to obtain the errors  $\Delta i_a$ ,  $\Delta i_b$ , and  $\Delta i_c$ . Based upon these errors, the current controller generates gating signals that commutate transistor power switches. This changes the phase voltages in the way which reduces the errors and eventually drives them toward zero. Ideally, the current controller achieves the operation with  $i_a(t) = i_a^*(t)$ ,  $i_b(t) = i_b^*(t)$ , and  $i_c(t) = i_c^*(t)$ .

There are six switching power transistors in a three-phase inverter. The switches are grouped in three phases, each one generating one of the phase voltages. The inverter is supplied by DC voltage  $E$ , obtained from the DC link circuit. By turning on the upper switch in one of the phases, the output phase is connected to the positive plate of the DC link capacitor  $C_{DC}$ . By turning on the lower switch, the output phase is connected to the negative plate of the DC link capacitor  $C_{DC}$ . Assuming that the phase windings are star connected, and that the star point has the potential in between the capacitor plates, turning on the upper or lower switch results in the phase voltage of  $+E/2$  or  $-E/2$ . Hence, by turning on the upper switch

$Q_1$  of phase  $a$ , the phase voltage  $u_a(t)$  becomes  $+E/2$ , while turning on the lower switch  $Q_2$  results in  $u_a(t) = -E/2$ .

During one commutation period of  $T \approx 100 \mu\text{s}$ , the upper switch in phase  $a$  is closed (on) during time interval  $t_{ON}$ , while in the remaining part of the commutation period,  $T - t_{ON}$ , the upper switch is opened (off) while the lower switch is closed (on). The phase voltage  $u_a(t)$  is equal to  $+E/2$  during time interval  $t_{ON}$ , and then it remains  $-E/2$  for the rest of the period ( $T - t_{ON}$ ). The width  $t_{ON}$  of the positive pulse can be varied continuously within the range  $0 < t_{ON} < T$ . The average value of the phase voltage during one period  $T$  is equal to  $u_{a(av)} = E \cdot (t_{ON}/T - 1/2)$ . Continuous change of the pulse width  $t_{ON}$  results in continuous change in the average value of the phase voltage which ranges between  $-E/2$  and  $+E/2$ . Variation of the instantaneous value of the phase current  $i_a(t)$  depends on the difference between the instantaneous value of the phase voltage and the electromotive force induced in the winding. Variation of the phase current during one commutation period  $T$  depends on the average phase voltage  $u_{a(av)}$  during one commutation period. The change in the phase current is based on the voltage balance equation  $u_a = R_S i_a + d\Psi_a/dt$ , where  $R_S i_a$  is the voltage drop across the stator resistance while  $d\Psi_a/dt$  is the electromotive force induced in one phase.

Distribution of magnetic field along the air gap depends on characteristics of permanent magnet and the way of their mounting. The air-gap field is also affected by the stator currents. The flux in the stator phase winding  $a$  is  $\Psi_a = L_S i_a + \Psi_{Rm(a)}$ , and it has two components. Component  $\Psi_{Rm(a)}$  is produced by permanent magnets, while the flux  $L_S i_a$  depends on the stator currents.<sup>2</sup> Component  $\Psi_{Rm(a)}$  depends on the amplitude and spatial orientation of the flux  $\Psi_{Rm}$ , which is generated by permanent magnets, which passes through the air gap, and which encircles the stator windings. Amplitude  $\Psi_{Rm}$  is determined by characteristics of permanent magnets, by the air gap  $\delta$ , and by properties of magnetic circuits. The spatial orientation of the permanent magnet flux is determined by the rotor position  $\theta_m$ . Therefore, the flux  $\Psi_{Rm(a)}$  depends on the amplitude  $\Psi_{Rm}$  and on the rotor position  $\theta_m$ . With  $\theta_m = 0$ , the rotor is aligned with magnetic axis of the phase  $a$ , resulting in  $\Psi_{Rm(a)} = \Psi_{Rm}$ .

The changes in the stator flux  $\Psi_a$  produce electromotive force  $d\Psi_a/dt$ . The later has two components,  $L_S di_a/dt$  and  $d\Psi_{Rm(a)}/dt$ . The second component  $e_a = d\Psi_{Rm(a)}/dt$  is caused by the flux of permanent magnets, and it depends on the rotor speed  $\Omega_m = d\theta_m/dt$ .

