Chapter 17
Variable Speed Induction Machines

This chapter discusses the means for the speed change of induction machines. The speed regulation is required in both generators and motors. Induction machines that serve as generators in wind power stations revolve at variable speed. Therefore, the machine and the associated equipment must ensure conversion of mechanical work in electrical energy at variable speed. The machines used as motors often serve in motion control applications, where the speed changes in continuous manner.

In the first part of this chapter, the means for altering the rotor speed of mains-supplied induction machines are discussed and explained. Possibilities are considered to adjust the rotor speed of induction machines operating with constant frequency of stator voltages. The analysis considers changes in mechanical characteristic and the rotor speed due to variations of the voltage amplitude. Variation of the rotor resistance is studied as the means of changing the rotor speed of wound rotor induction machines. The impact of the number of magnetic pole pairs p on synchronous speed Ω_e and rotor speed Ω_m is reinstated, and the change of the number of poles is looked upon as the means of changing the rotor speed. An introduction to electrical machines with multiple pole pairs is given by studying distribution of the magnetic field of an electrical machine with $2p = 4$ magnetic poles. The possibility of designing the stator winding so as to achieve the magnetic field with multiple pole pairs is studied on a sample winding that can be switched to produce either $2p = 2$ poles or $2p = 4$ poles. The expressions for synchronous speed, rotor speed, and slip frequency are given as functions of the number of magnetic pole pairs. Discussion on constant frequency-supplied induction machine closes with establishing deficiencies, limitations, energy losses, and design problems arising in mains-supplied induction machines.

The second part of this chapter deals with induction machines supplied from variable frequency sources such as the three-phase inverters with switching power transistors and pulse width modulation control. This chapter introduces basic aspects and problems of variable frequency supply. Operation of induction machines fed from variable frequency static power converter is introduced and studied. A short review of power converter topologies used for supplying induction machines is presented, along with methods for continuous change in the stator

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voltage amplitude and frequency, suited to accomplish desired rotor speed and desired flux. The effects of changing the supply frequency on mechanical characteristic are analyzed in both the constant flux region and in the field weakening region. The basic approaches the torque, flux, and power control are outlined for an induction machine fed from a variable frequency static power converter. Family of mechanical characteristics obtained by frequency variation is presented and explained. Based upon the study of operating limits of the machine and operating limits of associated three-phase inverter, steady state operating area and transient operating area are derived in T-Ω plane and studied for variable frequency operation of induction machines. The limits of constant power operation in field weakening mode are determined, explained, and expressed in terms of the machine leakage inductance. Finally, this chapter discusses the differences in construction and parameters of mains-supplied induction machines and inverter-supplied induction machines.

17.1 Speed Changes in Mains-Supplied Machines

In majority of applications of electrical machines, it is required to accomplish a continuous variation of the rotor speed. Some of examples are motion control tasks in production machines and industrial robots and propulsion tasks in electrical vehicles, fans, pumps, and similar.

The rotor speed of induction machines is different from the synchronous speed by the amount of the slip. When an induction motor is supplied from the mains, the stator current is maintained within rated limits under condition that the slip remains relatively low, $|s| \le s_n$, namely, for the speed range $\Omega_{en}(1 - s_n) \le \Omega_m \le \Omega_{en}(1 + s_n)$. Hence, continuous operation of mains-supplied induction machine is restricted to a rather narrow range of speeds. For medium- and high-power machines, the rated slip is lower than 1%; thus, the condition $I_S \le I_n$ is maintained within the range of speeds from 99% up to 101% of the synchronous speed. Operation at higher slip frequencies involves high losses and high currents in stator and rotor windings. The use of induction machine outside the zone where $|s| \le s_n$ and $I_S \le I_n$ leads to an increase of the machine temperature. For this reason, continued service of induction machines is possible only with relatively small values of slip. Therefore, it is justified to conclude that the speed of rotation Ω_m is close to the synchronous speed Ω_e. Synchronous speed $\Omega_e = \omega_e/p$ is determined by the angular frequency of the stator voltages ω_e, that is, by the frequency of electrical currents in stator windings. For two-pole machines, where the number of pole pairs p is equal to 1, the synchronous speed is equal to the angular frequency of the supply. For this reason, variation of the rotor speed of an induction machine requires a variable frequency of the stator voltages and currents. This can be accomplished by supplying the machine from a three-phase source of AC voltages having variable frequency. Most common solution to this is the three-phase inverter, a power converter which employs semiconductor power switches. Inverters are mostly used in conjunction with three-phase diode rectifiers. Three-phase diode rectifiers are power converters supplied from the mains with line voltages of 400 V and line frequency of $f = 50$ or 60 Hz. The rectifier converts the AC line voltages into DC voltage. This DC voltage is fed to the three-phase inverter, which converts the DC voltage into a set of three-phase AC voltage of variable frequency and variable amplitude. Finally, at the output terminals of the inverter, a three-phase system is available with frequency and voltage amplitude that can be changed to suit the needs of the induction machine. Contemporary inverters apply pulse width modulation control and make use of semiconductor switches such as bipolar transistors (BJT), MOSFET transistors, and IGBT transistors. Industrial use of such devices started in last decades of the twentieth century. At present, power converters using power transistors are standard industrial units for supplying induction machines and changing their speed (Fig. 17.10a).

Induction machines have been in use for more than 100 years. During the first century of industrial use of induction machines, there were no semiconductor switches suitable for designing the three-phase inverters. Therefore, another kind of components, devices, and techniques was being devised and used to achieve continuous variation of the rotor speed of induction machines. Induction machines were supplied from the mains providing the voltages of industrial frequency, whether $f_e = 50$ Hz or $f_e = 60$ Hz. There were no ways of altering the supply frequency and providing continuous change of the synchronous speed. Therefore, most induction machines were primarily used in constant speed applications. Particular procedures, methods, and devices were used in applications requiring variable speed operation, all of them conditioned and restricted by technology limits of the times. Traditional approaches to speed variations include:

1. Variation of stator voltage
2. Variation of rotor resistance
3. Variation of the number of poles

Besides, in some cases, induction motors were connected to mechanical load by means of transmission mechanism with gears or some other mechanical transducers. Using particularly suited transmission with variable transmission ratio, it was possible to change the load speed while the induction motor speed remained constant, close to the synchronous speed.

In further text, the effects of the traditional approaches (1), (2), and (3) will be reviewed.

17.2 Voltage Change

With constant frequency supply, mechanical characteristic of an induction machine crosses the abscissa Ω_m of T_{em}–Ω_m plane at $\Omega_m = \Omega_e = 2\pi f_e/p$, where $T_{em} = 0$ (Fig. 17.1). Upon loading, the slip increases and the speed decreases. In generator mode, where $T_{em} < 0$ and $\omega_{slip} < 0$, the torque amplitude $|T_{em}|$ increases as the

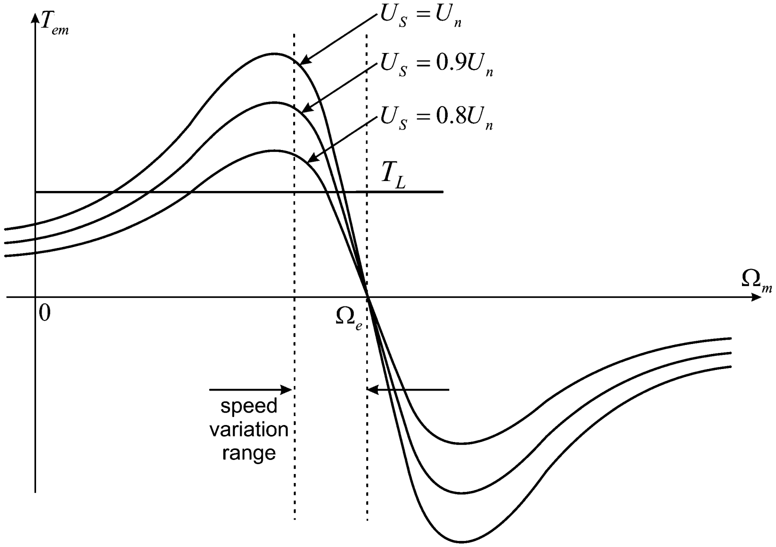

Fig. 17.1 Effects of voltage changes on mechanical characteristic

rotor speed goes beyond Ω_e. The electromagnetic torque is proportional to the slip and also proportional to the square of the stator flux Ψ_S. In turn, the flux amplitude Ψ_S is proportional to the quotient of the stator voltage and the angular frequency, $\Psi_S \approx U_S/\omega_e$.

Reduction of the stator voltage results in reduction of the flux. At the same time, the slope of the mechanical characteristic $S = \Delta T/\Delta\omega$ is reduced as well. As a consequence, the speed would exhibit larger reduction for the same torque. In the region of small slip, the electromagnetic torque varies according to the law $T_{em} = k\omega_{slip}\Psi_S^2/R_R$. Therefore, reduction of the flux Ψ_S causes an increase in the slip speed ω_{slip} and reduction of the rotor speed.

Speed can be changed by the voltage variations in a limited range. Namely, the speed can be varied within the range that does not exceed the breakdown slip s_b. A consequence of the speed reduction by the slip increase is an increase of the rotor losses $P_{Cu2} \approx sP_\delta$, which may lead to increase of the machine temperature.

Variation of the rotor speed of an induction machine obtained by changing the amplitude of the supply voltage is seldom used. One possible use of the voltage control is in fan drives, where mechanical power is proportional to the third power of the rotor speed. It takes a relatively small change of the rotor speed to produce significant change of the torque and power. Hence, the speed regulation based on reduction of the stator voltage does not produce significant losses $P_{Cu2} \approx sP_\delta$ since the air-gap power $P_\delta \sim \Omega_m^3$ reduces considerably even with relatively small drop in the rotor speed. For small fan drives with induction motors, voltage can be reduced by inserting variable resistors in series with the stator winding. The series connection of the stator windings and variable resistors is fed from the mains with constant voltages. An increase in series resistance reduces the voltage across the stator windings, increases the slip, and reduces the rotor speed. Disadvantage of this approach is the presence of Joule losses in series resistors.

17.3 Wound Rotor Machines

Most induction machines have the rotor winding made of cast aluminum, having the form of a cage with two short-circuiting rings. On the other hand, the rotor winding can be made in the same way as the stator winding by placing insulated copper conductors in the rotor slots and connecting these conductors in series, so as to make a star-connected three-phase winding. The three end terminals of such winding can be made available from the stator side. In most cases, the ends of the rotor winding are connected to three conductive rings that are fastened to the shaft and mutually isolated. These rings rotate with the shaft, but they are electrically isolated from the shaft too. From the stator side, there are three conductive brushes fastened to the stator and pressed against revolving rings. They touch the external surface of the rings and provide electrical contacts. The brushes slide along the circumference of the rings, which are also called *slip rings*. Induction machine with slip rings is also called *wound rotor* machine. The three brushes make the rotor winding ends available for stator side connections. A variable three-phase resistor can be connected to the brushes, providing the means for changing the equivalent resistance of the rotor circuit. The effects of inserting an external resistor into the rotor circuit are the same as the effects of hypothetical changes of the rotor resistance of the rotor with short-circuited cage. Variation of an externally connected three-phase resistor changes mechanical characteristic of the machine in the way shown in Fig. 17.2. Sample wound rotor machine is given in Fig. 17.3.

When the brushes are brought into short circuit, the wound rotor is short-circuited. In such case, behavior of the machine and its mechanical characteristic are the same as with a squirrel cage induction machine. By way of slip rings and brushes, additional resistance R_{ext} can be inserted in the rotor circuit, increasing in this way the value of equivalent rotor resistance $R_{Re} = R_R + R_{ext}$ which should substitute R_R in the steady state equivalent circuit.

Considering the expression for the breakdown torque, it is concluded that $T_b \sim \Psi_S^2/L_{\gamma e}$. Hence, an increase in the rotor resistance R_{Re} does not affect the value of the breakdown torque. On the other hand, the slope $\Delta T/\Delta\omega$ of the mechanical characteristic is inversely proportional to the rotor resistance, $T_{em} \sim k\omega_{slip}\Psi_S^2/R_{Re}$; thus, increasing the total rotor resistance R_{Re} decreases the slope of the mechanical characteristic $|\Delta T/\Delta\omega|$. Breakdown slip $s_b = R_{Re}/X_{\gamma e}$ is proportional to the rotor resistance; thus, increasing the external resistor R_{ext} increases the breakdown slip in both motor and generator modes. Total effects of variation of the rotor resistance on mechanical characteristic are shown in Fig. 17.2. While breakdown torque remains unchanged, breakdown slip increases, while slope of the mechanical characteristic gradually decreases.

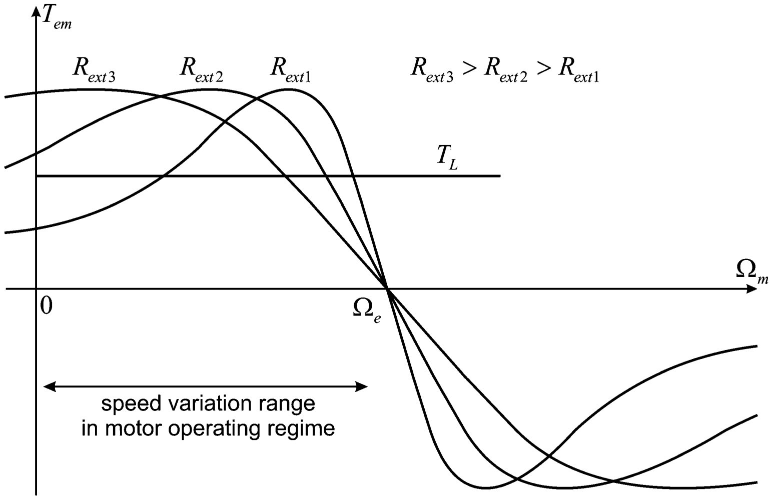

Fig. 17.2 Influence of rotor resistance on mechanical characteristic

The effects of the rotor resistance changes on the rotor speed are investigated by assuming that the load torque is constant as well as the supply frequency and the synchronous speed. In Fig. 17.2, the load characteristic becomes a horizontal line. The operating speed is obtained at the crossing of the load characteristic and the mechanical characteristic. It is observed in the figure that the intersection of the two characteristics occurs at lower speeds if the equivalent rotor resistance is larger. Hence, the rotor speed can be changed by changing the external resistor connected in the rotor circuit via slip rings and brushes. Continuous variation of the resistance enables continuous variation of the rotor speed. Unlike squirrel cage motors, wound rotor motors can operate with very large slips. When operating with slips in excess to the rated slip, the stator and rotor currents of the squirrel cage motors exceed the rated currents and result in overheating. An induction motor with wound rotor may operate with higher slip values. With elevated slip, resistance R_{Re}/s of the rotor branch in the equivalent circuit is reduced, but the equivalent resistance $R_{Re} = R_R + R_{ext}$ is increased due to external resistance, keeping the stator and rotor currents within acceptable limits.

