31 Subsurface Drainage Modeling

The original subsurface drainage model is the Hooghoudt equation, which is a one-dimensional steady-state simplification of the two-dimensional transient flow to parallel drains. It calculates the midpoint water table elevation between drains. Bower and van Schilfegaarde modified the Hooghoudt equation for transient analysis. The Bureau of Reclamation also developed drainage equations for transient analysis of midpoint water table elevation. Kirkham developed a Laplace analytic solution for the two-dimensional subsurface drainage geometry. He also adapted this solution for transient analysis with the concept of fixed streamtubes along the path of water flow. The advantage of this approach is that water table height can be simulated as a function of distance from the drain rather than just the midpoint water table elevation. The Kirkham streamtube approach is used in the *WINDS* drainage model, which enables *WINDS* to model water, salinity, and nitrogen in the soil profile as a function of distance from the drain. The chapter also includes an example of the economic analysis of drain spacing and depth.

Subsurface field drainage pipes are corrugated, black, high-density, polyethylene (HDPE) plastic pipe that is extruded in rolls with slits cut in the sides of the pipe to allow water to enter the pipe (Fig. 31.1). The corrugations support the walls of the pipe and prevent physical collapse. In regions with unstable soils in which particles can be carried by flowing water to the drain pipe, the pipes are wrapped with a fabric sleeve (sock) at the factory in order to prevent sediment from entering into the pipe during drainage.

Flow to subsurface drains takes two forms. Just after a heavy storm or irrigation, the entire water table may rise to the soil surface. Immediately after the event, the primary flow to the drains is from the region above the drains (Fig. 31.2, left). If the drain pipes are full due to excess flow in the system, then the water table above the drains will not decline and may even rise in portions of the field where drains have positive internal pressure (Fig. 31.2, right). Conventional drainage systems continue to remove water from the field until the water table is lowered to the elevation of the drain; this can result in insufficient water for plants during dry periods.

The water table above the drain generally declines within a day or two of the storm or irrigation event until the water table intersects the drain (Fig. 31.3). Subsequently, the water table forms an elliptical shape and flow to the drains is driven by the slope (energy gradient) of the water table.

The flow lines are not always toward the drain. Conventional (Fig. 31.4a) subsurface drainage systems can be modified to manage the water table elevation by installing structures in the drainage network such as flashboards or stoplogs that prevent water from draining through the pipe network. This is called controlled drainage (Fig. 31.4b). If a dry period is expected, then plant growth is maximized by preventing drainage and maintaining a high water table in order to carry the plant through the dry period. Subirrigation (Fig. 31.4c) adds water to the soil through the drainage system; water in the drainage canal and water flows into the drainage system.

Simulation of Yield Reduction

Wet stress and yield reduction take place during periods with an excessively high water table. Drainage simulation models typically report the average daily height of the water table at the midpoint between drains as a representative indication of the water content in the root zone. Figure 31.5 shows the daily water table depth computed by the DRAINMOD model, a popular drainage simulation program developed by Wayne Skaggs at North Carolina State University. During rainstorms, the elevation of the water table increases sharply as the soil is suddenly filled with water. Subsequently, the elevation of the water table slowly decreases as water is removed from the soil by plant roots and by subsurface drainage. The program calculates corn yield reduction based on the number of days when the water

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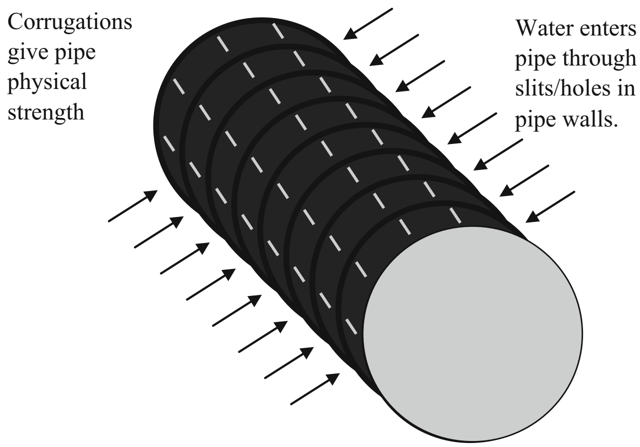


Fig. 31.1 Corrugated plastic drain pipe

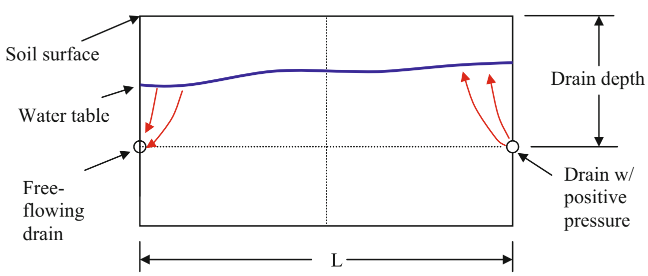


Fig. 31.2 Flow to drains just after a storm with water table above drain

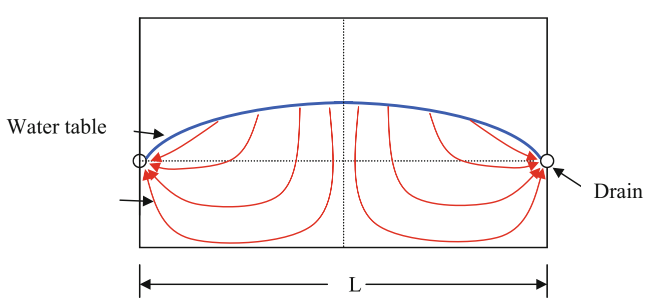


Fig. 31.3 Flow to subsurface drains between storms

table is within 30 cm of the ground surface (horizontal line in Fig. 31.5). This is done by calculating a wet stress coefficient and a weighted water table elevation.

In DRAINMOD, the wet stress during the growing season is computed as the weighted (greater weight when the crop is more susceptible to wet stress) sum of the distance from the 30 cm depth to the water table on each day that the water table is higher than 30 cm depth. The distance from the 30 cm depth to the water table is called the SEW30

$$\mathrm{SEW30} = 30 - \mathrm{DTWT} \qquad (31.1)$$

where

DTWT = depth to water table, cm.

The crop susceptibility to wet stress changes during the growing season and is quantified by the crop susceptibility factor, CS, in DRAINMOD. The crop susceptibility factor is typically higher during early to middle parts of the growing season. The summation of CS * SEW-30 is called the stress day index, SDI

$$SDI = \sum_{i=1}^{n} CS_i * SEW30_i \quad \text{for DTWT} < 30 \qquad (31.2)$$

Finally, yield reduction, YR, for the growing season is calculated based on the stress day index and the susceptibility of the crop to wet stress, expressed as DSLOPE.

$$\mathrm{YR} = \mathrm{YRDMAX} - \mathrm{DSLOPE} * \mathrm{SDI} \qquad (31.3)$$

where

YR = percent of maximum yield
YRDMAX = yield intercept
DSLOPE = ratio between SDI and YR

Example 31.1 Calculate the SDI for a corn crop. The planting date is DOY 115 (after spring rains). The time when the water table depth is less than 30 cm is between DOY 194 and DOY 209 (Table 31.1).

DOY 194–209 is approximately 90 days (yield formation) after planting and the crop susceptibility factor at that time is 0.08. The susceptibility is low because the crop is not very susceptible to wet stress during yield formation. The SDI is

$$\mathrm{SDI} = (30 + 18 + 30 + 12 + 10 + 30 + 8 + 2) * 0.08 = 11$$

For corn, YRDMAX = 102 and DSLOPE = 0.75

$$\mathrm{YR} = 102 - 0.75 * 11 = 94\%$$

Thus, the crop has a seasonal yield reduction of 6 % due to wet stress during this part of the growing season.