With constant rotor speed, the orientation of the permanent magnet flux is determined by  $\theta_m = \theta_0 + p\Omega_m t$ . The part of this flux which encircles the phase winding  $a$  is equal to  $\Psi_{Rm} \cos(\theta_m)$ . Besides, the phase winding has also the flux  $L_S i_a$ ; thus, the total flux of this phase is  $\Psi_a = \Psi_m \cos(\theta_m) + L_S i_a$ . The voltage balance equation becomes  $u_a = R_S i_a + L_S di_a/dt - \omega_m \Psi_m \sin(\theta_m)$ . By neglecting the

<sup>2</sup> Notice at this point that the flux  $\Psi_a$  in phase  $a$  depends on the phase current  $i_a$  but also on the phase currents  $i_b$  and  $i_c$ , due to a finite mutual inductance between the phase windings. Due to  $i_a = -i_b - i_c$  and with reasonable assumption that  $L_{ab} = L_{ac}$ , the flux component  $L_a i_a + L_{ab} i_b + L_{ac} i_c = L_a i_a + L_{ab}(i_b + i_c)$  can be rewritten as  $L_S i_a$ .

voltage drop across the stator resistance and by adopting notation  $e_a = d\Psi_{Rm(a)}/dt = -\omega_m \Psi_m \sin(\theta_m)$ , the voltage balance equation becomes  $u_a = L_S di_a/dt + e_a$ . With that in mind, the rate of change of the stator current  $i_a$  can be expressed as

$$\frac{di_a}{dt} = \frac{1}{L_S}(u_a - e_a) = \frac{1}{L_S}(\pm E - e_a). \quad (22.2)$$

In operating conditions where  $E \geq |e_a|$ , the phase voltage  $u_a$  prevails in the above expression. For  $u_a = +E/2$ , the rate of change of the stator current is positive, notwithstanding the electromotive force  $e_a$ . For  $u_a = -E/2$ , the rate of change of the stator current is negative. In other words, it is possible to obtain an increase of  $i_a$  by turning on the switch  $Q_1$  and a decrease of  $i_a$  by turning on the switch  $Q_2$ . These are the basic prerequisites for the current control. If the phase current  $i_a$  does not correspond to the reference value, the current controller detects the error  $\Delta i_a = i_a^* - i_a$ . Then, it generates command signals for the inverter commutation so that the error  $\Delta i_a$  is reduced and eventually removed. In cases where the error is positive ( $\Delta i_a > 0$ ), it is necessary to turn the upper switch  $Q_1$  on and to obtain  $di_a/dt > 0$ . This would reduce the error  $\Delta i_a = i_a^* - i_a$ . In cases where the error is negative ( $\Delta i_a < 0$ ), it is necessary to turn on the lower switch  $Q_2$  and to obtain reduction in the phase current ( $di_a/dt < 0$ ). This would drive the error  $\Delta i_a = i_a^* - i_a$  toward zero. A simple control law that proves efficient in controlling the stator currents of three-phase AC motors is expressed by the following equation:

$$u_a = \frac{E}{2} \cdot \text{sign}(i_a^* - i_a), \quad u_b = \frac{E}{2} \cdot \text{sign}(i_b^* - i_b), \quad u_c = \frac{E}{2} \cdot \text{sign}(i_c^* - i_c).$$

With the above control law, transistor power switches  $Q_1$ – $Q_6$  are commutated according to the sign of corresponding current error  $\Delta i$ . Whenever the error becomes positive, the upper switch is turned on, which increases the phase current and reduces the error. As soon as the error becomes negative, the lower switch is turned on to reduce the phase current and drive the error in positive direction. Hence, the straight application of the above control law may result in elevated commutation frequencies of the power transistors and increased commutation losses. In order to contain the commutation frequency, it is possible to introduce some hysteresis<sup>3</sup> into comparators which determine the sign of the current error. In this case, the commutation frequency is restrained by the hysteresis of comparators.