A shortcoming of the described approach is poor efficiency caused by additional losses in the external resistor. The speed $\Omega_m = \Omega_e(1-s)$ is controlled in the way which keeps synchronous speed Ω_e constant. The rotor speed Ω_m is lowered on account of an increased slip s. With an increase in slip, the rotor losses $sP_\delta = P_{Cu2}$ are increased as well. Increased losses P_{Cu2} do not cause overheating of the induction machine, because significant part of these losses is dissipated in the external three-phase resistor. Nevertheless, the efficiency of the system is significantly reduced. Mains-supplied two-pole induction motor has the synchronous speed of $n_e = 3{,}000$ rpm. In cases when the rotor speed is reduced to 1,500 rpm,

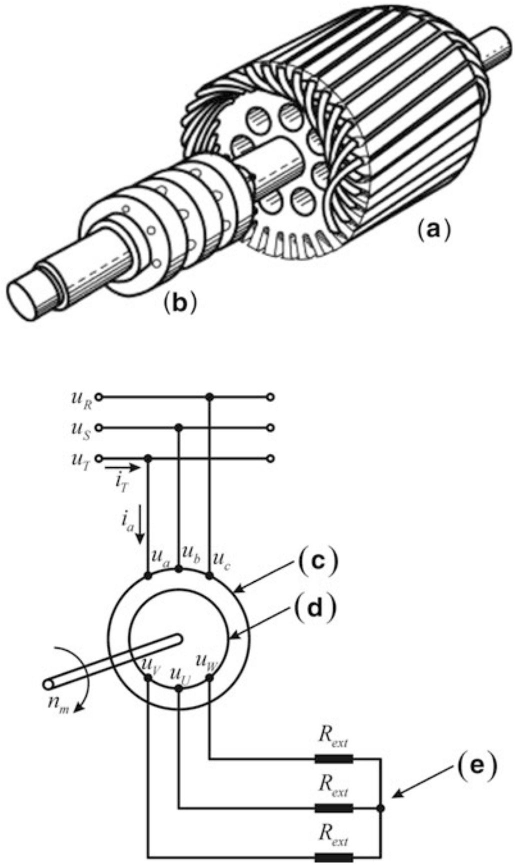

Fig. 17.3 Wound rotor with slip rings and external resistor. (**a**) Three-phase rotor winding. (**b**) Slip rings. (**c**) Stator. (**d**) Rotor. (**e**) External resistor

one half of the air-gap power is dissipated in the rotor circuit, while the other half is converted into mechanical power. The efficiency of this induction motor is lower than 50%. Poor efficiency is the consequence of a high value of sP_δ, also called *slip power*.

Efficiency of an induction machine with wound rotor and external resistor is poor due to large slip power sP_δ which is converted into heat. Efficiency can be increased by recovering the slip power back to the mains. In the middle of the twentieth century, semiconductor diodes and thyristors suitable for industrial applications have been developed and put to use. Static power converters have been designed comprising diodes and thyristors. Connecting a static power converter to the rotor circuit, the slip power sP_δ is transferred to the diode rectifier, shown in Fig. 17.4, which converts AC rotor currents to DC currents in the choke denoted by L_d. Further on, thyristor converter (C) converts DC currents to AC currents, the latter having the line frequency and being directed back into the mains through the transformer (D). In this way, the slip power sP_δ is returned to the mains

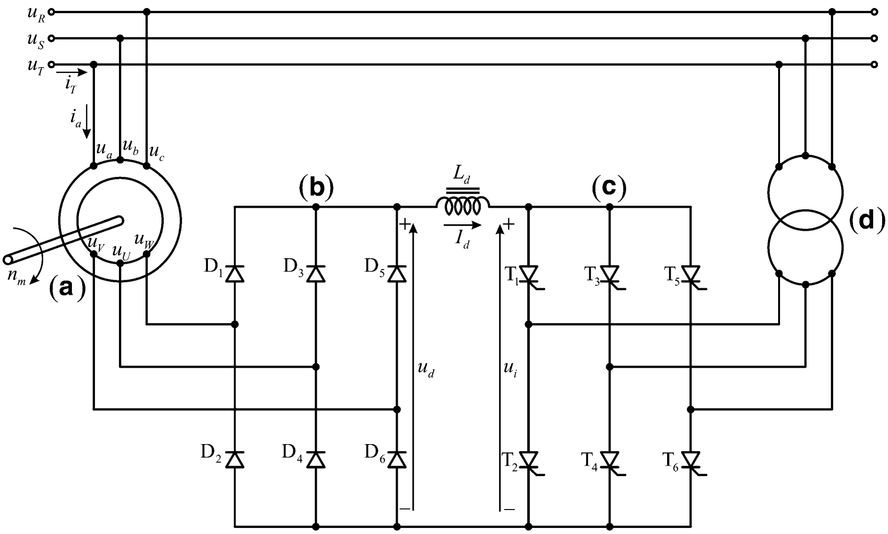

Fig. 17.4 Static power converter in the rotor circuit recuperates the slip power. (**a**) The converter is connected to the rotor winding via slip rings and brushes. (**b**) Diode rectifier converts AC rotor currents into DC currents. (**c**) Thyristor converter converts DC currents into line frequency AC currents. (**d**) Slip power recovered to the mains

instead of being wasted in heat. Converter structures connected into the rotor circuit are known as *synchronous cascades*.[1]

Development of power transistor suitable for building high-power inverters culminated in the last quarter of the twentieth century. It provided the means for supplying the induction machines with three-phase voltages of variable frequency. This allowed continuous change of synchronous speed as a viable way of controlling the rotor speed. Therefore, the need for wound rotor induction machines and cascade static power converters gradually declined.

[1] Slip rings' access to the rotor winding can be used to take the slip power out of the rotor circuit, as shown in Fig. 17.4. It is also possible to use the static power converter of different topology and to use it to supply the power to the rotor circuit. In this case, slip power and slip speed are negative, and the rotor revolves at the speed $\Omega_m > \Omega_e$. The two considered topologies are called *subsynchronous cascade* and *supersynchronous cascade*. Over the past century, there were also applications of wound rotor induction machines with slip rings and a four-quadrant (reversible) static power converter in the rotor circuit. With four-quadrant rotor converter, wound rotor machine can operate with $\Omega_m > \Omega_e$ as well as with $\Omega_e > \Omega_m$. Some early wind power solutions were conceived with wound rotor induction generators and static power converter in the rotor circuit. The advantage of this approach is relatively low slip power which results in relatively low voltage and current ratings of semiconductor power switches. More recent wind power generators are based on squirrel cage induction machines and full-power transistor-based static power converters that provide the interface between the constant frequency mains and variable frequency stator voltages.

17.4 Changing Pole Pairs

The rotor speed of a squirrel cage induction machine is close to the synchronous speed $\Omega_e = \omega_e/p$, where ω_e is the angular frequency of the power supply, while $2p$ is the number of magnetic poles of the machine. The mains-supplied machines operate with constant stator frequency, and their synchronous speed cannot be varied.[2] There is, however, a possibility to vary the number of poles $2p$. For a two-pole machine with $p = 1$ and with $f = 50$ Hz, the synchronous speed is $n_e = 3{,}000$ rpm. By increasing the number of poles to $2p = 4$, 6, or 8, one obtains synchronous speeds of 1,500, 1,000, and 750 rpm, respectively. Hence, the synchronous speed as well as the rotor speed can be changed in discrete steps by altering the number of magnetic poles.

It is of interest to discuss the number of magnetic poles in AC and DC machines. In DC machines, magnetic field is created by permanent magnets or by the excitation winding. The number of magnetic poles is determined by design of the machine, and it is equal to the number of main stator poles. For that reason, it cannot be changed during the operation. The number of magnetic poles in DC machines affects design and the construction of mechanical commutator, machine windings, and the form of magnetic circuits. For that reason, it is not possible to change the number of magnetic poles unless the whole construction of the machine is changed. In AC machines, magnetic field is created by electrical current in the stator windings. With suitable design of the stator windings, appropriate reconnection of the stator phases can be used to change the number of magnetic poles. Hence, the synchronous speed of an induction motor as well as the rotor speed can be changed by altering the electrical connections of the stator phases.

In many applications of electric drives, it is not required to accomplish a continuous variation of the rotor speed. Instead, it is sufficient to have two or three discrete values of the speed which can be selected as required. Then, the problem of regulation of the rotor speed can be solved by using an induction machine fed from a constant frequency source, under conditions that the number of poles can be varied. In the case of an induction machine, magnetic poles of the rotating field are not related to any particular part of the magnetic circuit. Instead, they rotate with respect to the stator and rotor magnetic circuits. The number of poles is dependent on distribution of electrical current in the stator slots. This distribution depends on the supply currents and on the method used to make the stator winding. So far, induction machines have been considered with two-pole magnetic field which has diametrically positioned north and south magnetic poles, as shown in the upper part of Fig. 17.5. The same figure also shows distribution of the stator currents which create magnetic field with two north and two south poles.

In the lower part of Fig. 17.5, electrical currents in diametrically positioned conductors have the same direction. In one pair of diametrically placed conductors,

[2] When the machine is supplied from an inverter with power transistors, the frequency of the supply can be varied. Consequently, the rotor speed can be varied too.

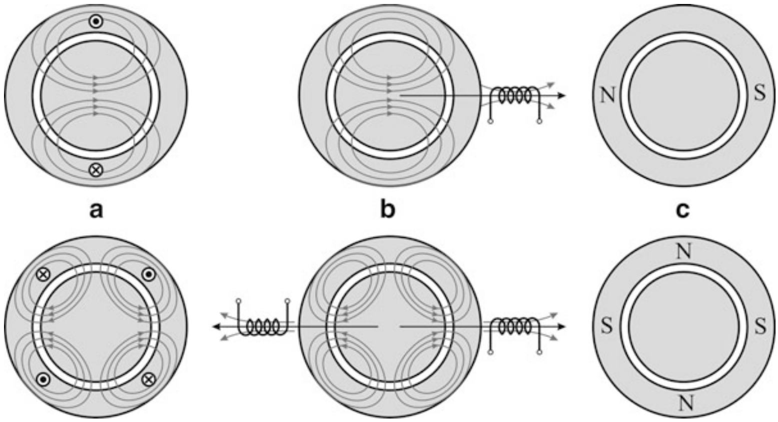

Fig. 17.5 Two-pole and four-pole magnetic fields. (**a**) Windings. (**b**) Magnetic axes. (**c**) Magnetic poles

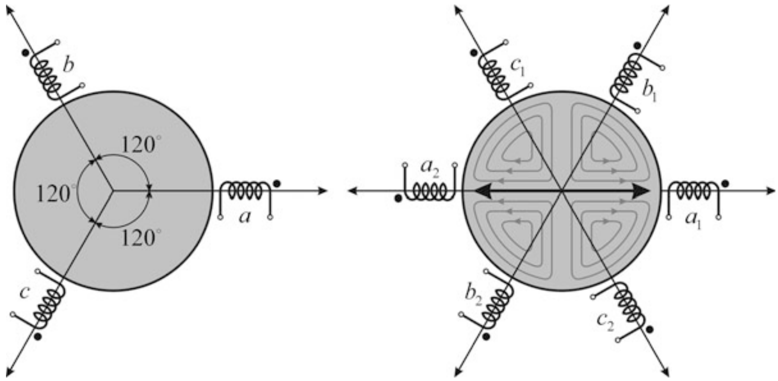

Fig. 17.6 Three-phase four-pole stator winding

direction is ⊙. The other pair of diametrically positioned conductors is displaced by π/2 with respect to the first pair, and it has electrical currents of the opposite direction ⊗. The magnetomotive force created by the system of four conductors comprises two zones where the lines of magnetic field pass from rotor to stator and two zones where the lines of magnetic field pass from stator to rotor. Therefore, stator currents in this example create magnetic field having two north poles and two south poles. This field is called a four-pole field. Magnetic field under consideration has four magnetic poles or two-pole pairs; hence, $p = 2$.

Example given in Fig. 17.5 shows that the three-phase windings with appropriate distribution of conductors can create a four-pole rotating magnetic field. On the left-hand side in Fig. 17.6, there is a three-phase stator winding with spatial displacement of 2π/3 between the phases, which creates a two-pole magnetic field in the air gap. A stator winding which creates a two-pole field is often called *two-pole winding*. The right-hand side of the same figure shows the method of forming a three-phase stator winding with magnetomotive force that creates a four-pole magnetic field in the air gap. Each of the phases a, b, and c is split into two sections, each with the same number of turns. The sections of the phase windings are spatially displaced by π, while the angle between the neighboring sections is $\pi/3$, which is one half of the spatial shift of a two-pole winding.

Phase a of the four-pole stator winding consists of two diametrically positioned sections. The sections are connected in series; thus, the conductors of both sections carry the same current $i_a(t)$. The reference terminals of each of the sections are marked by dots. Convention of marking the reference terminal is the following: when the current enters the dot-marked terminal of the considered section, then the section creates the magnetomotive force and field which start from the rotor magnetic circuit and propagate through the air gap toward the considered section. In the case when the two sections, actually the two halves a_1 and a_2 of the phase winding a, are connected in series and in such way that the phase current $i_a(t)$ enters the dot-marked terminals of both halves, magnetic field is created with two north and two south poles. Namely, at positions $\theta = 0$ and $\theta = \pi$, it creates the field with lines that pass from the rotor to the stator. Due to flux conservation law (div $\boldsymbol{B} = 0$), corresponding field lines must pass from the stator to the rotor at positions $\theta = \pi/2$ and $\theta = 3\pi/2$. In this way, a four-pole field is created, the field with two pairs of poles ($p = 2$), having two north magnetic poles and two south magnetic poles. With phase currents $i_a(t)$, $i_b(t)$, and $i_c(t)$ that have sinusoidal change of the same amplitude and frequency, and which are phase shifted by $2\pi/3$, consequential magnetic field revolves at the speed determined by the supply frequency, and it does not change the amplitude. It can be shown that the speed of rotation Ω_e of the four-pole field is equal to one half of the supply frequency ω_e. From Fig. 17.6, it should be noted that one of the magnetic poles is against the section a_1 at $t = 0$, when the phase current $i_a(t) = I_m \cos \omega_e t$ has the value $i_a(t) = +I_m$. Corresponding magnetic pole is marked by an arrow $\rightarrow$ which is directed toward the section a_1. The phase current $i_b(t) = I_m \cos(\omega_e t - 2\pi/3)$ is delayed, and it reached its maximum positive value at $t_1 = 2\pi/(3\omega_e)$. Over the interval $0 < t < t_1$, the considered magnetic pole is shifting from the section a_1 toward the section b_1, where it arrives at the instant t_1. At $t_2 = 4\pi/(3\omega_e)$, the phase current $i_c(t)$ reaches its maximum value. Then the considered magnetic pole is placed against the section c_1. At the end of one full period, at $t_3 = 2\pi/\omega_e$, the maximum of $i_a(t) = +I_m$ is repeated in the phase current $i_a(t)$. At the same time, the magnetic pole being tracked reaches position against the section a_2. During one period of voltages and currents (*electrical period*) $T = 2\pi/\omega_e$, the phase change of electrical currents is $\Delta\theta_e = 2\pi$, while the spatial displacement of the magnetic pole is $\Delta\theta_m = \pi$. Therefore, the speed of rotation of the field is one half of the supply frequency, $\Omega_e = \omega_e/2$.

It should be noted that the four-pole field shown in Fig. 17.6 has two diametrically positioned poles of the same polarity. During one period of variation of electrical variables, both poles shift by π; hence, they switch their places.

In general, when stator windings are arranged to make a rotating magnetic field with $2p$ poles, the synchronous speed of the field rotation is $\Omega_e = \omega_e/p$. The torque of multipole induction machines is determined on the basis of expression $T_{em} = P_\delta/\Omega_e = pP_\delta/\omega_e$, according to (17.3).

By changing the number of magnetic poles, windings obtain a different number of slots per phase, different winding factors such as belt factor and chord factor, and different magnetic induction, winding resistance, leakage inductance, and self-inductance.