If there is no outlet control for the drainage system (Fig. 31.4), then the drainage system must be designed for average conditions during the year. A well-designed depth and spacing of drains keeps the water table out of the critical root zone for the majority of growing seasons, but does not cause the water table to drop so low that dry stress becomes a problem. No drainage system can prevent the water table from ever reaching the ground surface during a heavy storm, but drainage systems will rapidly reduce the elevation of the water table after the storm as shown in Fig. 31.5.

After the water table directly over the drain drops to the drain level, the water table has an ellipse pattern and flow pattern to the drains is as shown in Fig. 31.6. The rate of flow to drains at the midpoint between drains is much slower than

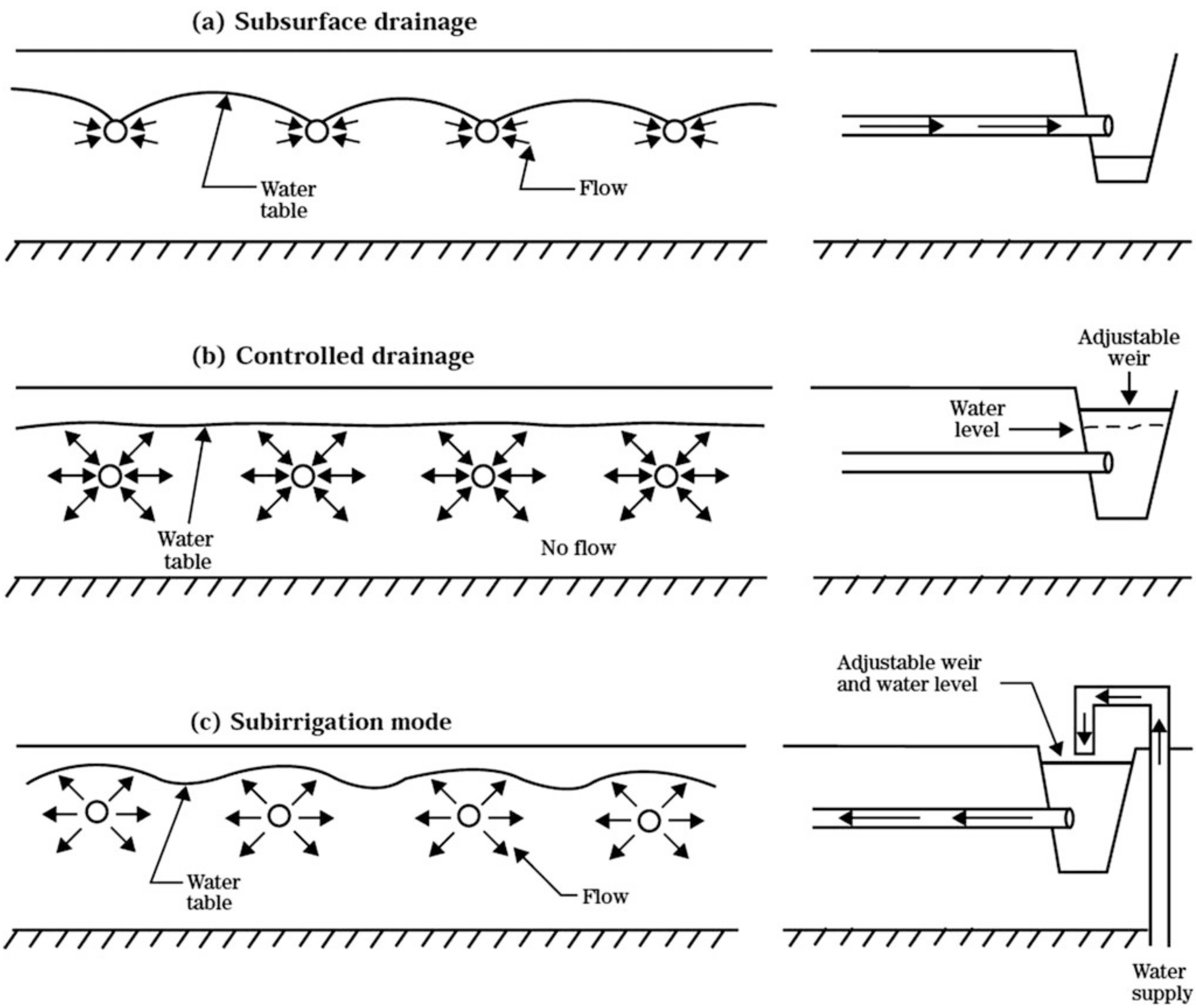


Fig. 31.4 Drainage alternatives (Credit NRCS, Part 650 Engineering Field Handbook National Engineering Handbook, Chapter 14)

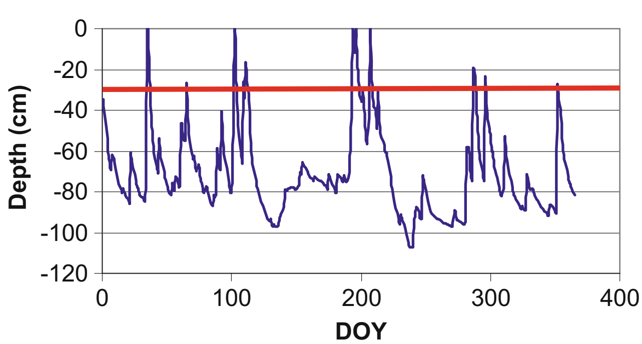


Fig. 31.5 DRAINMOD calculation of water table depth versus time

in the region close to the drain. This explains why the water table height, average soil water content, and salinity are normally higher between drains than near drains.

Drainage simulation models are based on a water volume balance for the region between drains (Fig. 31.7). The infiltration is normally assumed to be uniform across the region between drains; however, if water content and drain elevation are higher between drains, then infiltration may decrease with distance from the drain.

Hooghoudt Equation

The Hooghoudt equation was developed to calculate the midpoint water table elevation between drains with the assumption of steady state rainfall during the year: all of the precipitation for an entire year is averaged to calculate the average rate of precipitation per day. Steady state rainfall is not an unrealistic assumption in Hooghoudt's country, Holland. In order to develop a one-dimensional equation

Table 31.1 DTWT, SEW-30, and CS for data in Fig. 31.8

DOY	DTWT	SEW-30	CS	DOY	DTWT	SEW-30	CS
194	0	30	0.08	202	34	0	0.08
195	12	18	0.08	203	44	0	0.08
196	0	30	0.08	204	51	0	0.08
197	18	12	0.08	205	56	0	0.08
198	20	10	0.08	206	37	0	0.08
199	31	0	0.08	207	0	30	0.08
200	36	0	0.08	208	22	8	0.08
201	31	0	0.08	209	28	2	0.08

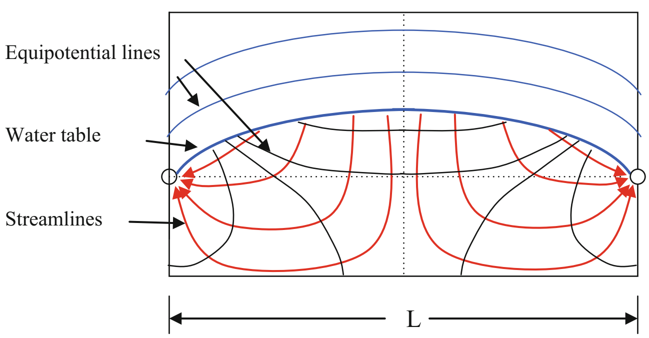


Fig. 31.6 Streamlines and equipotential lines for two-dimensional flow to parallel subsurface drains

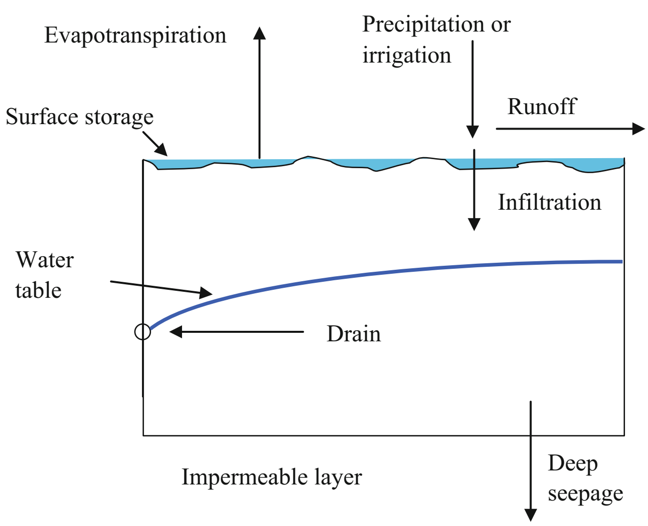


Fig. 31.7 Drainage control volume and components of drainage volume balance

for horizontal flow to drains, Hooghoudt assumed that all flow to drains below the water table was horizontal (the Dupuit-Forchheimer (D-F) assumption).