Practical current controllers operate in  $d$ – $q$  coordinate frame. This means that the errors  $\Delta i_d = i_d^* - i_d = 0 - i_d$  and  $\Delta i_q = i_q^* - i_q$  are calculated from references and feedback signals in  $d$ – $q$  coordinate frame. The current controller calculates the voltages  $u_d$  and  $u_q$  conceived to eliminate detected errors and drive the currents

<sup>3</sup> Comparator with hysteresis  $H$  generates the output on the basis of the previous comparison and on the new input. With previous output equal to +1, the input signal has to fall below  $-H$  in order to produce the new output of  $-1$ . With previous output equal to  $-1$ , the input signal has to climb above  $+H$  in order to produce the new output of  $+1$ .

$i_d$  and  $i_q$  to their references. The voltages  $u_d$  and  $u_q$  are transformed to the stationary  $\alpha$ - $\beta$  coordinate system by inverse Park transform, and the obtained voltages  $u_\alpha$  and  $u_\beta$  are passed to the inverse Clarke transform to obtain the voltage references for the phase voltages. Pulse-width-modulation-controlled three-phase inverter is used to supply the desired voltages to the three-phase motor.

The algorithm which calculates the control variable  $\underline{u}_{dq} = u_d + ju_q$  from the current error  $\Delta \underline{i}_{dq} = \Delta i_d + j\Delta i_q$  is called *control algorithm*, while the *regulator* or *controller* is the device which implements the algorithm and provides the output. Control algorithms are usually applied in digital form, by means of a program which is executed by digital signal processor. Regulator can often be described by its transfer function. The speed and quality of dynamic response depend on the transfer function of the controller.

The power converter with six switching power transistors supplies the three-phase stator winding by adjustable voltages. Therefore, it represents a controllable voltage supply. By closing the feedback loop which controls the stator phase currents, an appropriate current controller achieves that  $i_a = i_a^*$ ,  $i_b = i_b^*$ , and  $i_c = i_c^*$ . This turns the current-controlled three-phase inverter into a current source. Hence, the three-phase machine behaves as if it were supplied from a current source. The operation of synchronous permanent magnet motor and controlled stator currents is similar to the operation of DC machine with constant excitation with controller armature current. The speed and accuracy in delivering the torque are fully dependent on characteristics of the current controller.

## 22.4 Field Weakening

Current control can be accomplished in conditions where the electromotive force  $e$  induced in the stator phase windings does not exceed the available phase voltage  $E/2$ , where  $E$  is the DC voltage across the intermediate circuit called DC link circuit of the three-phase inverter. Condition  $E/2 \geq |e_a| = |\omega_m \Psi_m \sin(\theta_m)|$  must be fulfilled at every instant. In operation with  $i_d = 0$ , the stator flux in  $d$ -axis of the machine is equal to

$$\Psi_d = \Psi_{Rm} + L_d i_d = \Psi_{Rm}, \quad (22.3)$$

while the stator flux in  $q$ -axis is  $\Psi_q = L_q i_q$ . In synchronous machines with surface-mounted magnets, the stator inductances  $L_d$  and  $L_q$  are very low, and they range from 0.01 to 0.05 relative units. Therefore, the flux along  $q$ -axis is considerably lower than the flux  $\Psi_{Rm}$ . For this reason, it is justifiable to assume that the air-gap flux amplitude equals  $\Psi_m \approx \Psi_{Rm} + L_m i_d = \Psi_{Rm} = \Psi_n$ , where  $\Psi_n$  is the rated flux. At the rated rotor speed  $\Omega_n = \omega_n/p \approx E/\Psi_{Rm}/p = E/\Psi_n/p$ , induced electromotive force is equal to the rated voltage, which is at the same time the maximum voltage available from the three-phase switching inverter. With  $i_d = 0$ , permanent magnet synchronous motor cannot exceed the rated speed.

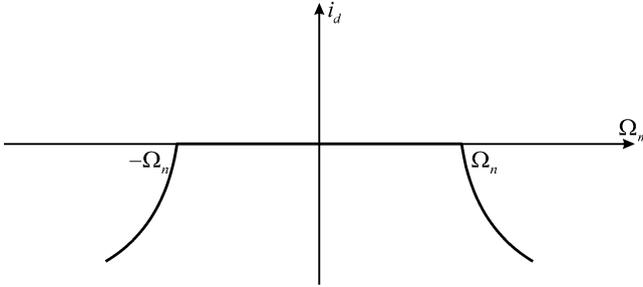


Fig. 22.2 Variation of defluxing current  $i_d$  in the field-weakening region

When approaching the rated speed, the electromotive force approaches the maximum available voltage. There is no more voltage margin and no possibility to control the stator current. In other words, there is no possibility to establish the stator currents required to obtaining the desired torque.