17.4.1 Speed and Torque of Multipole Machines

The number of magnetic pole pairs of an electrical machine is denoted by p. Synchronous speed of an induction motor is equal to

$$\Omega_e = \frac{\omega_e}{p}, \tag{17.1}$$

while the speed of rotor rotation is equal to

$$\Omega_m = \frac{\omega_m}{p} = \frac{\omega_e - \omega_{slip}}{p} = \Omega_e - \Omega_{slip}, \tag{17.2}$$

where ω_{slip} is the angular frequency of the rotor currents. The electromagnetic torque is

$$T_{em} = \frac{P_\delta}{\Omega_e} = p\frac{P_\delta}{\omega_e}. \tag{17.3}$$

17.5 Characteristics of Multipole Machines

Multipole machines ($p > 1$) make better use of the copper and iron compared to two-pole machines. There are applications of induction machines where selection of higher number of poles offers higher specific torque and power compared to the solutions involving two-pole machines. A rationale of these statements is presented here.

It is known that one turn of the phase winding of two-pole machine is positioned diametrically at angular distance $\Delta\theta \approx \pi$. When one of these conductors is under the north magnetic pole of the revolving field, the other is below the south pole. At the front and rear of the machine, the two conductors are connected by end turns. In two-pole machines, end turns are relatively long. Their length is one half of the machine circumference. Longer conductors contribute to increased consumption of copper, higher resistance of the winding, and higher losses in copper. In multipole machines, conductors making one turn are placed at angular distance $\Delta\theta \approx \pi/p$, which corresponds to the distance between the two magnetic poles. Length of the end turns in multipole machines is much shorter, which reflects favorably to the total mass of consumed copper, reduces winding resistance, and reduces power losses.

Moreover, multipole machines make a more efficient use of ferromagnetic materials. Magnetic field lines pass from the zone of the north magnetic pole of the stator, pass through the air gap, enter the rotor magnetic circuit, pass through the air gap for the second time, and reach the zone of the south magnetic pole of the stator. Following that, the field lines pass through the stator yoke and return to the north pole. Passing tangentially along the stator perimeter, the field lines in a two-pole machine cover the angular distance of $\Delta\theta \approx \pi$. Passing tangentially, the field lines do not pass through the zone where the stator slots are located. Instead, they pass through the outer part of the stator magnetic circuit called *yoke*. The yoke is required to reduce magnetic resistance on the flux path. On the other hand, the field passing through the yoke does not contribute to electromechanical conversion. Instead, it increases the iron weight and the total mass of the machine. In multipole machines, the path covered by the field lines between the two magnetic poles is shorter, and it is equal to $\Delta\theta \approx \pi/p$. Hence, the flux path through the yoke is shorter, and the iron usage is improved.

17.5.1 Mains-Supplied Multipole Machines

Synchronous speed of mains-supplied machines is determined by the line frequency f_e, and it is equal to $\Omega_e = 2\pi f_e/p$. Their specific power depends on the number of pole pairs. It can be determined by considering the relation between the electromagnetic torque and the size of the machine. It has been shown in the preceding sections that the available torque of the machine depends on its volume, resulting in proportion[3] $T_{em} \sim V \sim D^2L$, where D and L are diameter and axial length of the machine. With $T_{em} = P_\delta/\Omega_e = pP_\delta/(2\pi f_e)$, one obtains $V \sim pP_\delta$, namely, the size of mains-supplied multipole induction machine increases with the number of poles. Given the air-gap power P_δ, dimensions of the machine are proportional to $V \sim D^2L \sim pP_\delta$. In case where the power of the induction machine is predefined, its size is proportional to p. As an example, one can compare masses of standard induction motors with rated power of 1.1 kW, designed for the line voltage of $U_S = 400$ V and for the rated frequency of $f = 50$ Hz. While two-pole motor has a mass of $m \approx 8$ kg, four-pole motor has $m \approx 13$ kg, six-pole motor has $m \approx 16$ kg, and eight-pole motor has $m \approx 23$ kg. Practical values of motor masses are different from prediction $m \approx k \cdot p$ owing to the effects that were neglected in the preceding analysis.

[3] Electromagnetic torque depends on the fourth power of linear dimensions. Hence, $T_{em} \sim V^{4/3}$.

17.5.2 Multipole Machines Fed from Static Power Converters

In applications of induction machines fed from static power converters, it is possible to adjust the supply frequency and the synchronous speed according to needs. Static power converters are mostly transistorized inverters which use the pulse width modulation (PWM) to provide symmetric, three-phase system of voltages of variable frequency and variable amplitude. With the possibility of changing the stator supply frequency, the same synchronous speed $\Omega_e = \omega_e/p$ can be obtained with machines having different number of poles. Hence, there is a choice to select the number of poles in order to make a better use of the copper and iron. Two-pole machines have lower specific torque and lower specific power than equivalent four-pole and six-pole machines. In sample design where the induction machine is expected to run with the rotor speed Ω_m, it is necessary to supply the stator voltages which create magnetic field which revolves at the speed of $\Omega_e \approx \Omega_m$. One way to accomplish that is by selecting a two-pole machine and setting the supply frequency to $\omega_e = \Omega_e$ or by selecting a multipole machine ($p > 1$) and setting the supply frequency to $\omega_e = p\Omega_e$. In both cases, the machine has the same synchronous speed, develops the same electromagnetic torque, and gives the same power. With multipole machine (p =2, 3, or 4), specific power is higher due to improved usage of iron and copper. Therefore, multipole induction machine is smaller and lighter. It should be noted that these advantages of multipole machines are lost in the case when the number of poles is extremely high. In such cases, stator frequencies $\omega_e = p\Omega_e$ are exceptionally high, and this leads to a significant increase in iron losses. In designing magnetic circuits of induction machines operating with high frequencies, magnetic circuit cannot be made of iron sheets. Instead, ferrites or other special ferromagnetic materials have to be used.

17.5.3 Shortcomings of Multipole Machines

Angular frequency of the stator currents and voltages required for the given rotor speed depends on the number of poles, $\omega_e = p\Omega_e$. With the target speed of $\Omega_m \approx \Omega_e$, the stator frequency is approximately equal to $p\Omega_m$. Hence, with inverter-supplied machines that have variable supply frequency, the same rotor speed can be achieved with lower number of poles $2p$ and lower supply frequency or with multiple pole pairs and higher supply frequency. It is known that losses in magnetic circuit due to eddy currents are proportional to the square of the frequency, while hysteresis losses grow linearly with the frequency. Therefore, specific iron losses in multipole machines are larger than specific losses in two-pole machine. In the process of the machine design, it is necessary to envisage adequate cooling or to reduce the peak value of the magnetic induction B_m in order to keep the losses within permissible limits.

With multipole machines, it is more difficult to achieve quasisinusoidal distribution of stator conductors. Stator winding is formed by placing conductors of the same reference direction ⊗ under one magnetic pole and conductors of the opposite reference direction ⊙ under the opposite magnetic pole. In two-pole machines, the width of magnetic poles is close to $\Delta\theta \approx \pi$. In multipole machines, the width of magnetic poles is $\Delta\theta \approx \pi/p$. Therefore, conductors of the same reference direction are placed within the angular interval $\Delta\theta \approx \pi/p$, p times narrower than in the case of a two-pole machine. Conductors of the stator winding are placed in the stator slots. For a machine having N_Z slots, there are a finite number of discrete locations where the conductors could be placed. Therefore, the conductors cannot have an ideal sinusoidal distribution. Instead, they are distributed into a finite number of slots in a way that creates an approximate, quasisinusoidal distribution. For multipole machines, the range of $\Delta\theta \approx\pi/p$ comprises p times less slots than the range $\Delta\theta \approx\pi$ of two-pole machines. Hence, with $p > 1$, it is even more difficult to accomplish distribution of conductors which is close to sinusoidal. As a consequence, induced electromotive forces in multipole machines could have an increased amount of higher harmonics, increased cogging torque, and increased losses due to higher harmonics.

Question (17.1): The torque of multipole induction machines is determined by $T_{em} = P_\delta/\Omega_e = pP_\delta/\omega_e$. Considered is an induction machine of axial length L and diameter D. The stator windings can be made so as to create magnetic field with an arbitrary number of pole pairs p. Is it possible to increase the available torque by increasing the number of poles?

Answer (17.1): For a given value of maximum induction B_{max} and given value of permissible current density in conductors, the torque available from an electrical machine is proportional to l^4, the fourth power of linear dimensions, or $V^{4/3}$, where V is the machine volume. Therefore, for the machine of given dimensions, the way of making the stator winding cannot have major impact on the available torque. In other words, the available torque does not depend on the number of magnetic poles. This conclusion can also be derived by representing the torque generation process as the interaction of electrical current in rotor bars with magnetic field created by the stator. Due to a limited current density, electrical current in rotor bars cannot be increased, and their limit is unaffected by the number of poles. Magnetic induction B_m is determined by the characteristics of iron sheets. Electromagnetic torque depends on electrical currents in rotor bars, magnetic induction, the rotor length, and radius. By neglecting the secondary effects, it can be concluded that the torque of an induction machine of given dimensions does not depend on the number of poles, namely, it does not depend on the method of making the stator windings.

Question (17.2): A four-pole induction machine designed for mains supply of 3×400 V, 50 Hz, has the rated power P_n. The stator winding is removed from the stator slots. New stator winding is made, designed for the same power supply conditions, producing two magnetic poles. Make an estimate of the rated power of the new machine.

Answer (17.2): The power is product of the rotor speed and the electromagnetic torque. Rewinding a four-pole machine with a two-pole winding doubles the synchronous speed and, hence, the rotor speed. The torque depends on magnetic induction B and the sum of electrical currents in individual slots. Assuming that the flux density B in the air gap does not change much and that the current density remains the same, the two-pole machine will generate the torque comparable to the torque delivered by the original, four-pole machine. Hence, the two-pole machine has the potential of delivering $2P_n$.

Question (17.3): It has been shown that stator winding of an induction machine can be wound so that it creates a rotating magnetic field with four or more magnetic poles. Does the number of poles influence the rotor construction?

Answer (17.3): Rotor of a squirrel cage induction machine consists of a relatively large number of bars which are short-circuited by conducting rings at both rotor ends, at the front and the rear. The electromotive forces and currents in short-circuited rotor contours depend on the speed of the rotor relative to the field, that is, on the slip speed $\Omega_{slip} = \omega_{slip}/p$ and on magnetic induction in the air gap. Under the north magnetic pole of the rotating field, induced rotor currents have one direction, while under the south magnetic pole of the rotating field, they are of the opposite direction. Therefore, the number of poles of the consequential rotor field is determined by the number of poles of the stator field. In other words, the same rotor can be used within a two-pole machine as well as in a multipole machine.

17.6 Two-Speed Stator Winding

Change in the synchronous speed of an induction machines and change in the rotor speed can be accomplished by changing the number of poles. In order to accomplish that, it is necessary to have the possibility of changing the stator winding so as to change the number of magnetic poles of the stator magnetomotive force. In Fig. 17.7, stator winding of an induction motor is shown with each of the three-phase windings made of two sections. Sections a_1 and a_2 of phase winding a are made to create magnetomotive forces of the same course but of opposite directions. With connection shown on the right-hand side of the figure, the stator winding creates a four-pole magnetic field. In the left side of the figure, direction of phase currents in sections a_1, b_1, and c_1 is maintained, while direction of phase currents in sections a_2, b_2, and c_2 is changed. In each phase, both sections create magnetomotive forces in the same direction, and they create two-pole magnetic field.

The motion of the stator magnetomotive force vector during one third of the supply voltage period $T = 1/f_e$ is shown in Fig. 17.8 for two-pole and four-pole configurations.

Change of the rotor speed by means of changing the number of poles requires commutation of internal connections between the stator sections. This change is to

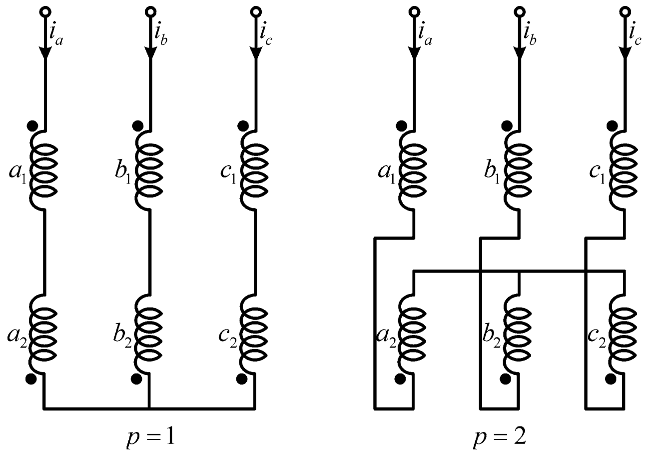

Fig. 17.7 A two-speed stator winding. By changing connections of the halves of the phase windings, two-pole (*left*) or four-pole (*right*) structures are realized

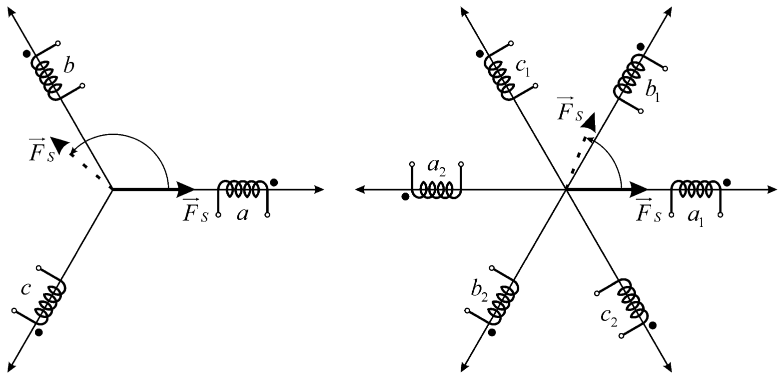

Fig. 17.8 Rotation of magnetomotive force vector in 2-pole and 4-pole configuration

be made in the course of the machine operation. In order to perform these changes, it is necessary to operate dedicated switches that make or break connections between individual sections and to achieve the winding configuration that results in desired number of poles and desired synchronous speed. The speed is usually changed automatically, in the absence of operator. Therefore, the state of the switches is controlled from a digital controller which issues command voltages. The need for a number of controlled switches makes the application relatively complex. In addition to that, additional shortcoming of the speed variation by changing the number of poles is discontinuous nature of this kind of control. Namely, the rotor speed cannot exhibit continuous change. Instead, one can select two or three discrete values of synchronous speed, and this is accomplished by connecting the stator windings in two or three configurations.

The appearance of transistorized inverters that operated on pulse width modulation principles opens the possibility for continuous change of the supply frequency, thus eliminating the need for changing the number of poles.

Question (17.4): A two-pole induction machine designed for mains supply develops the breakdown torque at the speed of $n_b = 2{,}000$ rpm. The stator winding is removed and a four-pole winding is built instead. Determine the speed where the four-pole machine develops the breakdown torque.

Answer (17.4): Electromagnetic torque is equal to the ratio of the air-gap power and synchronous speed. The air-gap power is the highest at the relative slip $s_b = R_R/(L_{\gamma e}\omega_e)$, that is, at the rotor frequency of $\omega_b = R_R/L_{\gamma e}$. Therefore, the breakdown torque is developed at the rotor frequency that does not depend on the number of poles. Relative value of the breakdown slip is equal to $s_b = (n_e - n_b)/n_e = 1/3$. For the two-pole machine, $n_b = 2{,}000$ rpm, while for the four-pole machine, the speed that results in the breakdown torque is $n_b = n_e(1 - s_b) = 1{,}500(1 - s_b) = 1{,}000$ rpm.