In-class Exercise 31.1 On a piece of paper, draw streamlines and equipotential lines below the water table for Dupuit-Forchheimer flow.

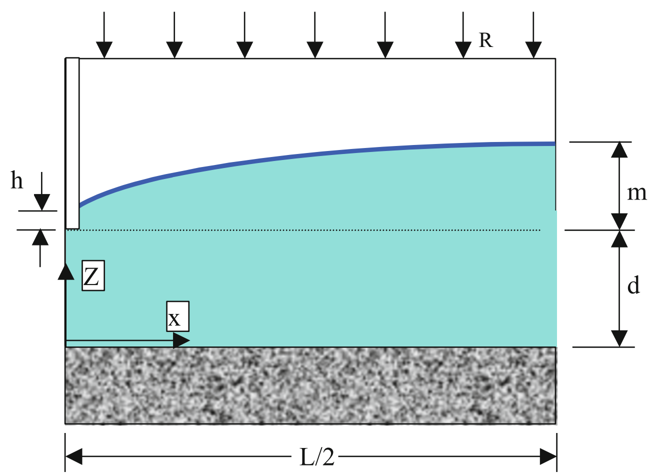


Fig. 31.8 Drainage geometry for Hooghoudt equation

Hooghoudt first derived an equation for flow to parallel ditches, which is simpler than calculating flow to subsurface drainage tubes. The parameters in the parallel ditch version of the Hooghoudt equation are shown in Fig. 31.8.

The law of conservation of mass can be used to find the total horizontal flow at any horizontal distance, x, from the drain. The flow toward the drain at distance x, q_x, is equal to the cumulative rainfall to the right of x. The direction of flow is against the positive x direction to the right.

$$\left(\frac{L}{2} - x\right)R = -q_x \tag{31.4}$$

where

q_x = flow rate per unit length of the drain at distance, x, from the drain, m^2/day,
L = drain spacing, m,
R = precipitation rate, m/day

Flow rate into the drain per unit length of drain tubing is the sum of flow into both sides of the drain

$$q_d = 2\frac{L}{2}R = LR \tag{31.5}$$

where

q_d = flow rate per unit length of drain tubing, m^2/day.

Total drain pipe flow rate is the product of flow rate per unit length and length of the drain.

$$Q = q_d{*}drain\ \ tube\ \ length\ \ (m) \tag{31.6}$$

where

Q = total drain flow, m^3/day

Example 31.2 Rainfall is 0.01 m/day, and drain spacing is 30 m. Calculate flow rate q_5, at a distance of 5 m from the drain. Find the flow rate into the drain per unit length of drain tubing.

$$q_x = \left(\frac{L}{2} - x\right)R = \left(\frac{30}{2} - 5\right)0.01\ \ m/day = 0.1\ \ m^2/day$$

The total flow rate per unit length, q_p, into the drain is

$$q_d = LR = 30{*}0.01 = 0.3\ \ m^2/day$$

The slope of the water table is the energy gradient that drives horizontal flow to the drain: the total energy at a distance x from the drain is the elevation of the water table, z, above the datum. Thus, the energy gradient along the x-axis, dH/dx, is dz/dx. The flow rate, q_x, is equal to the product of the Darcy velocity and the cross sectional area of flow, which is the elevation of the water table.

$$q_x = v_{\mathrm{Darcy}}z = -K\frac{dz}{dx}z \tag{31.7}$$

where

z = elevation of water table above impermeable layer, m.

Substitute Eq. 31.7 into Eq. 31.4.

$$\left(\frac{L}{2} - x\right)R = Kz\frac{dz}{dx} \tag{31.8}$$

Separation of variables gives:

$$\frac{L}{2}Rdx - xRdx = Kzdz \tag{31.9}$$

Integrate Eq. 31.9

$$\int_0^{L/2}\frac{L}{2}Rdx - \int_0^{L/2}xRdx = \int_{h_d+d}^{m+d}Kzdz$$

$$\left[\frac{RL}{2}x\right]_0^{L/2} - \left[\frac{R}{2}x^2\right]_0^{L/2} = \left[\frac{K}{2}z^2\right]_{h_d+d}^{m+d} \tag{31.10}$$

$$L^2 = \frac{4K\left(m^2 - h_d{}^2 + 2dm - 2dh_d\right)}{R}$$

where

m = maximum elevation of water table above drain, m.
d = elevation of drain above impermeable layer, m,
h_d = depth of water in drain, m,
K = hydraulic conductivity, m/day.

Subsurface drain tubes can be modeled with the Hooghoudt equation for open ditches (Eq. 31.10) with the assumption that h, the depth of water in the ditch, is zero. Parameters in the Hooghoudt modification for flow into subsurface drain pipes are shown in Fig. 31.9.

If h is zero, then Eq. 31.10 is written as

$$L^2 = \frac{4Km(m + 2d)}{R} \tag{31.11}$$

If the conductivity varies between layers in the soil, then the equivalent lateral conductivity must be used in Eq. 31.11. Only half of the elevation between the drain and the midpoint water table elevation, m, should be used in the equivalent conductivity calculation.

The Dupuit-Forchheimer assumption of horizontal flow is valid far from the drain, but flow converges and forms a radial

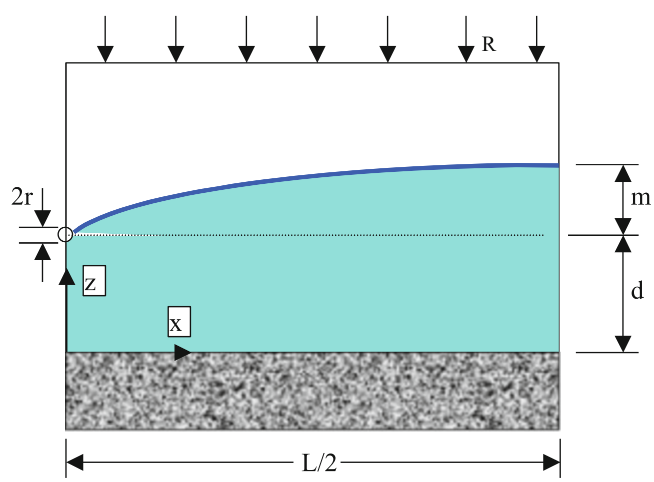


Fig. 31.9 Subsurface drain tube geometry

pattern near the drain (Fig. 31.6). Energy loss increases near the drain because the entire subsurface flow, q_p, to the drain must pass through a small area near the drain tube. Increased energy loss near the drain is accounted for in the Hooghoudt equation by reducing the depth, d, in Eq. 31.11. This reduced depth is called the equivalent depth, d_e. Equations 31.12 and 31.13 are used to calculate the equivalent depth. These equations include the effective diameter of the drain, r_e, which is less than the actual radius of the drain tube because water must move laterally along the drain into small holes in the drain. Skaggs reported that for conventional 11.4 cm OD drain tubing (4 in nominal diameter) the effective radius is 0.51 cm (DRAINMOD Manual). Selection between Eq. 31.12 or 31.13 depends on the ratio of aquifer depth (d in Fig. 31.9) to the distance between drains, L.