Operation at speeds above the rated speed requires the electromotive force to remain below the rated voltage. For that to achieve, it is necessary to reduce the flux, namely, to achieve the flux weakening. In order to keep the electromotive force  $\omega_m \Psi_m$  within the boundaries of the available voltage, the air-gap flux  $\Psi_m$  has to be changed in terms of the rotor speed. With flux that changes as  $\Psi_m(\Omega_m) = \Psi_n \cdot \Omega_n / \Omega_m = \Psi_{Rm} \cdot \omega_n / \omega_m$ , induced electromotive force at speeds above the rated speed retains its rated value  $\Psi_{Rm} \cdot \omega_n$ . It should be noted that the  $q$ -axis flux  $L_q i_q$  is considerably smaller than the flux of permanent magnets. The later assumption holds due to very small inductances  $L_q$  and  $L_d$  in synchronous machines with surface-mounted magnets. With  $\Psi_{Rm} \gg \Psi_q$  and  $\Psi_m \approx \Psi_d$ , desired change of the flux  $\Psi_m$  determines desired change of the flux  $\Psi_d$ , which changes according to

$$\Psi_d(\Omega_m)|_{\Omega_m > \Omega_n} = \Psi_{Rm} \frac{\Omega_n}{\Omega_m} = \Psi_{Rm} \frac{\omega_n}{\omega_m}. \quad (22.4)$$

With  $\Psi_m \approx \Psi_{Rm} + L_d i_d$ , it is concluded that the field weakening requires a certain negative current in  $d$ -axis,  $i_d < 0$ , which performs defluxing or demagnetization. The change of this defluxing current with the rotor speed is given in (22.5) and in Fig. 22.2. The rated flux of the machine  $\Psi_n$  is equal to  $\Psi_{Rm}$ :

$$i_d(\omega_m)|_{|\omega_m| \geq \omega_n} = -\frac{\Psi_{Rm}}{L_m} \left(1 - \frac{\Omega_n}{\Omega_m}\right) \approx -\frac{\Psi_n}{L_d} \left(1 - \frac{\omega_n}{\omega_m}\right). \quad (22.5)$$

**Question (22.1):** An isotropic synchronous machine with permanent magnets on the rotor has the stator inductance  $L_S = 0.05$ . Determine the maximum rotor speed in steady state, with no mechanical load attached to the shaft.

**Answer (22.1):** In operation with high speeds that exceed the rated speed, it is necessary to provide defluxing current in  $d$ -axis which has a negative value,  $i_d < 0$ . This current reduces  $d$ -axis flux and maintains the electromotive force within the limits of the available supply voltage. In the absence of the load torque, the electromagnetic torque and  $q$ -axis current are equal to zero. With  $i_q = 0$ , all of the available stator current can be used for weakening the field in  $d$ -axis. At steady state, the current in the stator windings must not exceed the rated current. Therefore, relative value of the defluxing current that can be maintained permanently is  $i_d = -1$ . By using the expression for the defluxing current  $i_d(\omega_m)$  in the region of field weakening, one obtains  $(1 - \omega_n/\omega_m) = L_S/\Psi_{Rm} = L_S/\Psi_n$ . Relative value of the rated flux is equal to one, thus resulting in  $\omega_n/\omega_m = 1 - 0.05 = 0.95$ , which gives  $\omega_m = 1.0526 \omega_n$ .