17.7 Notation

Preceding considerations use the lower case letter ω for denoting angular frequency of electrical currents and voltages. Mechanical speeds of the rotor, speed of revolving magnetic field and magnetomotive force, and other mechanical quantities are denoted by the upper case letter Ω. A survey of notation using the sample multipole machine is presented below. In the case of two-pole machines, where $p = 1$, all electrical quantities ω are equal to mechanical quantities Ω. Unless otherwise stated, all the quantities are expressed in rad/s:

- Ω_e – synchronous speed, angular speed of rotation of the magnetic field
- $\omega_e = p\Omega_e$ – angular frequency of power supply, frequency of the stator currents and voltages
- Ω_m – the rotor speed
- $\omega_m = p\Omega_m$ – electrical representation of the rotor speed
- $n = 9.54\Omega_m$ – the rotor speed expressed in rpm (revolutions per minute)
- $\omega_{slip} = \omega_e - p\Omega_m$ – angular frequency of the rotor currents
- $\Omega_{slip} = \Omega_e - \Omega_m$ – the slip speed of the rotor, the speed of lagging behind the synchronous speed

For a four-pole induction machine ($p = 2$) with rated supply frequency $f_e = 50$ Hz and rated speed of $n_n = 1{,}350$ rpm, characteristic angular frequencies and speeds in rated operating conditions are the following:

- Angular frequency of stator voltages – $\omega_e = 100\pi$
- Synchronous speed – $\Omega_e = 50\pi$, $n_e = 1{,}500$ rpm
- Angular frequency of rotor currents – $\omega_{slip} = 10\pi$
- Slip speed – $\Omega_{slip} = 5\pi$, $n_{slip} = 150$ rpm
- Rotor mechanical speed – $\Omega_m = 45\pi$, $n_n = 1{,}350$ rpm

17.8 Supplying from a Source of Variable Frequency

Preceding sections discussed traditional approaches for changing the rotor speed of induction machines, specifically:

- Variation of the stator voltage
- Variation of the rotor resistance
- Variation of the number of poles

Their drawbacks are:

- Difficult implementation
- High energy losses
- No possibility to achieve continuous speed change over a wide range

Continuous speed variation over a wide range relies on power converters which make use of transistor switches. They operate on the basis of pulse width modulation and provide the possibility for continuous variation of the supply frequency, which results in continuous variation of the synchronous speed and, hence, the rotor speed. In this way, variable speed is obtained without the need to operate the induction machine with increased slip frequencies. Therefore, there is no increase in conversion losses due to the speed change. Moreover, there is no need to use wound rotor, slip rings, or any special design of induction machine. Subsequent sections provide a brief introduction to variable frequency supply of induction machines.

17.9 Variable Frequency Supply

It is of interest to investigate the nature of the stator voltages in variable speed induction machines. In phase a of the stator winding, the current is $i_a(t)$, and the flux is $\Psi_a(t)$. Assuming that the leakage flux is small, the flux of the phase winding a comes as a consequence of the rotating magnetic field. At the instant when the vector of the rotating field is aligned with the axis of the phase a, the flux $\Psi_a(t)$ reaches its maximum value Ψ_m. With revolving field, the flux in phase a varies according to sinusoidal law. Phase voltage is equal to $u_a(t) = R_S\, i_a(t) + \mathrm{d}\Psi_a(t)/\mathrm{d}t$. The change of the phase voltage u_a is shown in Fig. 17.9. Neglecting the voltage drop across the stator resistance, one obtains

$$u_\alpha = u_a \approx \frac{\mathrm{d}\Psi_{\alpha s}}{\mathrm{d}t} = \omega_e \Psi_m \sin(\omega_e t - \varphi). \tag{17.4}$$

Slip of induction machines is relatively small; thus, the frequency of power supply is determined from the rotor speed, $\omega_e = p\Omega_e = p(\Omega_m + \Omega_{slip}) \approx p\Omega_m$. Therefore, variation of the rotor speed requires the power supply for the stator

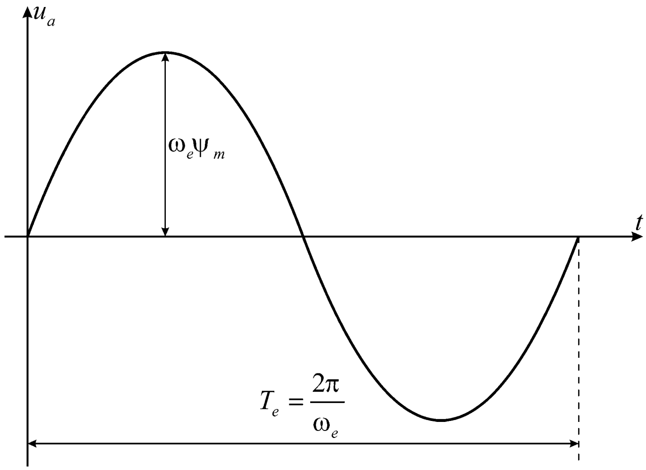

Fig. 17.9 Desired shape of the phase voltage

winding which provides a three-phase system of voltages with variable frequency and variable amplitude ($\Psi_m \omega_e$). Continuous change in the rotor speed requires continuous change of the voltage amplitude and frequency. Therefore, PWM controlled three-phase inverters are required to provide continuous change of both the voltage amplitude $\Psi_m \omega_e$ and the period $T_e = 2\pi/\omega_e$.

17.10 Power Converter Topology

Simplified schematic diagram of the power converter intended for supplying induction machines is given in Fig. 17.10a, along with the shape of the line voltage obtained at the output terminals (Fig. 17.10b). The change of the voltage across the output terminals comprises a train of voltage pulses. Averaged value of this pulse-shaped waveform has sinusoidal change with adjustable amplitude and frequency. Such waveforms are fed to the stator terminals of induction machines and used for supplying three-phase machines by voltages of variable frequency and amplitude. Converter topology includes a three-phase diode rectifier with six diodes, shown in the left side of the figure. It converts AC voltages and currents, provided from the three-phase AC mains, into DC voltages and currents across the parts L_{DC} and C_{DC}. These parts are placed in the middle of the converter, between the rectifier and the inverter, and they are called *intermediate DC circuit*, *DC link*, or *DC bus*. DC voltage E across the capacitor C_{DC} is fed to the three-phase inverter, the switching structure which makes the use of six power transistors. Each transistor is used in *switching mode*, namely, it is either *opened* (off, $i_{CE} \approx 0$) or *closed* (on, $u_{CE} \approx 0$). Transistor power switches are organized in three groups, called *inverter phases* or *inverter arms*. Each arm has two transistor switches, connected in series and

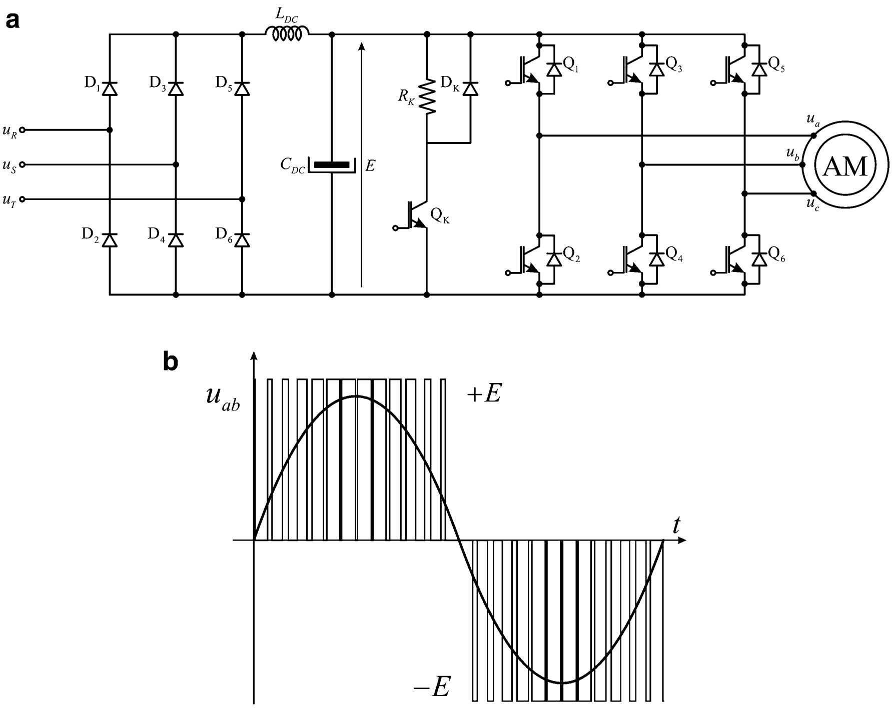

Fig. 17.10 (**a**) Three-phase PWM inverter with power transistors. (**b**) Typical waveform of line-to-line voltages

attached between the plus rail and the minus rail of the DC bus. At each instant, only one switch in each arm is turned on. Turning on both switches would result in a short circuit across the DC bus circuit. Turning the upper switch on brings the output phase to the potential of positive DC bus rail. Turning the lower switch on brings the output phase to the potential of negative DC bus rail.

17.11 Pulse Width Modulation

By taking the negative rail of the DC link circuit for the reference potential, turning on of the upper switch results in phase voltage $u_a = +E$, while turning on the lower switch results in $u_a = 0$. The same applies for the phase voltages u_b and u_c. Hence, the phase voltages take discrete values $u \in \{0, +E\}$. At the same time, line-to-line voltages such as $u_{ab} = u_a - u_b$ may take the values $u \in \{-E, 0, +E\}$. Hence, the instantaneous value of the line voltage cannot be changed in continuous manner. Instead, it takes one of the three discrete values. However, a fast exchange of the switching states results in a train of pulses of variable width. The width of the voltage pulses affects the average value of the voltage waveform. A continuous change of the pulse width results in a continuous change of the average value of the voltage. With a fast sequencing of the available discrete values $\{-E, 0, +E\}$, the line voltage becomes a train of pulses. The pulses are of variable width, and they can have either positive or negative value. Variation of the width of these pulses results in variation of the average line voltage. With sinusoidal change of the pulse width, behavior of the electrical machine supplied from the three-phase PWM inverter is very much the same as behavior of the same machine fed from an ideal voltage source with smooth, sinusoidal change of instantaneous voltages brought across the stator terminals.

17.12 Average Value of the Output Voltage

Power transistors in three-phase inverters commutate a number of times within each period of the supply voltage. The frequency of commutation f_{PWM} of semiconductor power switches in a three-phase transistor inverter is usually close to 10 kHz or higher. Phase voltage $u_a(t)$ is a train of pulses that repeat each $1/f_{PWM}$. During each period $T = 1/f_{PWM} = 100$ μs, the switching state where the upper switch is turned on is maintained over time interval t_{ON}, where $0 < t_{ON} < T$, while the switching state where the lower switch is turned on is maintained during the rest of the period. The shape of the phase voltage is shown in Fig. 17.11. The average voltage within each commutation period T is proportional to the pulse width t_{ON}. When the potential of the negative rail of the DC link is taken as the reference potential, the phase voltage over the interval $0 < t < t_{ON}$ is equal to $+E$, while the voltage during the remaining part of the period T is $u_a = 0$. Continuous variation of t_{ON} over the range $0 < t_{ON} < T$ results in average value $u_a^{av} = E(t_{ON}/T)$ change from 0 to $+E$.

$$U_{av} = \frac{1}{T} \cdot \int_{NT}^{(N+1)T} u_a \mathrm{d}t = \frac{t_{ON}}{T} E. \tag{17.5}$$

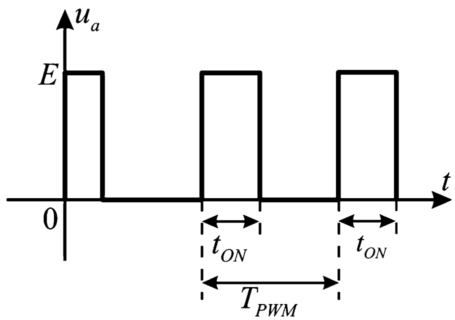

Fig. 17.11 Pulse width modulation: upper switch is on during interval t_{ON}

17.13 Sinusoidal Output Voltages

The width of the voltage pulses that constitute the phase voltage can be varied or *modulated*. Variation of the pulse width is called *pulse width modulation* (PWM). Starting from the expression $u_a^{av} = E(t_{ON}/T)$, desired average voltage u_a^{av} can be used to calculate the time $t_{ON,}$ which determines the average value of the phase voltage within one switching period $T = 1/f_{\mathrm{PWM}}$.

In order to achieve variation of the average voltage value $u_a^{av}(t)$, that is, to obtain the phase voltage that changes the average value in successive switching periods T, the pulse width t_{ON} should change as $t_{ON}(t) = T\, u_a^{av}(t)/E$. As a matter of fact, it is not correct to write $t_{ON}(t)$, as the pulse width assumes one discrete value in each switching period T. Namely, the pulse width over the period $[nT..(n + 1)T]$ is determined by discrete value $t_{ON}(n) = T\ u_a^{av}(n)/E$. If $f_e = 50$ Hz is the desired frequency of the phase voltage, while φ is the initial phase, and the number $0 < A < 1$ determines the desired amplitude, the pulse width should be varied according to

$$t_{ON}(n) = \frac{T}{2} + \frac{T}{2} A \sin(2\pi f_e \cdot nT - \varphi). \tag{17.6}$$

With this change of the pulse width, the phase voltage u_a is obtained with average value over successive intervals T that change according to expression

$$u_a^{av}(n) \approx \frac{E}{2} + \frac{E}{2} A \sin(2\pi f_e \cdot nT - \varphi). \tag{17.7}$$

The frequency component f_e of the phase voltage has the amplitude which can be varied by changing parameter A, while the frequency and phase are determined by parameters f_e and φ. Commutation frequency f_{PWM} has to be considerably higher than the desired frequency of the phase voltage, $f_e << f_{PWM}$. In the expression for $u_a^{av}(n)$, there is a DC component of the voltage which is equal to $E/2$. This is the consequence of selecting the negative rail of the DC bus for the reference potential V_0. By turning on the upper switch Q_1 in Fig. 17.10a, the phase voltage u_a is equal to E. With Q_2 turned on, $u_a = 0$. Other choices for reference potential result in different values of the phase voltage u_a.

DC component in the phase voltages is equal in all three phases. It has no impact on the operation of the induction machine. Namely, the stator winding is connected to three-phase inverter by three conductors, and the operation of the machine is determined by line-to-line voltages. Line-to-line voltage u_{ab} is equal to u_a-u_b. Subtracting the two-phase voltages removes the DC components. It is of interest to point out that the choice of the reference potential is arbitrary one. Therefore, it cannot have an impact on the operation of the electrical machine. In order to confirm this statement, one can calculate the line voltage $u_{ab}^{av}(t)$ as the difference between the phase voltages $u_a^{av}(t)$ and $u_b^{av}(t)$. The voltage $u_a^{av}(t)$ is given in (17.7) while the voltage $u_b^{av}(t)$ is equal to

$$u_b^{av}(t) \approx \frac{E}{2} + \frac{E}{2} A \sin(2\pi f_e \cdot nT - \varphi - 2\pi/3),$$

and it lags behind u_a by $2\pi/3$. By calculating $u_{ab}{}^{av}(t) = u_a{}^{av}(t) - u_b{}^{av}(t)$, DC components $E/2$ are canceled, resulting in line-to-line voltage $u_{ab}{}^{av}$ which does not have a DC component:

$$u_{ab}^{av}(t) \approx \frac{AE\sqrt{3}}{2} \sin(2\pi f_e \cdot nT - \varphi + \pi/6).$$

Digital implementation of the pulse width modulation implies that parameters A, f_e, and φ are the numbers represented by binary record in RAM memory. They can be adjusted to the needs of the actual operating regime of the induction machine.