For $d/L < 0.3$

$$d_e = \frac{d}{1 + \frac{d}{L}\left(2.55\ln\left(\frac{d}{r_e}\right) - c\right)} \qquad c = 3.55 - 1.6\frac{d}{L} + 2\left(\frac{d}{L}\right)^2 \tag{31.12}$$

Where

r_e = the effective radius of the drain, m.

For $d/L > 0.3$

$$d_e = \frac{L}{\left(2.55\ln\left(\frac{L}{r_e}\right) - 2.93\right)} \tag{31.13}$$

Equation 31.11 is thus modified to include equivalent depth, d_e, and effective conductivity, K_e.

$$L = \sqrt{\frac{4K_e m(m + 2d_e)}{R}} \tag{31.14}$$

The procedure for calculating the water table elevation with the Hooghoudt equation is as follows:

- Specify the drain elevation above the impermeable layer, d.
- Specify the required midpoint water table elevation, m, based on the crop.
- Make an initial guess for the correct drain spacing, L.
- Solve for d_e with Eq. 31.12 or Eq. 31.13.
- Solve for a new L with Eq. 31.14.
- Calculate a new d_e by using the actual d and the new guess for L in Eq. 31.12 or Eq. 31.13: selection between Eqs. 31.12 and 31.13 for calculation of effective radius is always based on the new L and the original d.
- If the new iteration of d_e and the previous iteration have a greater than 5 % difference, then continue the iterative procedure. If not, then the solution has converged and the final L is the required drain spacing.

Example 31.3 Calculate the required spacing between drains if the farmer wants to maintain the water table at least 0.5 m below the soil surface. Yearly rainfall is 1.8 m/yr. The impermeable layer is 3.6 m below the soil surface. Drains are standard 4 in (10 cm) nominal diameter drains. Hydraulic conductivity within 1.1 m of the soil surface is 1.5 m/day, and hydraulic conductivity below the 1.1 m depth is 3.0 m/day. The drains are installed at 1.1 m depth below the soil surface. Effective drain radius for a 4 in (10 cm) nominal diameter drain is 0.51 cm = 0.0051 m. Drainage parameters are shown in Fig. 31.10.

$$\text{Average rainfall} \quad (1.8\ \text{m/year})/(365\ \text{days/year}) = 0.005\ \text{m/day}.$$

The drains are 1.1 m below the soil surface. Thus, the water table elevation above the drain, m, is

$$\text{m} = 1.1\ \text{m} - 0.5\ \text{m} = 0.6\ \text{m}.$$

The elevation of the drain, d, above the impermeable layer is

$$\text{d} = 3.6\ \text{m} - 1.1\ \text{m} = 2.5\ \text{m}.$$

Find the effective conductivity with Eq. 7.12. Let D_1 be equal to half of the elevation, m, above the drain, 0.3 m.

$$K_e = \frac{K_1 D_1 + K_2 D_2}{D_1 + D_2} = \frac{1.5*0.3 + 3.0*2.5}{0.3 + 2.5} = 2.84\ \ m/day$$

Make an initial guess of L = 40 m and solve for d_e with Eq. 31.12 because d/L is much less than 0.3.

$$c = 3.55 - 1.6\frac{d}{L} + 2\left(\frac{d}{L}\right)^2 = 3.55 - 1.6\frac{2.5}{40} + 2\left(\frac{2.5}{40}\right)^2 = 3.46$$

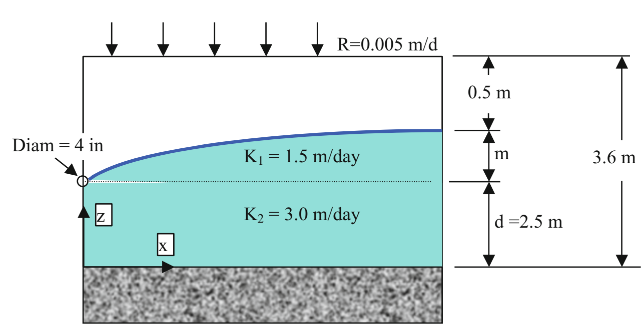


Fig. 31.10 Drainage dimensions for Example 31.3

$$d_e = \frac{d}{1 + \frac{d}{L}\left(2.55\ln\left(\frac{d}{r_e}\right) - c\right)}$$
$$= \frac{2.5}{1 + \frac{2.5}{40}\left(2.55\ln\left(\frac{2.5}{0.0051}\right) - 3.46\right)} = 1.41$$

Solve for L with Eq. 9.11

$$L = \sqrt{\frac{4K_e m(m + 2d_e)}{R}} = \sqrt{\frac{4(2.84)(0.6)(0.6 + 2(1.41))}{0.005}}$$
$$= 68.3 \ m$$

The new d_e with L = 68.3 and d = 2.5 is 1.72 m.

Solve for L with the new equivalent depth

$$L = \sqrt{\frac{4K_e m(m + 2d_e)}{R}} = \sqrt{\frac{4(2.84)(0.6)(0.6 + 2(1.72))}{0.005}}$$
$$= 74.3 \ m$$

The new d_e with L = 74.3 and d = 2.5 is 1.77 m.

Solve for L with the new equivalent depth

$$L = \sqrt{\frac{4K_e m(m + 2d_e)}{R}} = \sqrt{\frac{4(2.84)(0.6)(0.6 + 2(1.77))}{0.005}}$$
$$= 75.1 \ m$$

The change in d_e was less than 5 % (1.72–1.77 m) so the solution has converged and the final drain spacing, L, is 75 m.

If the conductivity above the drain is different from the conductivity below the drain (as in Example 31.3), then Eq. 31.14 can be modified to account for the varying conductivity.

$$L = \sqrt{\frac{4K_1 m^2}{R} + \frac{8K_2 m d_e}{R}} \quad (31.15)$$

where

K_1 = conductivity above the drain, m/day,
K_2 = conductivity below the drain, m/day.

Example 31.4 Redo Example 31.3 with Eq. 31.15. Let initial d_e = 1.41 (initial L = 40).

Solve for L

$$L = \sqrt{\frac{4K_1 m^2}{R} + \frac{8K_2 m d_e}{R}}$$
$$= \sqrt{\frac{4(1.5)(0.6^2)}{0.005} + \frac{8(3.0)(0.6)(1.41)}{0.005}} = 67.1 \ m$$

After several iterations, the solution converges to 74 m, approximately equal to the 75 m calculated in Example 31.3.

$$L = \sqrt{\frac{4K_1 m^2}{R} + \frac{8K_2 m d_e}{R}}$$
$$= \sqrt{\frac{4(1.5)(0.6^2)}{0.005} + \frac{8(3.0)(0.6)(1.76)}{0.005}} = 74.2 \ m.$$

Transient Drainage Models

This section presents two different models of transient water table response to storm events and dry periods between storms: the Bower and Van Schilfegaarde (1963) modification of the Hooghoudt equation and the United States Bureau of Reclamation (USBR) transient drainage model. The Bower and Von Schilfegaarde model is used in the DRAINMOD model.

Bower and Van Schilfegaarde rearranged Eq. 31.14, and solved for rainfall based on a known drain depth and spacing. Rainfall is replaced by unit flow rate to the drain, q_d, divided by L.

$$R = \frac{4K_e m(m + 2d_e)}{L^2} \quad (31.16)$$

$$\frac{q_d}{L} = \frac{4K_e m(m + 2d_e)}{L^2} \qquad q_d = \frac{4K_e m(m + 2d_e)}{L} \quad (31.17)$$

Bower and Van Schilfegaard added a constant, C, to account for the difference between the assumption of uniform infiltration in the Hooghoudt equation and the varying infiltration observed in actual drainage response to storm and irrigation events.

$$q_d = \frac{4K_e m(m + 2d_e)}{CL} \quad (31.18)$$

Even though the C term is probably necessary for an accurate solution, the DRAINMOD model assumes that C is 1.0 because no values for the C term for different soils are available in the literature. Therefore, C is dropped from the equation.