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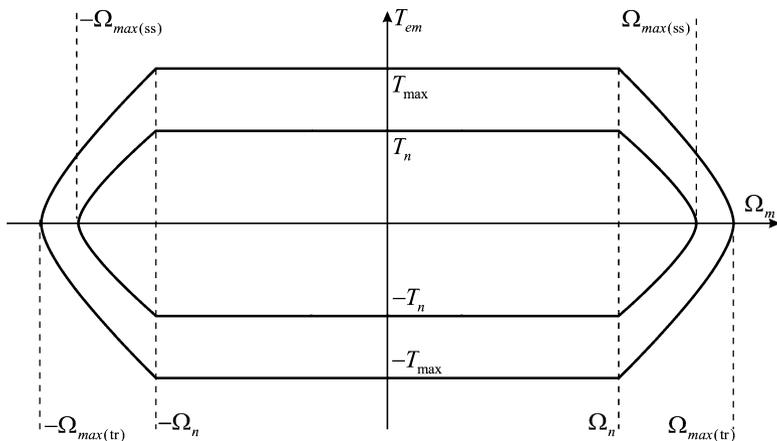
Previous example shows that synchronous motors with surface-mounted magnets and with very low stator inductance cannot perform any significant increase of the rotor speed above the rated value. Therefore, they cannot be used in the field-weakening operation. On the other hand, induction servomotor can operate in field-weakening mode and achieve the rotor speeds that exceed the rated speed by several times. This is due to the fact that the flux in induction motors gets produced by the stator current and not by permanent magnets. This shortcoming of synchronous servomotors is the principal reason that induction servomotors are still in use. Namely, other characteristics of these two types of servomotors go in favor of synchronous motors. Induction machines have the power of losses significantly larger than synchronous machines. While the rotor losses in synchronous machines are virtually zero, there are considerable rotor losses  $P_{Cu2} = sP_n$  in induction machines, proportional to the slip. The absence of the rotor losses facilitates cooling of synchronous motors and allows larger current and flux densities. Therefore, synchronous servomotors have superior specific power and torque, and they are smaller than the equivalent induction servomotors. Moreover, their rotors have much lower inertia  $J$ , which is beneficial in most motion control applications. There are, however, applications of electrical actuators where it is very important to provide the operation in the field-weakening region and to provide the rotor speed that goes well beyond the rated speed. In these applications, induction servomotors are advantageously used.

Synchronous motors with surface-mounted permanent magnets have a very low stator inductance. This proves as an advantage in motion control applications such as industrial robots and manipulators. With a low inductance  $L_S$ , the rate of change of the stator current  $di_d/dt = (u_a - e_a)/L_S$  is very large and it reaches  $di_d/dt \approx 10^4 I_n/s$ , which allows the electromagnetic torque  $T_{em} = k |\Psi_{Rm} \times i_S|$  to make the change from zero to the rated value in some 100  $\mu s$ . Fast torque response contributes to an increased bandwidth and improved closed loop performance in speed control and position control applications. For this reason, synchronous motors with surface-mounted magnets are applied in industry automation, robotics, and many other motion control applications that employ speed and position control loops. The absence of the field-weakening operations in these applications is not considered a significant shortcoming.

### 22.5 Transient and Steady-State Operating Area

Due to a very low inductance of the stator winding of  $L_S = 0.01 \dots 0.05$  relative units, the operation of synchronous motors with permanent magnets mounted on the rotor surface is limited to the range of  $\Omega_m \in [-\Omega_n \dots +\Omega_n]$ . The maximum no load speed at steady state is  $\Omega_{max(ss)} = \Omega_n/(1 - L_S)$  which exceeds the rated speed by several percents. The crossing of the steady-state operating limits and the abscissa in  $T_{em} - \Omega_m$  diagram is at the speed  $\Omega_n/(1 - L_S)$ , which is just slightly above the rated speed. Regarding the torque steady-state operating limits, continuous torque is determined by the rated stator current. The steady-state torque has to remain within the limits of the rated torque,  $-T_n \leq T_{em} \leq +T_n$ . Steady-state operating area and transient operating area are shown in Fig. 22.3.

The transient operating limits depend on the maximum available instantaneous value of the stator current. The stator current may exceed the rated current for a brief interval of time. This does not have to result in excessive motor temperature, provided that in between the current pulses there is sufficient time with reduced current, so that the motor can release the heat and cool down. All the motion control tasks are usually performed in cycles. Within each cycle, the torque of relatively large value is delivered within relatively short interval of time, in order to perform desired acceleration or deceleration. These short intervals are mostly followed by prolonged intervals with reduced torque and reduced stator current. For this reason, the ratio between the peak torque and the rms value of the torque within one motion cycle is very large. The same conclusion holds for the peak and rms values of the stator current. Therefore, the transient operating area is limited by the peak torque which exceeds the rated torque by several times. Synchronous permanent magnet motors are designed and manufactured to sustain overloads of  $I_{max}/I_n$  in excess to five.