Within each period $T_e = 1/f_e$ of phase voltages, there are a finite number of pulses. The width of these pulses is modulated (changed) in the way to obtain sinusoidal change of the average value $u^{av}(t)$.

The process of approximating sinusoidal change of the phase voltage by means of a finite train of pulse width-modulated impulses has similarities with the procedure of making distributed windings with quasisinusoidal distribution of conductors placed in a finite number of slots.

17.14 Spectrum of PWM Waveforms

Sinusoidal PWM is used to obtain a sequence of variable width pulses. Averaged pulses provide the phase voltages of the desired frequency f_e. When the pulse width has sinusoidal change, according to expression $t_{ON}(n) = (T/2)\,[1+ A\sin(2\pi f_e\, nT-\varphi)]$, the average values with each switching period $T = 1/f_{PWM}$ change according to expression $u^{av}(n) \approx (E/2) + (E/2)\, A \sin(2\pi f_e\, nT-\varphi)$.

The commutation frequency f_{PWM} has to be considerably higher than the desired frequency of the phase voltage, $f_e << f_{PWM}$, so as to obtain a smooth, gradual change of average voltage between the successive switching periods $T = 1/f_{PWM}$ and to obtain the operation similar to feeding the machine from an ideal source. The frequency f_e of the phase voltages determines the synchronous speed, and it is called *basic* or *fundamental*. It ranges from several tens to several hundreds of cycles per second. The frequency f_{PWM} is called *commutation* or *switching* frequency, and it ranges from 5 to 20 kHz.

The spectrum of a pulse width-modulated sequence of phase voltage pulses contains:

- DC component $E/2$
- Slowly varying AC component of frequency f_e, created by sinusoidal variation of the pulse width, called the *basic* or *fundamental* frequency component

- Frequency component at the commutation frequency $f_{PWM} = 1/T$, created by the train of variable width voltage pulses that keep repeating each T
- A series of frequency components with smaller amplitudes and with frequencies $m{\cdot}f_{PWM}$ that are integer multiples of the switching frequency f_{PWM}
- A series of frequency components at frequencies $m{\cdot}f_{PWM} \pm n{\cdot}f_e$, produced by interaction between the switching frequency f_{PWM} and the basic frequency f_e

DC component of the phase voltage is the same in all phases; thus, it has no influence on line voltages. The fundamental AC component at the basic frequency f_e is the desired result, the voltage required across the stator terminals. It has adjustable amplitude and adjustable frequency. Assuming that the high-frequency content of the spectrum can be neglected, the three-phase inverter with PWM control can be regarded as the source of sinusoidal voltages with adjustable amplitude and adjustable fundamental frequency f_e.

Spectral component at the switching frequency $f_{PWM} = 1/T \approx 10$ kHz is the consequence of the pulsating nature of the three-phase inverter. Component of the voltage at the switching frequency has the amplitude determined by the DC link voltage E. Due to considerable amplitude, the effects of this frequency component cannot be neglected. The effects of the pulsed supply on an induction machine should be analyzed in order to establish the impact of pulsating voltage on the machine and verify whether this way of supplying the stator winding is acceptable. Subsequent discussion proves that the stator current, the electromagnetic torque, and the rotor speed of an induction machine, supplied from PWM controlled three-phase inverter, resemble the current, torque, and speed obtained with an equivalent source of smooth, sinusoidal phase voltages. Therefore, the operation with PWM power supply can be considered equivalent to the operation with an ideal source of sinusoidal waveforms providing the voltages of instantaneous value such as $u(t) = (E/2) + (E/2){\cdot}A\sin(2\pi f_e t - \varphi)$.

17.15 Current Ripple

The voltage balance in the phase winding a is given by the equation $u_a(t) = R_S i_a(t) + \mathrm{d}\Psi_a/\mathrm{d}t$. The flux Ψ_a can be represented as sum of the mutual flux Ψ_{ma}, which passes through the air gap and encircles both stator and rotor windings, and the leakage flux of the stator winding, which is proportional to the leakage inductance L_γ. The voltage balance equation assumes the form $u_a(t) = R_S i_a(t) + L_\gamma \mathrm{d}i_a(t)/\mathrm{d}t + \mathrm{d}\Psi_{ma}/\mathrm{d}t$, where $L_\gamma i_a(t)$ is the leakage flux in the phase a of the stator winding. Rotating magnetic field changes its relative position with respect to the phase winding a; thus, the flux Ψ_{ma} exhibits sinusoidal change of the frequency determined by the synchronous speed. For this reason, the electromotive force $\mathrm{d}\Psi_{ma}/\mathrm{d}t$ is sinusoidal, and it has the frequency $\omega_e \approx \omega_m$ and the amplitude $\Psi_m \omega_e$, where Ψ_m represents the maximum value of the mutual flux. With $f_e << f_{PWM}$, the change in the considered electromotive force is slow compared to the switching phenomena.

While considering the effects of switching power supply on the machine behavior, slow variations of the electromotive force can be neglected; thus, the voltage balance equation reduces to $u_a(t) = R_S i_a(t) + L_\gamma \mathrm{d}i_a(t)/\mathrm{d}t$.

By applying Laplace transform on previous differential equation, one obtains complex image of the stator current, $I_a(s) = U_a(s)/(R_S + sL_\gamma)$. Function $W(s) = 1/(R_S + sL_\gamma)$ represents *transfer function* of the stator winding, which is the ratio of the complex image of the stator current and the complex image of the stator voltage. Transfer function $W(s)$ provides the means to calculate the stator current response to excitation by the stator voltage of known amplitude and frequency. In the case of interest, the voltage excitation has the switching frequency f_{PWM}. The function is obtained by neglecting slow-varying electromotive force; thus, it is applicable for calculating the response at frequencies as high as f_{PWM}, but it cannot be used for the analysis of the machine response to lower excitation frequencies, such as the fundamental frequency f_e.

The ratio of the current and voltage at the frequency $\omega = 2\pi/T = 2\pi f_{PWM}$ is obtained by introducing $s = \mathrm{j}\omega$ in function $W(s)$; thus, the function assumes the form $W(\mathrm{j}\omega) = 1/(R_S + \mathrm{j}\omega L_\gamma)$. At switching frequencies of the order of several kHz, it is justified to introduce the assumption $R_S << \omega L_\gamma$ and to obtain relation $|I(\mathrm{j}\omega)/U(\mathrm{j}\omega)| \approx 1/(L_\gamma \omega)$. This expression shows that electrical machines behave as low-pass filters. When exposed to high-frequency voltages, the stator current response to such excitation is smaller as the excitation frequency increases. In other words, low-frequency excitation has much larger impact on the stator currents than high-frequency excitation. At frequencies close to $f_{PWM} \approx 10$ kHz, reactance $L_\gamma \omega$ is so large that the voltage pulses have a very small influence on the stator currents. Typical change of the phase current of an induction machine supplied from three-phase PWM inverter is shown in Fig. 17.12.

With pulsed supply, the phase voltages exceed the desired sinusoidal waveform during t_{ON} and then fall below during the reminder of the PWM switching period. Therefore, the stator currents oscillate around their mean values. These oscillations have the frequency of the switching bridge $f_{PWM} = 1/T = \omega/(2\pi)$. When the switching frequency is sufficiently high, the amplitude of these oscillations is rather small, and their effect on the operation of the induction machine can be neglected. An estimate of the amplitude of oscillations of the stator current can be obtained by using expression $|I(\mathrm{j}\omega)/U(\mathrm{j}\omega)| \approx 1/(L_\gamma \omega)$. This expression is applicable for excitation by sinusoidal voltages of the frequency $\omega = 2\pi\ f_{PWM}$. The three-phase switching inverter does not output sinusoidal waveforms at the switching frequency. Instead, it provides rectangular voltage pulses of the period $1/f_{PWM}$. Nonetheless, the formula can be used to obtain a rough estimate of the stator current oscillations, also called *ripple*. In most cases, the current ripple amounts from 1% to 5% of the rated current.

Question (17.5): Induction machine of rated frequency $f_{en} = 50$ Hz has an equivalent leakage reactance $x_{\gamma e} = 20\%$. The machine is supplied from a three-phase transistor inverter of the switching frequency $f_{PWM} = 10$ kHz. Provide an estimate of the stator current ripple that appears due to pulsed power supply.

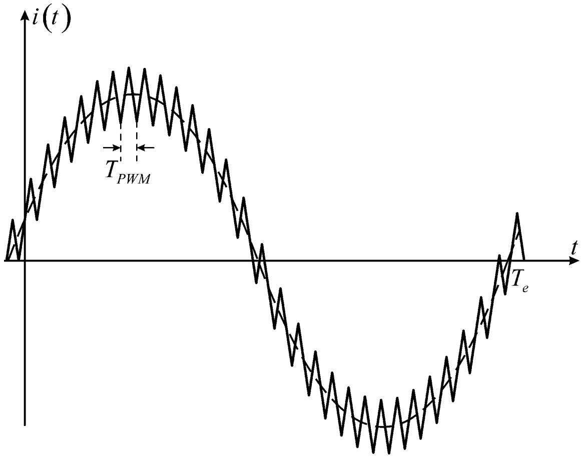

Fig. 17.12 Stator current with current ripple

Answer (17.5): The amplitude of the pulses generated by the switching source and supplied to the stator winding terminals is equal to the voltage E of the DC link circuit. Voltage E must be sufficient to provide the rated AC voltage at the inverter output. Hence, for the purpose of making an estimate, the DC link voltage can be assumed to have the relative value of $E \approx 1$. The leakage reactance $X_{\gamma e}$ at the rated frequency f_{en} has relative value of $x_{\gamma e} = 0.2$. The switching frequency is 200 times higher than the rated frequency. Reactance is proportional to the frequency. At the switching frequency, the relative value of the reactance is $x_{\gamma e(\mathrm{PWM})} = x_{\gamma e}(f_{PWM}/f_e) = 40$, that is, 4,000%. The ripple current comes as the quotient of the voltage, having the relative value of 1, and the leakage reactance at the switching frequency. Therefore, the relative value of the current ripple due to pulsed supply is estimated as $\Delta I \approx 1/x_{\gamma e(\mathrm{PWM})} = 2.5\%$.

Question (17.6): Induction motor is supplied from a three-phase transistor inverter with DC link voltage E and with the switching frequency of $f_{PWM} = 1/T$. The speed of rotation is equal to zero; thus, the electromotive force induced in the stator winding can be neglected. Resistance of the stator winding is also negligible. The leakage inductance $L_{\gamma e}$ of the motor is known. It can be assumed that potential of the star point node is in between the positive and negative DC bus rails and it is chosen for the reference potential. Determine the shape and amplitude of oscillations of the stator current in the case when $t_{ON} = T/2$.

Answer (17.6): With reference potential point in between the DC bus rails, the output phase voltage can be either $+E/2$ or $-E/2$. Having neglected the electromotive force and the voltage drop across the stator resistance, the voltage balance equation reduces to $u_a = L_{\gamma e}\, \mathrm{d}i_a/\mathrm{d}t$. During the first half of the period T, that is, during the interval t_{ON}, the upper switch of the inverter phase a is turned on, thus $L_{\gamma e}\, \mathrm{d}i_a/\mathrm{d}t = E/2$. Therefore, the change of current is linear. Similar conclusion applies for the second half of the period, when the voltage is $u_a = -E/2$. The current oscillates around its average value I_{av} with an amplitude of ΔI. During first half of the period, it increases from $I_{av} - \Delta I$ to $I_{av} + \Delta I$. In the second half period, it slides back to $I_{av}-\Delta I$. Hence, it changes by $2\Delta I$ within each half period. The change of the current is linear. Therefore, the slope $\mathrm{d}i_a/\mathrm{d}t$ is equal to $2\Delta I/(T/2) = (E/2)/L_{\gamma e}$. From this expression, $\Delta I = ET/(8L_{\gamma e})$.

17.16 Frequency Control

By varying the frequency of the stator winding power supply, one varies the synchronous speed Ω_e of induction machine. When operating with $T_{em} = 0$ and $s = 0$, the rotor revolves at the synchronous speed, thus $\Omega_m = \Omega_e$. Therefore, continuous change of the supply frequency contributes to continuous change of the no load speed. Considering the mechanical characteristic $T_{em}(\Omega_m)$, the stator supply frequency determines the intersection with the abscissa. For that reason, the frequency changes would affect as well the rotor speed of loaded induction machines. A family of mechanical characteristics obtained by varying the stator frequency is shown in Fig. 17.13.

In addition to the no load speed, mechanical characteristic of an induction machine is also characterized by the breakdown torque, breakdown slip, and stiffness $S = |\Delta T_{em}/\Delta \Omega_m|$. Breakdown slip $\Omega_b = p\omega_b = pR_R/L_{\gamma e}$ is determined by the machine parameters, and it does not depend on the power supply frequency. The breakdown torque $T_b = (3p/4)\Psi_S{}^2/L_{\gamma e}$ and the stiffness of the mechanical characteristic $S = k\Psi_S{}^2/R_R$ both depend on the square of the stator flux $\Psi_S{}^2$. The amplitude of the stator flux depends on the ratio of the power supply voltage and frequency. While operating with the rotor speeds lower than the rated speed Ω_n, it is desirable to maintain the rated flux, that is, the maximum flux that can be achieved within the machine. At high speeds, it is necessary to perform the field weakening and to operate with the flux inversely proportional to the speed. With $\Omega_m > \Omega_n$, the flux has to be reduced in order to maintain the stator voltage U_S within the limits of the rated voltage U_n.