For different hydraulic conductivities above and below the drain, Eq. 31.18 becomes:

$$q_d = \frac{4K_1 m^2}{L} + \frac{8K_2 m d_e}{L} \quad (31.19)$$

The depth of drainage in the field during time Δt is

$$d_{drainage} = \frac{q_d}{L}\Delta t \quad (31.20)$$

Table 31.2 Two-week water table depths

Time (days)	Initial m (m)	q_d (m^2/day)	$d_{drainage}$ (m)	Δm	m_{final} (m)	DTWT (m)
0	0.9	0.58	0.0078	– 0.078	0.82	0.2
1	0.82	0.52	0.0070	– 0.070	0.75	0.28
2	0.75	0.47	0.0064	– 0.064	0.69	0.35
3	0.69	0.43	0.0058	– 0.058	0.63	0.41

where

Δt = time, days.
$d_{drainage}$ = average depth of drainage, m.

For the case of two-dimensional drainage geometry, the ratio of the decline of the midpoint water table elevation, d_{mid} to the drainage depth, $d_{drainage}$, can be defined as the specific yield.

$$SY_{mid} = -\frac{d_{drainage}}{\Delta m} \tag{31.21}$$

where

SY_{mid} = specific yield during decrease in midpoint water table elevation,
Δm = change in midpoint water table elevation, m.

Example 31.5 For the drainage design in Examples 31.3 and 31.4 (d = 2.5 m and L = 74.2 m), calculate the decrease in the midpoint water table elevation over time. The initial depth of the water table at the midpoint between drains is 0.2 m. The specific yield, SY_{mid}, is 10 %. Assume that C = 1.0 (equal drainage over the length, L, between drains).

Note that the effective depth, d_e, is not a function of water table elevation, m, so d_e is constant if the depth and spacing of the drains are fixed. The drain depth is 1.1 m, and the midpoint water table is 0.2 m below the soil surface.

$$d_e = 1.76 \text{ m} \quad m = 1.1 - 0.2 = 0.9 \text{ m}.$$

Find the volume drained (per unit length of drain) during the first day

$$q_d = \frac{4K_1m^2}{L} + \frac{8K_2md_e}{L} = \frac{4(1.5)(0.9^2)}{74.2} + \frac{8(3.0)(0.9)(1.76)}{74.2} = 0.58 \ \ m^2/day$$

The depth drained during the first day, decline in elevation and final height are calculated.

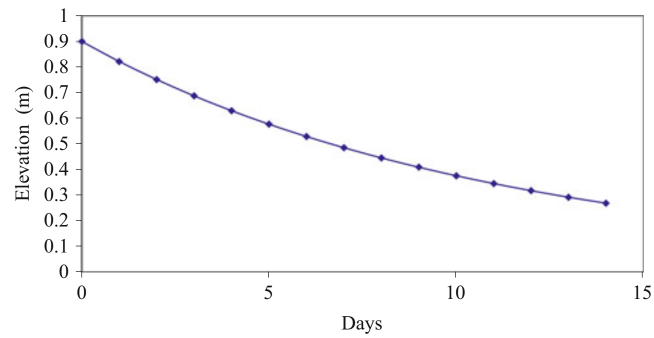


Fig. 31.11 Midpoint water table elevation versus time

$$d_{drainage} = \frac{q_d}{L}t = \frac{0.58}{74.1}(1 \ \ day) = 0.0078 \ \ m$$

$$\Delta m = -\frac{d_{drainage}}{SY_m} = -\frac{0.0078}{0.1} = -0.078 \ \ m = -7.8 \ \ cm$$

$$m_{final} = m_0 + \Delta m = 0.9 \text{ m} - 0.078 \text{ m} = 0.822 \text{ m}$$

The depth of the water table below the soil surface after the first day is 1.1 – 0.822 = 0.278 m. The process is repeated with 0.822 m as the initial height of the water table above the drain at the beginning of the second day. Based on the SEW-30 rule, the crop yield would decrease on days 0 and 1, but there would be no yield decrease on the following day with the depth to the water table at 0.349 m. Water table depths and other parameters for a two-week period are shown in Table 31.2.

The water table height, m, is shown for a two-week period in Fig. 31.11.

The USBR developed large irrigation and water resources projects in the western United States. The assumption of steady rainfall was not appropriate in the Western climate with periodic irrigations and storm events. In the 1950s, the USBR developed drainage equations and charts that allowed engineers to model the transient behavior of the water table and resultant flow to drains during the year. The USBR calculations were observed to generally fall within 10 % of observed field data.

The USBR method calculates the ratio of final and initial midpoint water table elevations, m/m_0, based on hydraulic conductivity, time, water table depth, specific yield and drain spacing. Parameters for the USBR equations are

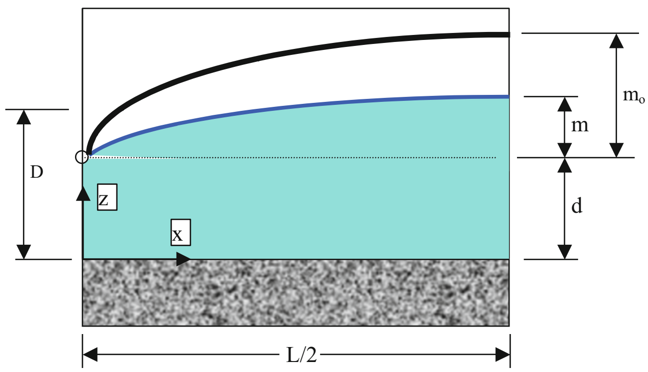


Fig. 31.12 Bureau of reclamation drainage parameters

shown in Fig. 31.12. Equation 31.22 is a regression fit of the "Drain above barrier" curve in Fig. 5.4 from the USBR Drainage Manual (http://www.usbr.gov/pmts/wquality_land/DrainMan.pdf). This equation calculates the ratio of the initial and final midpoint water table elevation over a specified period. In order to remain consistent with the previous derivation, m and m_0 are used instead of the USBR parameters, y and y_0.

$$\frac{m}{m_0} = 10^{0.01569 - 4.335\frac{K_e D t}{SY_{FC-S} L^2}} \tag{31.22}$$

where

K_e = effective hydraulic conductivity, m/day,
m_0 = initial depth of the water table above the drain, m,
m = final depth of the water table above the drain, same as m_{final}, m,
D = weighted average elevation of the water table ($d_e + m_0/2$), m,
SY_{FC-S} = specific yield defined as difference between field capacity and saturation,
t = time, days,
L = distance between drains, m.

The depth, D, in Eq. 31.22 is the average elevation of the water table above the impermeable layer and is the sum of the equivalent depth and m/2.

$$D = d_e + m_0/2 \tag{31.23}$$

Equation 31.22 is valid when the ratio of d/m_0 is greater than 0.8. When d/m_0 is less than 0.1, drains near impermeable layer, then Eq. 31.24 is used. For in-between values, $0.1 < d/m_0 < 0.8$, an approximation is made between the two equations.

$$\frac{Z}{H} = f\left(\frac{K_e H t}{SY_{FC-S} L^2}\right) \tag{31.24}$$

Table 31.3 Deep percolation percentage from surface irrigation as a function of soil infiltration rate (Table 5.1 in USBR drainage manual)

Infiltration rate (mm/hr)	Deep percolation percent (f_{dp}) 100 %	Infiltration rate (mm/hr)	Deep percolation percent (f_{dp}) (100 %)
1.3	3	25	20
2.5	5	32	22
5	8	38	24
7.6	10	51	28
10	12	64	31
13	14	76	33
15	16	102	37
20	18		

where

H = Initial midpoint water table elevation above the impermeable layer.
Z = Final midpoint water table elevation above the impermeable layer.