**Fig. 22.3** The transient and steady-state operating limits of synchronous motors with permanent magnet excitation

The peak current capability is limited by the motor construction but also by the characteristics of the three-phase inverter which supplies the stator windings. Semiconductor power switches within the inverter comprise tiny silicon crystals with very low thermal capacity and with limited density of electrical current. In cases where the current density within semiconductor exceeds certain limit, there is an abrupt increase in temperature which changes the structure of the crystal and causes permanent damage to the semiconductor power switch. Semiconductor power switches can withstand electrical currents that exceed declared limits but only for relatively short intervals of time, measured in milliseconds. Large peak currents can also damage the synchronous servomotor. Large stator current may damage the permanent magnets. The stator currents produce the stator magnetomotive force, which depends on the current amplitude and on the number of turns. With large currents, the stator magnetomotive force produces large demagnetization field within the magnets. In  $B$ - $H$  characteristic of permanent magnets, the operating point ( $B, H$ ) of the magnets is moved closer to coercive field  $H_C < 0$ , where the induction  $B$  of permanent magnets reduces to zero. For most permanent magnet materials, reaching the coercive field  $H_C$  damages the magnet. Having reached this operating point, their remanent induction cannot return to the initial value. Instead, the remanent induction is decreased by two or three times. Therefore, the maximum permissible stator current is set to the value that does not bring the risk of damaging the magnets. At the same time, it has to be compatible with peak current capability of semiconductor power switches that are used within the three-phase inverter attached to the stator windings.

The maximum torque  $T_{max}$  which defines the limits of the transient operating area is determined by the peak current  $I_{max}$ . The maximum no load speed  $\Omega_{max(tr)}$  that can be reached over short time intervals is determined by expression  $\Omega_{max(tr)} = \Omega_n / (1 - I_{max} L_S)$ , where the stator inductance and the peak current are both expressed in relative units.

**Question (22.2):** An isotropic machine with permanent magnets on the rotor has stator self-inductance of  $L_S = 0.05$ . Find the maximum rotor speed that can be reached for a short interval of time. The peak current capability is  $I_{max} = 5$ , while the mechanical load attached to the shaft is equal to zero.

**Answer (22.2):** By using expression for demagnetizing current  $i_d(\omega_m)$  in the field-weakening region, one obtains that  $(1 - \omega_n/\omega_m) = I_{max} L_S / \Psi_{Rm} = I_{max} L_S / \Psi_n$ . Relative value of the rated flux is equal to one; thus, the ratio of the rated speed and the maximum speed is determined by  $\omega_n/\omega_m = 1 - 5 \cdot 0.05 = 0.75$ , resulting in  $\Omega_m = 1.33 \cdot \Omega_n$ .

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Synchronous permanent magnet machines are also used in applications that require high efficiency, low losses, and high specific power, but where it is not necessary to effectuate quick changes of the electromagnetic torque. These applications do not include motion control tasks, and they do not use synchronous machines in speed control and position control loops. Some of the examples are the

motors used in blowers, household appliances, and HVAC systems but also the generators in renewable sources of electrical energy, such as the wind turbines, the motors in electrical vehicle propulsion, auxiliary drives in automotive field, and similar. Superior efficiency, lower weight, and lower inertia of synchronous permanent magnet motors are the reasons for their use instead of corresponding induction machines. In order to improve the field-weakening performance of synchronous permanent magnet machines, it is possible to remove the permanent magnets from the rotor surface and to bury them deep into the rotor magnetic circuit. This results in larger values of the stator self-inductance. According to (22.5), larger  $L_S$  improves capability of synchronous machines to operate above the rated speed. Yet, the field-weakening performance of synchronous permanent magnet machine is inferior to that of induction machines. Synchronous permanent magnet machines that operate in the field-weakening range require considerable demagnetization current  $i_d < 0$  which does not contribute to the torque, while increasing the amplitude of the stator current  $I_S = \sqrt{i_d^2 + i_q^2}$  and increasing the copper losses. For that reason, the applications requiring prolonged operation in field-weakening mode with considerable ratio  $\Omega_m/\Omega_n$  call for an induction machine.