To achieve flux control, it is necessary to perform simultaneous change of the supply voltage and the supply frequency. Relation between the voltage, frequency, and flux is derived from the equivalent circuit. The voltage balance equation of the stator winding of an induction machine which operates at steady state is

$$\underline{U}_s = R_s \underline{I}_s + j\omega_e (L_{\gamma s} \underline{I}_s + \underline{\Psi}_m). \tag{17.8}$$

By neglecting the voltage drop across the stator resistance, voltage balance equation of the stator winding at steady state becomes $\underline{U}_S \approx \mathrm{j}\omega_e \Psi_S = \mathrm{j}\omega_e L_{\gamma S} + \mathrm{j}\omega_e \Psi_m = \mathrm{j}\omega_e L_{\gamma S}\, \underline{i}_S + \mathrm{j}\omega_e L_m \underline{I}_m$. The flux amplitude is determined by the ratio of the maximum voltage and the supply frequency, $\Psi_S \approx U_S/\omega_e$. When the stator voltages are obtained from a three-phase transistor inverter, the frequency ω_e of the basic (fundamental) component determines the synchronous speed, while the quotient of the voltage amplitude and the angular frequency ω_e determines the amplitude of the

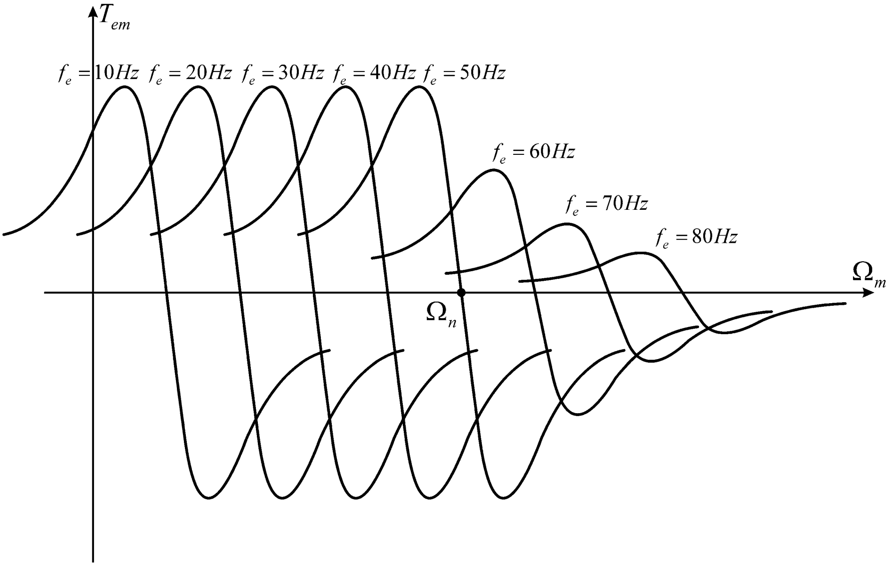

Fig. 17.13 Family of mechanical characteristics obtained with variable frequency supply

stator flux. In DC machines, the excitation flux depends on electrical currents in a separate excitation winding, while the electrical power subject to electromechanical conversion is supplied through the armature winding. Hence, DC machines have two electrical ports, and these are the excitation winding terminals and the armature winding terminals. Induction machines are supplied from the stator side only. Hence, both the machine excitation and the electrical power subject to electromechanical conversion pass from the three-phase inverter into the stator winding terminals. The flux, torque, and power of induction machines all depend on the voltages supplied to the stator terminals.

It is of interest to recall criteria for selecting the flux amplitude in various operating conditions. The torque developed in any electrical machine can be calculated as vector product of the flux and the current. Given the target torque T_{em}, electrical current required for the torque generation is proportional to the ratio T_{em}/Ψ_S, that is, it is inversely proportional to the flux. Lower currents are preferred as they lead to lower losses. Thus, it is beneficial to use higher values of the flux, whenever possible. With three-phase inverter supply, the flux is determined by the ratio of the voltage and the frequency, $\Psi_S \approx U_S/\omega_e$. The flux can be increased up to the value $\Psi_{max} \approx \Psi_n$ which marks the knee of the magnetizing characteristic shown in Fig. 17.14.

The flux values in excess to Ψ_{max} result in saturation of the magnetic circuit. The difference between the air-gap flux (i.e., mutual flux) and the stator flux is considered negligible for the discussion in course. Therefore, with $\Psi_m \approx \Psi_S$, the flux is determined by the magnetizing current i_m which circulates in the magnetizing branch of the equivalent circuit. This current is on the abscissa of the magnetizing characteristic

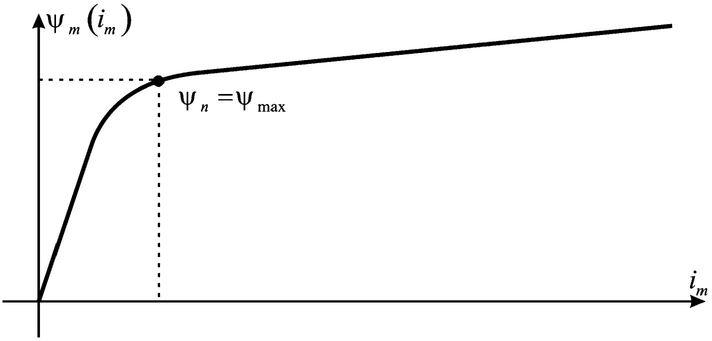

Fig. 17.14 Magnetizing curve

$\Psi_m(i_m)$. When going above the knee point and entering the zone of magnetic saturation, which extends in the upper right of the $\Psi_m(i_m)$ curve, any further increase of the flux is very small, and it requires considerable magnetizing current i_m. In all practical uses, the flux increase above the knee point is of little significance, as it requires significant increase in electrical current. Hence, with $\Psi_m > \Psi_n$, a marginal flux increment would require very high currents, accompanied by consequential copper losses. For this reason, the flux is maintained at the rated value Ψ_n by keeping the ratio U_S/ω_e equal to Ψ_n, unless other reasons and specific circumstances require the operation with reduced flux:

$$\underline{\Psi}_m \approx \underline{\Psi}_s \approx \frac{\underline{U}_s}{j\omega_e}. \tag{17.9}$$

17.17 Field Weakening

Operation with $U_S/\omega_e = \Psi_n$ is not always possible. At higher rotor speeds, it is necessary to reduce the flux. In operation with $\Omega_m > \Omega_n$, it is necessary to increase the stator frequency above the rated value. In order to keep the flux at its rated value, it is necessary to have the stator voltage of $U_S = \omega_e \Psi_n$. With $U_S/\omega_e = \Psi_n$ and $\omega_e > \omega_n$, it is necessary to increase the stator voltage above the rated level.

At steady state, the stator voltage must not exceed the rated value. Otherwise, electrical insulation of windings will be damaged. For this reason, three-phase inverters designed for the stator winding power supply are made to produce voltages within the range $0 < U_S < U_n$. It would not make sense to make the power supply capable of providing the voltages that can cause damage to the machine. Therefore, three-phase inverters such as the one shown in Fig. 17.10 cannot produce the output voltage which exceeds the rated voltage of the motor. Considering the operation above the rated speed, where $U_S \leq U_n$ and $\omega_e > \omega_n$, the flux cannot be maintained at the rated level, and it has to be reduced. With the stator voltage equal to the rated value and, hence, constant, and with the operating frequency of the stator power supply in progressive rise, the flux of the machine is inversely proportional to the rotor speed. The expression describing the flux change at high speeds can be derived from the following discussion.

With rated flux, the electromotive force in the machine is equal to $\omega_e \Psi_n$. At the rated speed $\Omega_m \approx \Omega_n$, the electromotive force reaches the rated voltage $U_n \approx \omega_n \Psi_n$. This discussion neglects the slip and overlooks the difference between the air-gap flux and the stator flux. With limited voltage, the rated flux cannot be maintained in operation above the rated speed. The highest sustainable voltage that is applicable to the stator terminals is the stator rated voltage U_n. At speeds $\Omega_m \approx \Omega_e > \Omega_n$, the electromotive force must not exceed the rated voltage. For that to achieve, the flux must not exceed $\Psi_S(\omega_e) = \Psi_n(\omega_n/\omega_e)$. In this case, the electromotive force would be equal to the rated voltage. At higher speeds, the flux is inversely proportional to the rotor speed and, hence, inversely proportional to the supply frequency, $\Psi \sim 1/\omega$.

For an induction machine supplied from the three-phase inverter, the stator voltage and frequency have to be changed in order to obtain desired rotor speed. Calculation of the stator voltage amplitude and frequency in terms of the rotor speed is described below.

In operation below the rated speed, the stator frequency is $\omega_e < \omega_n$, the stator voltage $U_S \approx \omega_e \Psi_n$ is proportional to frequency and lower than the rated voltage, while the flux in the machine is constant and it has rated value:

$$\Psi_m = \frac{U_s}{\omega_e} = \frac{U_n}{\omega_n} = const. \quad \Rightarrow \quad \frac{U}{f} = const. \tag{17.10}$$

During operation at the speeds above the rated speed, stator voltage is maintained at rated value, which is the highest voltage available from the three-phase inverter. The stator frequency increases with the speed, and the flux decreases according to the law $\Psi \sim 1/\omega$. The machine operates in the field weakening regime:

$$\Psi(\omega)\big|_{|\omega|>\omega_n} = \frac{\omega_n}{\omega} \cdot \Psi_n. \tag{17.11}$$

Finally, variation of the flux is determined by

$$\Psi(\omega) = \begin{cases} \omega \le \omega_n & \Rightarrow \quad \Psi_n \\ \omega > \omega_n & \Rightarrow \frac{\omega_n}{\omega} \cdot \Psi_n \end{cases}. \tag{17.12}$$

With that in mind, it is possible to envisage the family of mechanical characteristics obtained by frequency variation. In the following diagrams (Fig. 17.15) and expressions, legibility is helped by assuming that $\Omega_m \approx \Omega_e$, as well as $\omega_e \approx \omega_m = p\Omega_m$. Hence, the subscript may be omitted due to assumption $\omega \approx \omega_e \approx \omega_m$.

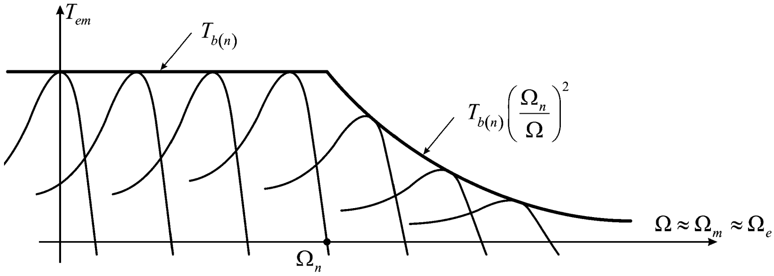

Fig. 17.15 The envelope of mechanical characteristics obtained with variable frequency

At speeds below the rated speed, the ratio U_S/ω_e does not change, and the flux is constant. Consequently, all mechanical characteristics obtained for the supply frequencies $\omega_e < \omega_n$ have the same value of the breakdown torque and the same slope. By changing the supply frequency over the range of $0 < \omega_e < \omega_n$, a family of mechanical characteristics is obtained having the same slope and the same breakdown torque. The characteristics can be drawn by starting with natural characteristic, obtained with the rated frequency, and performing translation toward the origin of the $T(\Omega)$ diagram. At speeds above the rated speed, induction machine operates in field weakening regime. The stator voltage amplitude is maintained at the rated value, while the flux decreases according to the law $\Psi_S(\omega) = \Psi_n(\omega_n/\omega_e)$. No load speed of the resulting mechanical characteristics is determined by the stator supply frequency, $\Omega_e = \omega_e/p$. Since the breakdown torque T_b and the slope S are proportional to the square of the flux, they decrease proportionally to the square of the speed, $T_b \sim 1/\omega^2$. The breakdown torque obtained at operation with the rated flux is denoted by $T_{b(n)}$. In field weakening region, the breakdown torque is $T_b(\omega_e) = T_{b(n)}(\omega_n/\omega_e)^2$. Therefore, the envelope of mechanical characteristics obtained with variable frequency supply in field weakening regime decreases with the square of the speed and frequency, $T_b \sim 1/\omega^2$.

17.17.1 Reversal of Frequency-Controlled Induction Machines

The rotor speed Ω_m of induction machines is close to the synchronous speed Ω_e. In order to change direction of the rotor speed, it is necessary to change direction of the revolving field and invert the synchronous speed. In mains-supplied machines, changing the phase sequence results in $\Omega_e = -\omega_e/p$. Inverter-supplied machines can be reversed without rewiring the phases.

The power supply frequency ω_e may take a negative value. The number ω_e resides in RAM memory of digital controller, and it is used to calculate t_{ON} intervals according to expressions similar to (17.6). Entering a negative value for ω_e leads to generation of three-phase system of stator voltages which create magnetic field that revolves in the opposite direction. In such cases, the synchronous speed and the rotor speed change direction.

Speed reversal by means of negative supply frequency does not require any change of the wiring. It is not necessary to exchange the two-phase conductors for the induction machine to change direction of rotation. It is sufficient to insert a negative value of ω_e in the expression that calculates the pulse width $t_{ON}(n) = (T/2)[1+ A\sin(\omega_e nT - \varphi)]$ of the voltage pulses created by the switching action of the three-phase inverter. During the operation with $\omega_e < 0$, mechanical characteristic is transferred to the second and third quadrant of the T_{em}-Ω_m plane.

17.18 Steady State and Transient Operating Area

By varying the frequency and amplitude of the stator voltages, it is possible to achieve operation in all four quadrants of the T_{em}-Ω_m plane. It is of interest to establish the steady state operating area, that is, the region of T_{em}-Ω_m plane that encircles all of the operating points where the machine can operate permanently and with no damage. In a like manner, transient operating area contains the operating points that can be reached within short time intervals. Steady state operating limits of induction machine operating in the first quadrant are given in Fig. 17.16 and explained henceforth.

Continuous operation of an induction machine at certain operating mode requires that the voltages and currents are within the rated limits. At the same time, the sum of the losses should be within the rated limits in order to avoid overheating and damage to the machine.

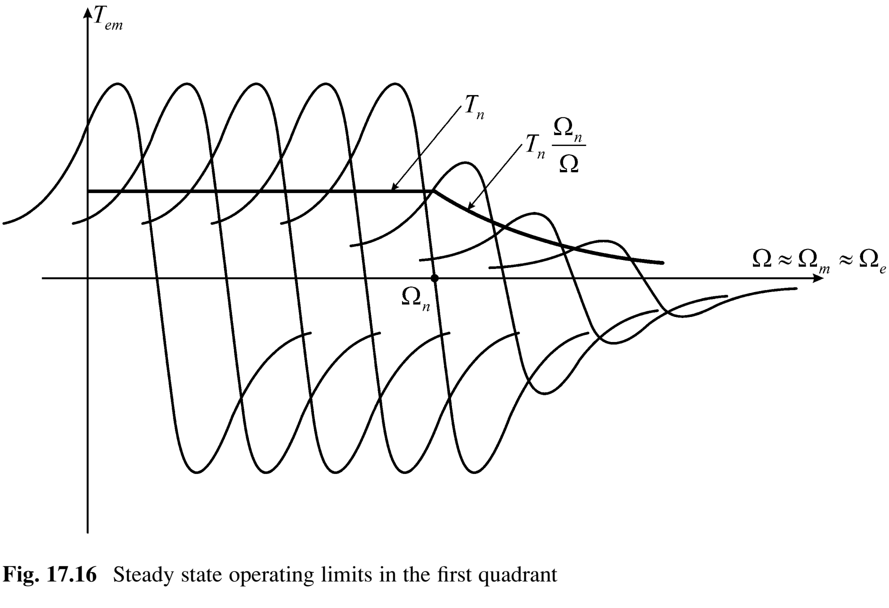

Fig. 17.16 Steady state operating limits in the first quadrant

During operation at speeds below the rated speed, the flux is maintained at the rated value. With rated currents in stator windings and with the rated flux, induction machine provides the rated torque T_n at the shaft. The rated torque is available in continuous service at all speeds where the stator frequency remains within the rated limits, $|\omega_e| \leq \omega_n$.

In the field weakening region, the stator frequency exceeds the rated value, $|\omega_e| > \omega_n$. The flux varies according to the law $\Psi_S(\omega) = \Psi_n(\omega_n/\omega)$. For this reason, the torque available in continuous service in the field weakening regime is inversely proportional to the rotor speed, that is, inversely proportional to the stator frequency. The torque available in the field weakening operation can be represented by expression

$$T_n \frac{\Omega_n}{\Omega} \tag{17.13}$$

which defines boundaries of the steady state operating area in the zone of higher speeds (Fig. 17.16).