In most irrigated areas, irrigation is the major water input to the soil. Because of poor irrigation efficiency or because of the need to leach salts below the root zone, some of the water applied during irrigation leaches below the root zone and contributes to build up of the water table. This water is called deep percolation. Only the wasted water lost below the root zone to deep percolation is added to the USBR water table model (unlike the DRAINMOD model). Average values of deep percolation as a function of soil infiltration rate during surface irrigation events were established by the USBR (Table 31.3) through field evaluations.

Deep percolation percentage increases with infiltration rate because more water is wasted to deep percolation at the upper end of fields while water advances to the ends of the field. On the other hand, excessive water can be wasted to runoff from a field with a very low infiltration rate; however, this fact is not reflected in Table 31.3 because it would run off the farm and not add to the water table. The depth of deep percolation during an irrigation event is

$$d_{dp-i} = i(1 - f_{RO})(f_{dp}) \tag{31.25}$$

where

i = gross depth of irrigation, m,
d_{dp-i} = depth of deep percolation from irrigation event, m,
f_{RO} = fraction of runoff,
f_{dp} = fraction of deep percolation from Table 31.3.

The rise in the midpoint water table elevation in response to deep percolation from irrigation is

$$\Delta m = \frac{d_{d\,p-i}}{SY_{FC=S}} \qquad (31.26)$$

Specific yield, SY_{FC-S}, as defined by the USBR is the difference between the saturated water content and field capacity and is estimated based on hydraulic conductivity with a graph in the USBR Drainage Manual. An equation that approximates the graph is on the *USBR relationships* page in *Chapter 31 Drainage*: SY =10^(−0.175*(LOG10(A3))^2 + 0.7214*LOG10(A3) + 0.7188)

The USBR also developed a relationship between rainfall and infiltrated water as a function of magnitude of precipitation. This equation is also included in the *USBR relationships* worksheet:=IF(A6>2.5,IF(A6<80,−0.007673*A6^2+1.1755*A6−2.2401,42.5),0).

The change in depth of the water table due to deep percolation from rainfall is the calculated in the same way as for irrigation (Eq. 31.26)

$$\Delta m = \frac{d_{d\,p-r}}{SY_{FC=S}} \qquad (31.27)$$

where

d_{dp-r} = deep percolation due to rainfall, m.

Example 31.6 Use the USBR equations to calculate the water table depth vs. time for a farm that is surface irrigated with an average irrigation depth per application of 180 mm. Average surface runoff during irrigation is 25 %. Soils have a silty clay loam texture with an infiltration rate of 12.7 mm/hr (1.27 cm/hr, 0.3 m/day with hydraulic conductivity of the soil equal to 0.3 m/day). Average precipitation per rainfall event is 31 mm. The depth from the soil surface to the impermeable layer is 5 m. The equivalent drain radius, r_e, is 0.02 m. The drain depth is 1.2 m. Drain spacing is 40 m. Minimum acceptable depth to the water table is 0.48 m.

The following irrigation, snowmelt, and rainfall events are expected based on historical records.

Snowmelt: April 22
Irrigation dates: May 28, Jun 6, Jun 20, Jul 2, Jul 14, Jul 26, Aug 10, Aug 22.
Rainfall events: Nov 1, Dec 15, Mar 1, Mar 15

Seasonal snowmelt has the same depth of deep percolation as one irrigation event.

Calculate the equivalent depth, d_e.

Calculate depth, d, the distance from the drain to the impermeable layer

$$d = 5\ \text{m} - 1.2\ \text{m} = 3.8\ \text{m}.$$

The ratio of depth to distance, d/L = 3.8/40 = 0.10: use Eq. 31.12 because d/L < 0.31. The effective radius of the drain is 0.02 m.

$$c = 3.55 - 1.6\frac{3.8}{40} + 2\left(\frac{3.8}{40}\right)^2 = 3.41$$

$$d_e = \frac{3.8}{1 + \frac{3.8}{40}\left(2.55\ln\left(\frac{3.8}{0.0051}\right) - 3.41\right)} = 1.95\ m$$

Calculate the increase in the water table elevation during irrigation events.

The fraction of deep percolation for soil with infiltration rate 12.7 mm/hr is 0.14 (Table 31.3).

$$d_{d\,p-i} = i^*(1 - f_{RO})^*\left(f_{d\,p}\right) = 0.18\ \text{m}^*(1 - 0.25)(0.14) = 0.0189\ \text{m}.$$

Specific yield, SY_{FC-S}, is calculated in the *USBR relationships* worksheet. For K = 1.27 cm/hr, SY_{FC-S} = 7 %. Change in water table elevation during each irrigation event is

$$\Delta m = \frac{d_{d\,p-i}}{SY_{FC=S}} = \frac{0.0189}{0.07} = 0.27\ m$$

Thus, the midpoint water table elevation increases by 0.27 m during irrigation events.

Calculate the increase in the water table elevation during rainfall events.

If rainfall depth is 31 mm, then infiltration (deep percolation depth) = 25 mm (*USBR relationships* worksheet).

Deep percolation depth = 0.025 m.

$$\Delta m = \frac{d_{d\,p-r}}{SY_{FC=S}} = \frac{0.025}{0.07} = 0.357\ m$$

Calculate drainage position vs. *time of year. Start with August 22 as the first day.*

Because the water table height is typically a maximum at the end of the irrigation season, start the simulation with the water table at the maximum acceptable height at the end of the growing season. The minimum acceptable depth to the water table (from soil surface) is 0.48 m.

Calculate initial m_0.

$$1.2\text{m} - 0.48\text{m} = 0.72\text{m}.$$

Thus, on the day just after the last irrigation of the growing season, August 22, the water table elevation above the drains, m_0 = 0.72 m.

Calculate the initial D

$$D = d_e + m_0/2 = 1.95 + 0.72/2 = 2.31\text{m}$$

Calculate

$$\frac{K_e Dt}{SY_{FC-S}L^2} = \frac{0.30*2.31*1}{0.07*40^2} = 0.00629$$

Calculate

$$\frac{m}{m_0} = 10^{0.01569-4.335(0.00629)} = 0.974$$

Calculate water table depth after 1 day: $m = (m/m_0)(m_0) = (0.974)(0.72\ m) = 0.701\ m$.

The process is repeated for each day of the year. For days with water table buildup such as rainfall and irrigation days, the deep percolation is added instantaneously at the beginning of the day, and the calculation for change in water table height is conducted with the increased water table elevation. For example, on November 1, the water table elevation before the storm is 0.157 m, and the water table elevation after the storm is 0.154 + 0.357 = 0.511 m. The change in water table elevation is based on the 0.511 m water table elevation.

$$D = D + m_0/2 = 1.95 + 0.511/2 = 2.208m$$

Calculate

$$\frac{K_e Dt}{SY_{FC-S}L^2} = \frac{0.30*2.208*1}{0.07*40^2} = 0.00601$$

Calculate

$$\frac{m}{m_0} = 10^{0.01569-4.335(0.00601)} = 0.976$$

$$m = m/m_0*m_0 = 0.976*0.511 = 0.499m.$$

Water table elevation fluctuation during the year is shown in Fig. 31.13.

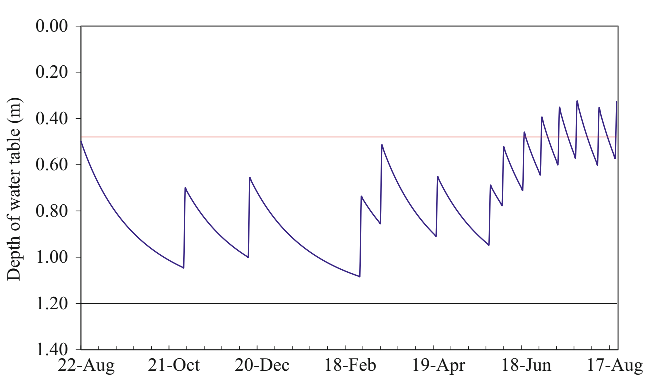


Fig. 31.13 Water table elevation versus time for L = 40 m and drain depth = 1.2 m

The water table (Fig. 31.18) elevation at the end of the growing season exceeds the criteria of 0.48 m distance from the soil surface during the growing season for 26 days, and the water table is higher at the end of the year than at the beginning of the year. Thus, the drains are spaced too far apart.