17.19 Steady State Operating Limits

Steady state operating limits of relevant variables are shown in Fig. 17.17. Any value that does not exceed the limits is sustainable in continuous operation. The limits are given for the voltage, current, stator frequency, torque, flux, and power of an

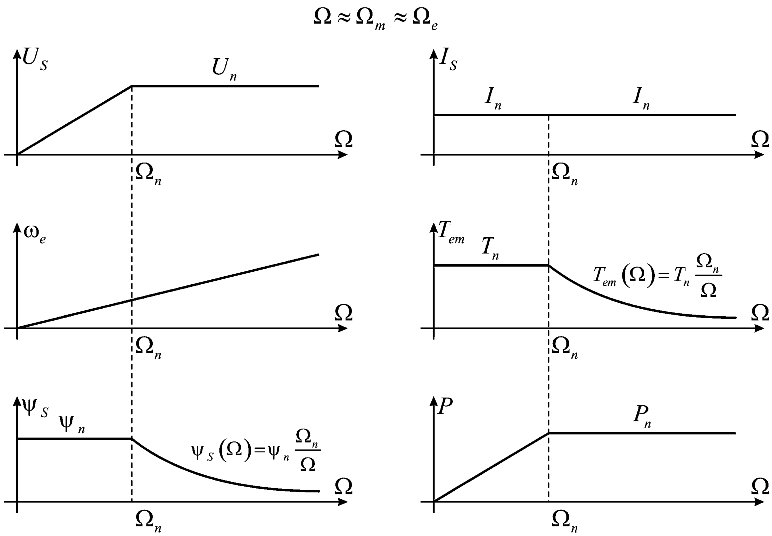

Fig. 17.17 Steady state operating limits for the voltage, current, stator frequency, torque, flux, and power. The region $\Omega_m < \Omega_n$ is with constant flux and torque, while the field weakening region $\Omega_m < \Omega_n$ is with constant power

induction machine supplied from variable frequency, variable voltage source. For clarity of diagrams, it is assumed that $\Omega \approx \Omega_e \approx \Omega_m$. The region $\Omega_m < \Omega_n$ below the rated speed is called *constant flux* or *constant torque* region. The region $\Omega_m > \Omega_n$ above the rated speed is called *flux weakening region* or *constant power region*.

Diagrams in Fig. 17.17 represent the change of steady state operating limits for U_S, I_S, T_{em}, P, and Ψ_S. Therefore, they comprise only the first quadrant with $\Omega > 0$. However, the limits such as $T_{em}(\Omega_m)$ apply to all the four quadrants of T_{em}-Ω_m plane. Namely, the same limits of the continuous service apply for both directions of the speed, and they are equally valid in motor operating mode as well as in generator operating mode. It is shown in the figure that the stator frequency ω_e increases proportionally to the rotor speed. By neglecting the slip, relation between the rotor speed and the stator frequency is $\omega_e \approx p\Omega_m$. In constant flux zone, at speeds lower than the rated speed, the ratio U_S/ω_e is kept constant. Upon reaching the rated speed, the voltage is maintained at the rated value. Further increase of the rotor speed gets the machine in the field weakening mode, where the voltage remains constant while the angular frequency of the power supply keeps increasing. This results in flux decrease. In the field weakening regime, the flux changes according to $\Psi_S(\omega) = \Psi_n(\omega_n/\omega_e)$.

In constant flux region, the available torque is constant, while in field weakening region, the torque is limited by $T_{em}(\Omega) \approx T_n(\Omega_n/\Omega_e)$. The power available in constant flux region ($|\Omega_e| \leq \Omega_n$) increases linearly with the speed. In field weakening, the available torque drops according to $T_{em} \sim 1/\Omega$. Therefore, the power available in field weakening has a constant value, $P(\Omega) \approx \Omega T_n(\Omega_n/\Omega) = P_n$. For this reason, the field weakening region is called *constant power region*. By recognizing the secondary effects, which have been neglected in the first approximation, it can be shown that the power available in the field weakening regime is somewhat higher than the rated power.[4]

17.19.1 RI Compensation

In the process of calculating the stator voltage, the voltage drop across the stator resistance has been neglected. For clarity, approximation $\underline{U}_S = R_S\underline{I}_S + \mathrm{j}\omega_e\Psi_S \approx \mathrm{j}\omega_e\Psi_S$ has been made. The stator resistance has a very low relative value. Therefore, this approximation does not introduce any significant error in calculations, provided that the rotor speeds are sufficiently high and that the electromotive force $\omega_e\Psi_S$ has the value significantly higher than the voltage drop R_SI_S. At very small speeds, where the electromotive force is comparable to the voltage drop R_SI_S, this approximation cannot be justified.

[4] Due to flux decrease in the field weakening regime, the magnetizing current I_m is lower than the rated magnetizing current. This allows for a slight increase in the rotor current liable for the torque generation.

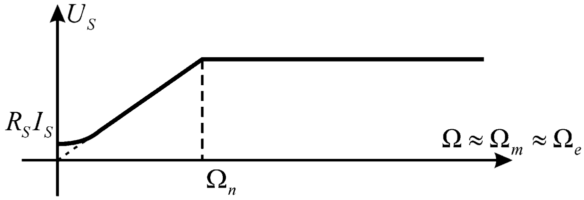

Fig. 17.18 *RI* compensation – the voltage increase at very small speeds

If an induction machine operates at very small speed, the angular frequency is very small as well. Maintaining the rule that the stator voltage U_S is proportional to the supply frequency, the flux of the machine is obtained below the rated flux. Consider the case where the ratio *U/f* is retained even at very low speeds, notwithstanding the resistive voltage drop. The stator voltage is equal to $U_S = \omega_e \Psi_n$. Assuming that the mechanical load T_L is close to zero, the machine operates with the slip of $s = 0$ and with $\underline{I}_R = 0$. With $\underline{U}_S = U_S$, the stator current is equal to $\underline{I}_S = U_S/(R_S + j\omega_e L_S)$, and the stator flux is $\Psi_S = L_S \underline{I}_S = L_S(U_S/(R_S + j\omega_e L_S)) = \Psi_n \cdot (jL_S\omega_e/(R_S + j\omega_e L_S))$. In cases where $\omega_e L_S >> R_S$, the stator flux amplitude is equal to Ψ_n, and it does not depend on the parameter R_S. At very low speeds, the flux amplitude decreases due to the voltage drop $R_S I_S$.

According to diagram $U_S(\Omega)$ in Fig. 17.17, the operating point $\Omega_e = \omega_e/p = 0$ results in the stator voltage $U_S = \omega_e \Psi_n = 0$. With $U_S = 0$ and $R_S \neq 0$, the stator flux is equal to zero as well. With low supply frequencies and with $U_S = \omega_e \Psi_n$, the actual stator flux is lower than the rated value. A low flux amplitude at very low frequencies reduces the start-up torque and has adverse effect on the operation of induction machines at low speeds. These effects can be reduced by changing the control law $U_S = \omega_e \Psi_n$ and increasing the supply voltage amplitude at low speeds in the manner shown in Fig. 17.18.

17.19.2 Critical Speed

According to Fig. 17.17, induction machine operating in the field weakening region is capable of providing a constant rated power. This figure has been derived based upon certain assumptions. One of them is neglecting the difference between the stator flux and the air-gap flux, namely, neglecting the voltage drop across the leakage inductance. The leakage reactance is proportional to the supply frequency. At very high speeds, the operating frequency increases up to the levels where the leakage reactance cannot be neglected. Therefore, there is a limit to the constant power operation. The speed Ω_{cr} is the maximum speed where the machine is still capable of delivering the rated power. The operation above this speed is feasible but with a power lower than the rated power. The speed Ω_{cr} is called *critical speed*. It denotes the intersection of functions $T_s(\omega) = T_n(\omega_n/\omega)$ and $T_b(\omega) = T_{b(n)}(\omega_n/\omega)^2$. An approximate value of the critical speed will be determined in the subsequent analysis. To keep the discussion simple, it is assumed that $p = 1$ and that electrical frequencies ω have the same values as the relevant speeds Ω. At the same time, the slip frequency and the slip speed are considered negligible ($\omega_{slip} << \omega_e$, $\Omega_{slip} << \Omega_e$) allowing the rotor speed Ω_m to be replaced with the synchronous speed Ω_e.

The operation at a constant, rated power in the zone of field weakening requires the torque $T_s(\omega) \approx T_n(\omega_n/\omega)$. The function $T_s(\omega)$ delimits the steady state operating limit for the torque. Namely, the curve $T_s(\omega)$ expresses the maximum steady state torque at the given speed. The limit torque $T_s(\omega)$ is obtained with rated stator current I_n. On the other hand, the envelope of breakdown torques varies according to the law $T_b(\omega) \approx T_{b(n)}(\omega_n/\omega)^2$, where $T_{b(n)}$ is the breakdown torque obtained at the rated power supply conditions, with the rated flux. The function $T_b(\omega)$ represents the maximum transient torque available at the given speed. This transient torque T_b can be maintained only for a short interval of time, as it requires the stator currents $I_S > I_n$, and therefore cause increased losses and temperature rise. The function $T_s(\omega)$ crosses the function $T_b(\omega)$ at the speed $\omega_{cr} = \omega_n(T_{b(n)}/T_n)$.

For speeds above ω_{cr}, the available breakdown torque $T_b(\omega)$ is smaller than the torque $T_s(\omega) \approx T_n(\omega_n/\omega)$ which is permissible in continuous service. Hence, the transient torque limit falls below the torque limit in continuous service, which appears a contradiction. For the proper understanding, it is of interest to understand the difference between the functions $T_s(\omega)$ and $T_b(\omega)$. The curve $T_s(\omega)$ represents the torque T_s which is available at the given speed of ω provided that the current in the stator windings is $I_S = I_n$. Hence, in a way, the curve $T_s(\omega)$ represents the limit $I_S < I_n$ expressed in $T(\omega)$ plane. It is of interest to notice that the torque values $T_s(\omega)$ are feasible only in cases where the stator current can actually reach the rated current. On the other hand, the curve $T_b(\omega)$ is the actual limit for the instantaneous torque. At the given speed ω, the function $T_b(\omega)$ provides the peak torque available from the induction machines of the given parameters. Above critical speeds, the curve $T_b(\omega)$ provides the values of the breakdown torque available for $\omega > \omega_{cr}$. At the same time, the values indicated by $T_s(\omega)$ cannot be reached for speeds $\omega > \omega_{cr}$. At elevated supply frequencies, the leakage reactance increases. With the stator voltage limited to the rated value and with an increased leakage reactance, the stator current cannot reach the rated values. Hence, for the speeds $\omega > \omega_{cr}$, the stator current falls below I_n in both transient and steady state service. This leads to situation where $T_s(\omega) > T_b(\omega)$.

Induction machines with variable frequency supply can operate above critical speed, but their power will fall below the rated power. In this range of speeds, the available torque will drop proportionally to the square of the rotor speed, while the available power will drop proportionally the speed, $P \sim 1/\omega$. Transient and steady state operating limits of an induction machine are given in Fig. 17.19.

With the assumed approximations, the critical speed is

$$\Omega_{cr} = \Omega_n \frac{1}{2x_{\gamma e}} = \frac{T_{b(n)}}{T_n}, \tag{17.14}$$

where

$$x_{\gamma e} = \frac{X_{\gamma e}}{Z_n} = \frac{L_{\gamma e}\omega_n}{Z_n} = \frac{L_{\gamma e}\omega_n I_n}{U_n}.$$

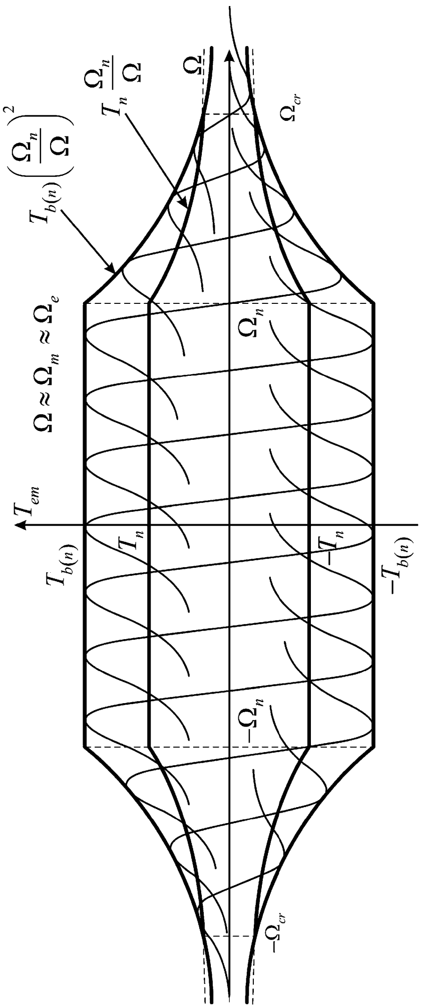

Fig. 17.19 Transient and steady state operating limits

Question (17.7): Induction motor connected to the voltage source with rated frequency and rated voltage amplitude develops the stator current $I_P = 5I_n$ in locked rotor conditions ($\Omega_m = 0$). Provide an estimate of relative values of the breakdown torque and the critical speed.

Answer (17.7): The breakdown torque obtained with the rated power supply is determined by expression

$$T_{b(n)} = \frac{3p}{\Omega_e} \frac{U_{Sn}^2}{2X_{\gamma en}},$$

where U_{Sn} is the rated rms value of the phase voltage, while $X_{\gamma en}$ is the leakage inductance at the rated stator frequency. By introducing approximations $R_S = 0, L_m >> L_{\gamma e}$, and $(R_R/s_n) \approx U_n/I_n >> X_{\gamma en}$, the rated torque can be represented by the following expression:

$$T_{nom} = \frac{3p}{\Omega_e}\frac{R_R}{s_n}\frac{U_{Sn}^2}{\left(\frac{R_R}{s_n}\right)^2 + X_{\gamma en}^2} \approx \frac{3p}{\Omega_e}\frac{R_R}{s_n}\frac{U_{Sn}^2}{\left(\frac{R_R}{s_n}\right)^2} \approx \frac{3p}{\Omega_e}\frac{U_{Sn}^2}{\left(\frac{R_R}{s_n}\right)} \approx \frac{3p}{\Omega_e} U_{Sn} I_{Sn},$$

where s_n represents the rated value of the relative slip.

Relative value of the breakdown torque is equal to the ratio of the two preceding expressions,

$$m_{b(n)} = \frac{T_{b(n)}}{T_n} = \frac{\frac{U_{Sn}^2}{2X_{\gamma en}}}{U_{Sn} I_{Sn}} = \frac{1}{2x_{\gamma en}},$$

where $x_{\gamma en}$ is the relative value of the leakage reactance. An approximate value of the leakage reactance can be determined from the start-up current, $x_{\gamma en} \approx 1/I_P = 0.2$. Relative value of the initial torque is equal to $m_{b(n)} = 2.5$. Critical speed $\Omega_{cr} = \Omega_n(T_{b(n)}/T_n)$ is the highest speed at which the rated power can still be obtained. Relative value of the critical speed (Ω_{cr}/Ω_n) is equal to the relative value of the breakdown torque, thus $\omega_{cr} = 2.5\omega_n$.

17.20 Construction of Induction Machines

Induction machines have been in use since the end of the nineteenth century. During the first hundred years of their application, the switching power transistors and other components required for variable frequency supply were not available. For that reason, induction machines were supplied from the mains, with the voltages having a constant, line frequency. Therefore, all the induction machines used in this period were designed and optimized for constant frequency operation. Starting up of the induction motors was performed by connecting them to a three-phase network of industrial frequency 50/60 Hz.