Economic Analysis

The following parameters can be incorporated into engineering economic analysis for drainage systems:

1. Capital investment
2. Maintenance
3. Crop yield or improvement in crop yield with the new drainage system
4. Environmental cost or benefit

In the following example, the two primary drainage installation methods are compared: trenching and pulling. Trenched drains are placed in a trench with a gravel pack around the drain tube. The gravel pack increases the conductivity of the region around the drain and as a result, the effective radius is increased. The increased effective radius increases the equivalent depth (Eq. 31.12) and makes it possible to have wider drain spacing. Pulled drains are installed by a large tractor that pulls a shank through the field. The drain tube is threaded down the shank and inserted in the soil at the shank depth. Soil is often compressed around the shank, and there is no gravel pack around the drain, so the effective radius is small. The maximum depth of pulled drain installation is 1.2 m. The current practice is to pull drains into the soil because trenched drains are more expensive and are only installed when the required drain depth is greater than is possible to install by pulling.

Example 31.7 Select between the following three drainage alternatives, and determine the correct spacing for each alternative. Use soil and weather parameters from Eq. 31.6. Use 4 inch nominal diameter drain tubing. Effective radius of the tubing for pulled in drains is 0.02 m and is 0.183 m for trenched drains with gravel pack. Place the USBR equations into an Excel spreadsheet in order to solve the problem.

Pulled drain at 1.2 m depth
Trenched drain at 2 m depth
Trenched drain at 2.4 m depth.

Design criteria:

The depth to the water table may not be less than 48 cm for more than 14 days during the growing season. The water

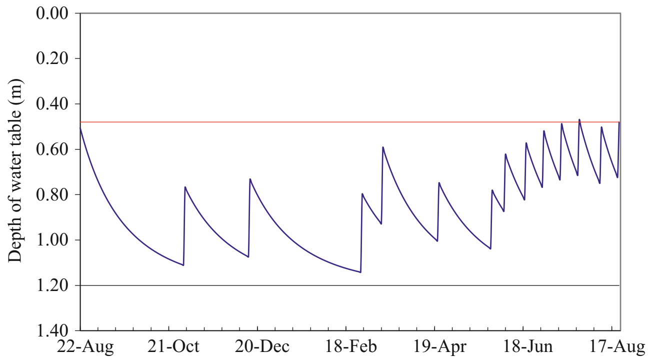


Fig. 31.14 Water table elevations versus time for L = 36.5 m and drain depth = 1.2 m

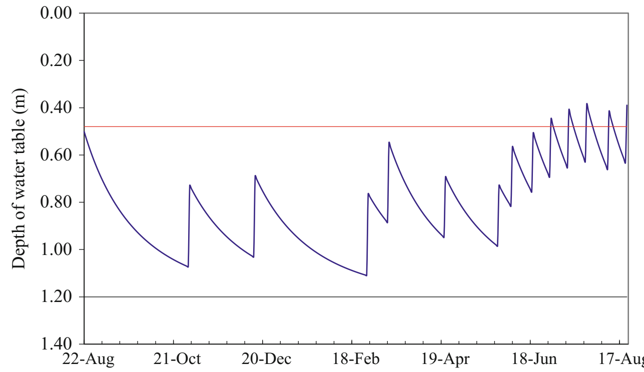


Fig. 31.15 Water table elevations versus time for L = 38.7 m and drain depth = 1.2 m

table elevation immediately following the last irrigation of the growing season cannot be higher than the water table elevation on the same day during the previous year.

Material costs and maintenance:

The cost of the collector drains and drainage structures is $200/ha for both trenched drains and pulled drains. For this analysis, assume that the cost of drain maintenance is equal to the cost of field maintenance without drainage so it is not necessary to include it in the analysis. The cost of the 4 in drain tubing is $3.00/m.

Economics

The price of corn is $2.64 per bushel. Without drainage, the farmer has an average yield of 250 bushels per hectare. With a drainage system, the expected average corn yield is 320 bushels per hectare. The required rate of return is 10 % and project life is 20 years.

Trenching costs

Trenched drains: $4.84/m of drain tube length for depths less than 2 m. There is an additional cost of $.10/cm for trenching deeper than 2 m. Thus, there is no cost saving for trenching shallower than 2 m. The gravel pack around the drain is $1.84/m.

Pulled drains: The cost of pulling in the drains at 1.2 m depth is $1.69/m.

Pulled drain at 1.2 m depth.

The 40 m drain spacing was not satisfactory (Example 9.5) because the final water table depth at the end of the period (August 22) was less than the initial depth.

An iterative solution or Goal Seek in Excel can be used to find the correct solution, L = 36.5 m, where the water table elevation, m_0, is 0.72 m at the end of the growing season. At a drain spacing of 36.5 m, the water table only rises to within 48 cm of the soil surface for 1 day, July 26 (Fig. 31.14).

Goal Seek was also used to find a drain spacing that results in the water table not entering the root zone more than 14 days during the growing season, L = 38.7 (Fig. 31.15). The water table elevation at the end of the year is 0.81 m. Although the criteria for water not entering the root zone for more than 14 days is met, the first criterion, that the water table is the same elevation at the end of the year at the beginning of the year is not satisfied. Thus, a drain spacing of 36.5 m is selected in order to satisfy both criteria.

Trenched drain at 2 m depth

Desired water table depth is 0.48 m. Thus, initial m_0 is 2 m − 0.48 m = 1.52 m.

$$d = 5\text{m} - 2\text{m} = 3\text{m}.$$

Test a drain spacing of 50 m.

The ratio of depth to distance, d/L = 3/50 < 0.31. The effective radius of the drain is 0.183 m.

$$c = 3.55 - 1.6\frac{3}{50} + 2\left(\frac{3}{50}\right)^2 = 3.46$$

$$d_e = \frac{3}{1 + \frac{3}{50}\left(2.55\ln\left(\frac{3}{0.183}\right) - 3.46\right)} = 2.46\ m$$

Example calculation of water table elevation at the end of the day on August 23:

Initial m_0 = 1.52 m

$$D = d_e + m_0/2 = 2.46 + 1.52/2 = 3.22\text{ m}$$

Calculate

$$\frac{K_e D t}{SY_{FC-S} L^2} = \frac{0.30*3.22*1}{0.07*50^2} = 0.00561$$

Calculate

$$\frac{m}{m_0} = 10^{0.01569 - 4.335(0.00561)} = 0.980$$

Calculate water table depth at the end of the day.

$$m = m/m_0 * m_0 = 0.980*1.52 = 1.49\text{m}.$$

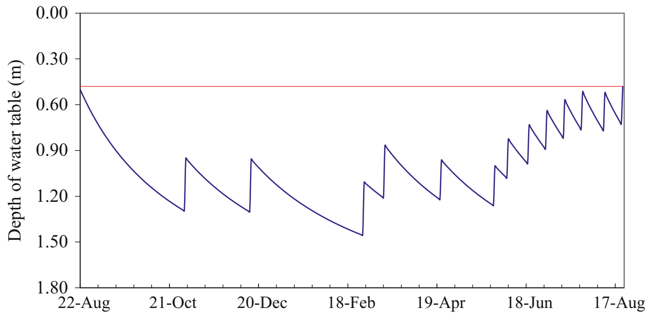


Fig. 31.16 Water table elevations versus time for L = 52.6 m and drain depth = 2 m

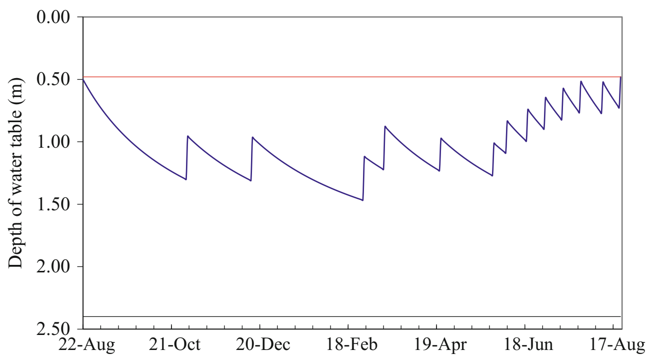


Fig. 31.17 Water table elevations versus time for L = 54.1 m and drain depth = 2.4 m

Goal seek was used to find the optimal distance, L = 52.6 m (Fig. 31.16) based on the criteria that the water table at the end of the period not exceed the water table depth at the beginning of the period.