17.20.1 Mains-Supplied Machines

At start-up time, a mains-supplied induction motor has the rotor speed $\Omega_m = 0$ and the stator voltages of the rated amplitude and frequency. The start-up current in the stator windings is $I_P \approx U_{Sn}/X_{\gamma e}$, where $X_{\gamma e} \approx X_{\gamma S} + X_{\gamma R}$ is the equivalent leakage reactance. Small values of the leakage reactance would result in high start-up

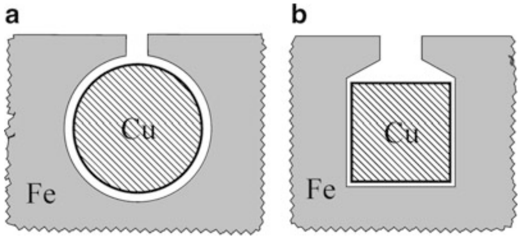

Fig. 17.20 (**a**) Semi-closed slot. (**b**) Open slot

currents. A reactance of $x_{\gamma e} \approx 10\%$ gives the start-up current which is 10 times higher than the rated current. Such current results in the stator copper losses that exceed the losses under rated condition by 100 times. At the same time, large start-up currents result in considerable drop in the mains voltage and affect other electrical loads that are connected to the same line. The start-up mode of an induction motor lasts until the rotor speed comes close to the synchronous speed, where the relative slip s comes down and the impedance R_R/s in the equivalent circuit obtains the value $R_R/s >> X_{\gamma e}$, therefore causing the stator current to decrease and come down to acceptable levels. The acceleration time depends on the load inertia J, and it can last from several hundreds of milliseconds up to several seconds. Losses in the windings during the start-up are proportional to the square of the initial current, and they cause a steep rise of the motor temperature. Therefore, the acceleration cannot last long. Unless the machine approaches the synchronous speed in a short time, it has to be disconnected from the mains in order to avoid dangerous temperatures and preserve the machine integrity. The start-up current is much higher than the rated current, and this may pose a problem for the installations, fuses, and cabling. Mains-supplied induction machines have to be designed to sustain large start-up currents without damage.

In order to reduce the start-up current of mains-supplied induction machines, it is necessary to design such machines so as to provide higher leakage inductances. The leakage inductance is proportional to the ratio N^2/R_μ, where N is the number of turns of the relevant winding, while R_μ is magnetic resistance along the path of the leakage flux. By reducing the magnetic resistance R_μ, it is possible to increase the leakage inductance and leakage reactance. This would contain the start-up current of an induction machine. One of the ways to achieve this is the use of the semi-closed and closed slots in the rotor magnetic circuit (Fig. 17.20).

Making the rotor slot opening toward the air-gap narrower reduces the magnetic resistance along the path of the leakage flux. With narrow top of the rotor slot, the leakage flux path through the air is made shorter, which reduces the magnetic resistance. An increase in the leakage inductance and reactance would decrease the start-up current. There are also side effects. An increase in leakage inductance reduces the breakdown torque, which is inversely proportional to the leakage reactance. In the process of designing an induction machine intended for constant frequency operation, the choice of leakage reactance is the result of a compromise. The final value should result in acceptable start-up currents, but it should not make an unacceptable reduction of the breakdown torque.

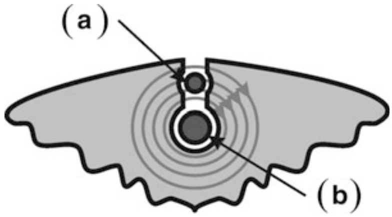

Fig. 17.21 Double cage of mains-supplied induction machines. (**a**) Brass cage is positioned closer to the air gap. (**b**) Copper or aluminum cage is deeper in the magnetic circuit

At start-up of an induction motor supplied from the mains, it is necessary to develop the start-up torque T_P as high as possible. This would prevail over the motion resistances T_L and provide for acceleration. At start-up, the acceleration is equal to $d\Omega_m/dt = (T_P - T_L)/J$, and it strongly depends on the start-up torque. Higher values of T_P result in higher acceleration, resulting in short acceleration times, lower amount of heat caused by losses, and lower increase in temperature. Shortening the start-up reduces the thermal stress and extends the lifetime of the machine. The start-up torque $T_P = (3p/\Omega_{en})R_R {I_P}^2$ is dependent on the square of the start-up current, and it is proportional to the rotor resistance R_R. In order to increase the start-up torque, it is necessary to increase the rotor resistance R_R. This can be accomplished by making the rotor bars with a smaller cross section or making them of materials with higher specific resistance, such as brass. However, an increase in the rotor resistance affects the steady state losses in the rotor windings. In rated operating condition, the copper losses in the rotor would be higher due to elevated resistance R_R. This would reduce the coefficient of efficiency of the machine, increase the temperature, and eventually reduce the rated power. High efficiency during steady state regimes requires the rotor resistance to be as small as possible. At the same time, the need to maximize the start-up torque requires the use of rotor resistances as high as possible. This contradiction was resolved by making the rotor cage so as to obtain frequency dependence of the rotor resistance.

The rotor winding can be designed so as to have a frequency-dependent resistance. Parameter R_R can be made dependent on the frequency of the rotor currents, namely, on the slip frequency. In start-up mode, the slip frequency is high. It is equal to the line frequency due to the relative slip being equal to one. At steady state, the machine operates with the speeds that are close to the synchronous speed, and the slip frequency is very low, of the order to 1 Hz. The change in the slip frequency can be used to obtain variable rotor resistance that would suit the needs of mains-supplied induction machines.

At start-up, it is necessary to have high values of R_R in order to have a high start-up torque. Then, the frequency of the rotor currents is equal to that of the stator currents, $f_{slip} = f_e = 50$ Hz. Hence, it is necessary to make the rotor cage so that it pays relatively high resistance to electrical currents of the line frequency.

At steady state, the speeds are close to the rated value and to the synchronous speed. The frequency of rotor currents is much lower, and it is close to $f_{slip} \sim 1$ Hz. Equivalent resistance of the rotor cage at low frequencies should be as low as possible.

By building a double cage, like the one shown in Fig. 17.21, the rotor winding can be made with frequency-dependent resistance. Low-resistance, large cross-sectional bars made of copper or aluminum are placed deeper into the rotor magnetic circuit. This inner cage (B) has much smaller resistance to DC currents. Closer to the surface, there are brass bars of smaller cross section (A). Their resistance is much higher. At very low frequencies, such as the rated slip frequencies, the rotor current has a low-resistance path through the inner cage. At line frequency of 50 Hz, the inner cage reactance prevents the rotor circuits from passing through the inner cage. Therefore, the start-up current in the rotor circuit passes through the brass cage, which is closer to the air gap, which has a much higher resistance and therefore provides a higher start-up torque.

It is of interest to notice that the leakage reactance of the inner cage is much higher than the leakage reactance of the brass cage. The figure shows the lines of the magnetic field of the leakage flux. The copper bars are encircled by a large number of field lines. For this reason, the leakage flux, leakage inductance, and leakage reactance are relatively high. The brass cage, placed much closer to the air gap, is encircled by a smaller number of field lines. Its leakage flux and leakage reactance are considerably smaller. At start-up, the frequency of rotor currents is $f_{slip} = f_e$ =50 Hz, which increases the values of rotor reactance $X_\gamma = L_\gamma\, \omega_{slip} = L_\gamma\, \omega_e$. Due to relatively high frequency, reactances of both cages prevail over resistances, and the impedance of each of the cages is mainly reactive, $X_\gamma >> R_R$. Since reactance of the brass cage is considerably smaller, the rotor start-up current passes mainly through the brass cage, the cage with higher resistance. When the motor enters the steady state, the frequency of the rotor currents is considerably smaller, and it is close to $f_{slip} \sim 1$ Hz. Therefore, the impedance of both cages is mainly resistive, as the resistances prevail over reactances, $X_\gamma = L_\gamma\, \omega_{slip} << R_R$. Since the resistance of the lower (copper) cage is considerably smaller, the rotor current at steady state passes mainly through the low-resistance copper bars.

It can be concluded that the rotor currents in a double-cage rotor pass through the upper, brass cage during start-up, while they get shifted to the lower, copper cage during operation at steady state, where the speed is close to the synchronous speed and the slip frequency is low. In this way, the rotor resistance is made frequency dependent. At steady states close to rated operating conditions, equivalent rotor resistance is low, while during the start-up, equivalent rotor resistance is high. Manufacturing double cage increases complexity of the production process and increases the costs. Therefore, it is used mainly for machines with larger rated power and/or larger start-up torque requirements.

The effects similar to those created by double cage can be obtained by designing rotor slots with an increased depth and decreased width. An example of such deep slot is shown in Fig. 17.22. The slot contains a rotor bar of rectangular cross section. With very low rotor frequencies, where the leakage reactances are of no importance, the rotor current is distributed equally across the cross section of the rotor bar. The currents that pass at the bottom of the slot are encircled by a number of lines of the magnetic field, that is, by relatively large leakage flux. On the other hand, there are also currents next to the surface, facing the air gap. They are encircled by considerably lower number of field lines and have much smaller leakage flux.

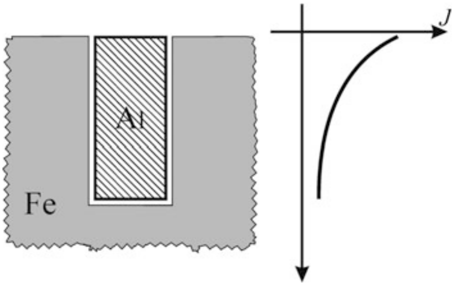

Fig. 17.22 A deep rotor slot and distribution of rotor currents

When the rotor bars have current of relatively high slip frequency, variable leakage flux creates electromotive force which opposes to electrical currents. The current passing through the upper part of the slot is encircled by a small leakage flux. Therefore, the reactive electromotive force for this current is smaller. On the other hand, the current passing through the lower part of the slot is buried into the rotor magnetic circuit, and it is encircled by a larger leakage flux. Thus, the reactive electromotive force for this current is much higher. It impedes the current flow and pushes the rotor current toward the air gap. An example of an uneven distribution of the rotor current is given in the right-hand side of the figure. Since the currents of relatively high slip frequency pass through a smaller part of the rotor bar cross section, the equivalent resistance of the rotor is increased. On the other hand, the current distribution at low slip frequencies is even, and the equivalent rotor resistance is much lower.

17.20.2 Variable Frequency Induction Machines

In previous section, different approaches to designing mains-supplied induction machines have been outlined. They were focus on resolving the problems of constant frequency induction machines. Most important problems include limiting the start-up current, providing sufficient start-up torque, and providing a satisfactory efficiency in steady state conditions.

Modern induction machines are supplied from three-phase switching inverters which make use of power transistors. They produce three-phase voltage system of variable frequency and variable amplitude. Parameters of the power supply are suited to serve the target operating modes. The start-up of frequency-controlled induction machine does not imply a large start-up current. Instead, the stator frequency is reduced to the value close to the rated slip frequency, and the voltage amplitude is determined so as to produce the rated flux. In this way, development of the start-up torque does not require the stator currents that exceed the rated current, unless the motion resistances do exceed the rated torque. Hence, frequency-controlled induction machines are never exposed to rated voltage in locked rotor condition. Therefore, they do not need to have an increased leakage inductance, since there is no need to limit the start-up current. Instead, they can be designed to have open slots of both stator and rotor magnetic circuits, which results in a smaller leakage flux, smaller leakage inductance, and larger breakdown torque. Example of an open slot is shown in Fig. 17.20b.

In addition to higher breakdown torque, the advantage obtained by decreasing leakage inductance is the possibility to achieve faster changes of the stator current, which results in quicker torque changes. Since the electromagnetic torque of an electrical machine depends on electrical currents in the windings, the rate of change of the electromagnetic torque is dependent on the first derivative of the current, $\mathrm{d}i_S(t)/\mathrm{d}t$. The voltage balance equation in the stator winding can be represented by $u_S = R_S i_S + L_{\gamma e}\mathrm{d}i_S/\mathrm{d}t + e$. Therefore, the first derivative of the stator current $\mathrm{d}i_S/\mathrm{d}t = (u_S - R_S i_S - e)/L_{\gamma e}$ is inversely proportional to the leakage inductance. With lower leakage inductances, it is possible to achieve larger rate of change of the electromagnetic torque.

Reduction of leakage inductance can also have negative consequences. Due to a finite number of slots carved into magnetic circuits and due to non-sinusoidal distribution of conductors, the windings of an induction machine contain electromotive forces that have higher harmonics. These harmonics cause electrical currents of the same frequency. The amplitude of such currents in rotor bars is directly proportional to the amplitudes of the relevant frequency component of the electromotive force, and inversely proportional to the winding impedance. The winding impedance at higher frequencies is determined primarily by the leakage reactance. For this reason, reduction of the leakage inductance leads to increased amplitudes of winding currents caused by higher harmonics, and increases the current ripple caused by the PWM supply. The adverse effects can be avoided by careful design of the rotor slots by shaping the stator and rotor magnetic circuits so as to reduce non-sinusoidal distribution of the field and to design the stator and rotor windings so as to reduce the electromotive forces induced due to distortions and higher harmonics.

Question (17.8): Rotor bars are placed in slots which are separated by teeth. The lines of the magnetic field are directed along the path of smaller magnetic resistance. Therefore, magnetic induction is high in rotor teeth and significantly lower in rotor slots. Conductors carrying rotor currents are placed in slots, where magnetic induction B is close to zero. Considering the force exerted on conductors, it depends on magnetic induction and electrical current. Apparently, this force is going to be very low. Explain the fact that, notwithstanding the abovementioned, induction machine does generate considerable torque.

Answer (17.8): The torque generation can be represented as the result of forces acting on conductors. With conductors placed in relatively deep slots, magnetic induction within the iron teeth exceeds by far the magnetic induction within aluminum-filled slots. Ratio of magnetic induction in slots and magnetic induction in teeth is close to μ_0/μ_{Fe}. Therefore, only a very small force is acting on conductors. Instead, forces act on rotor surfaces that separate ferromagnetic materials, such as iron in magnetic circuits, from nonmagnetic materials, such as aluminum conductors or the air gap. They act on the surface walls between slots and teeth. The forces acting on the rotor teeth can be explained by using the concept called *equivalent magnetic pressure*.[5] Calculation of spatial distribution of the magnetic field in relatively complex, three-dimensional structure such as the slotted rotor is rather involved. In most cases, it requires automated software tools. Once completed, this calculation provides the information on the equivalent magnetic pressure which is acting upon surfaces that separate iron from nonmagnetic domains. With the equivalent magnetic pressure readily available, the forces and torque can be calculated by performing surface integration along all the relevant surfaces. The outcome of such calculations is the torque value equal to the value obtained by assuming that the force *LIB* acts on each conductor and that magnetic field lines pass equally through slots as they do through teeth. The last assumption is equivalent to considering the machine where the rotor surface is smooth cylinder with no slots, while the rotor conductors reside in the air gap, attached to the rotor surface. In such hypothetical case, the expression $F = LIB$ is more obvious.

[5] Magnetic field creates forces acting on surfaces delimiting different domains. These forces can be described by introducing equivalent pressure p(N/m^2). The force acting on surface S is equal to $F = pS$. The energy density of the magnetic field in the first domain, next to the boundary, is $w_1 = \mu_1 H_1^2/2$. Across the boundary, in the second domain, the energy density is $w_2 = \mu_2 H_2^2/2$. Equivalent pressure is equal to $w_1 - w_2$.