Goal Seek was used to find the optimal distance for the 2.4 m drain depth: 54.1 m (Fig. 31.17).

Economic analysis

Economic analysis of drainage alternatives is performed in two steps. First, the drainage alternative with the lowest cost is selected. Second, a present value cost/benefit analysis of the lowest cost drainage alternative is made. If benefits are greater than costs, then the project is approved.

Comparison of the three drainage alternatives:

Costs are calculated on a per hectare basis. The length of drain tubing installed per hectare (m/ha) is the area (m^2/hectare) divided by the drain tube spacing (m). For the drains installed at 1.2 m depth, the spacing L is 36.5 m.

$$(\mathrm{m/ha}) = 10{,}000/36.5 = 274\mathrm{m}$$

The cost per m of drain tubing is the sum of materials and installation costs:

$$\begin{aligned}\$/\mathrm{m} &= \mathrm{pullingcost} + \mathrm{draintubecost} = \$1.69/\mathrm{m} + \$3.00/\mathrm{m}\\ &= \$4.69/\mathrm{m}\end{aligned}$$

The cost per ha is the product of m/ha and $/m

$$\$/\mathrm{ha} = 274{*}4.69 = \$1{,}285/\mathrm{ha}$$

The installation cost of the other two alternatives was calculated in the same manner except that the cost of gravel was added for the trenched alternatives (Table 31.4).

The lowest cost alternative is the pulled drain at 1.2 m depth.

The present value of installation of the drain tubes ($1,285) and collector drains and structures ($200/ha) is

$$\$1{,}285/\mathrm{ha} + \$200/\mathrm{ha} = \$1{,}485$$

The benefit of the drainage system is increased yield. The yield without drainage is 250 bushels/ha and the yield with drainage is 320 bushels/ha. Thus, the increase in yield is 320 − 250 = 70 bushels/ha. The amount of money received each year because of drainage is the product of the increased yield due to drainage and the crop price:

$$70\mathrm{bushels/ha/yr}{*}\$2.64/\mathrm{bushel} = \$184.80/\mathrm{ha/year}$$

The Excel present value formula, PV, can be used to find the present value of 20 annual payments of $184.80/yr at 10 % interest rate: = PV(0.1, 20, 184.8). The calculated present value with the PV formula is $1,573/ha.

The present value of the benefits of drainage ($1,573) are higher than the cost ($1,485/ha) at a 10 % rate of return. Thus, the decision is made to invest in a drainage system.

The value of a crop will have a significant influence on whether the decision is made to invest in drainage; thus, the NRCS states that drain spacings closer than 40 ft (12 m) can be justified economically for high value crops. Another key factor in the economics of drainage design is possible work at the discharge end. If a channel must be deepened in order to carry the drainage flow from a subsurface drainage network, then that would add cost to the system.

Streamtube Model

The previous models presented in this chapter modeled the two-dimensional flow geometry below the water table with one-dimensional equations. Finite element, finite difference, or analytic solutions of the La Place equation are needed to model the flow in two dimensions. Kirkham developed a two-dimensional analytic solution of the La Place equation that calculates the position of the water table over time. He simplified the geometry by assuming that the soil above the drain has infinite conductivity. Thus, he only modeled the rectangular region between the impermeable layer and the drain. He assumed that the water above the drain flowed

Table 31.4 Costs of drainage alternatives

Alternative.	Spacing, L (m)	Length (m/ha)	Pipe cost ($/m)	Installation ($/m)	Gravel ($/m)	Total cost ($/ha)
Cost – pulling 1.2 m	36.5	274	3.00	1.69	–	1,284.93
Cost – 2 m trench	52.6	190	3.00	4.84	1.84	1,840.30
Cost – 2.4 m trench	54.1	185	3.00	8.84	1.84	2,528.65

vertically downward to the drain elevation in streamtubes. The Kirkham (1958) solution to the LaPlace equation calculates the elevation of the water table over the drain vs. distance away from the drain for a given steady state recharge rate, R.

$$h = \frac{LR}{K\pi}\left[\ln\frac{\sin(\pi x/L)}{\sin(\pi r/L)} + \sum_{m=1}^{\infty}\frac{1}{m}\left(\cos\frac{m\pi r}{L/2} - \cos\frac{m\pi x}{L/2}\right)\frac{e^{-m\pi d/(L/2)}}{\sinh(m\pi d/(L/2))}\right] \tag{31.28}$$

where

h = height of water table above drain at any distance x from the drain, m,
r = effective radius of the drain, m,
m = the number of terms in the series solution.

The formula inside the parenthesis is labeled as F(x,0) by Kirkham and is solely dependant on the geometry of the region between the drain and the midpoint between drains.

A Visual Basic program was written in the *Chapter 31 Drainage* program that calculates the Kirkham F. Although the sinh(z) function is not available in VBA, it is equal to (exp(z) − exp(−z))/2, as in the following code.

```
Function Kirkham_F(num_series, x, L, re, d) As Single
    Dim sum, m, f
    f = Log(Sin(3.14159 * x / (L)) / Sin(3.14159 * re / (L)))
    sum = 0
    For m = 1 To num_series
      sum = sum + (1 / m) * (Cos(m * 3.14159 * re / (L / 2)) - Cos(m * 3.14159 * x / (L / 2))) * (Exp(-m * 3.14159 * d / (L / 2)) / (Exp(m * 3.14159 * d / (L / 2)) - Exp(-m * 3.14159 * d / (L / 2))))
    Next m
    f = (f + sum) / 3.14159
    Kirkham_F = f
End Function
```

Kirkham developed a method based on the streamtube model to calculate the rate of fall of the water table at each distance between drains.

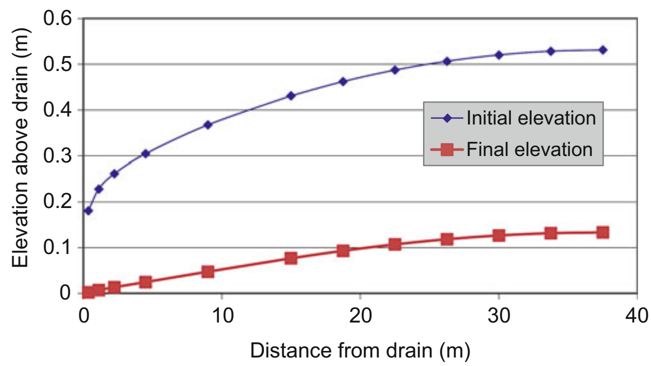


Fig. 31.18 Initial and final water table elevations versus distance from the drain calculated with Kirkham solution (0 and 14 days after equilibrium)

$$\frac{db}{dt} = \frac{-K/f}{1 + LF/b} \tag{31.29}$$

where b is the elevation of the water table at any distance x and *f* is water table elevation at that distance over specific yield (b/SY).

Example 31.8 Calculate the water table elevation vs. distance from the drain using the parameters in Example 31.3. Calculate the rate of fall of the water table specified in Example 31.8. Compare the rate of fall of the midpoint water table elevation to the Bower and Von Schilfegaarde solution in Example 31.5. For the Bower and Von Schilfegaarde solution, assume that the initial water table elevation is 0.53 m. Plot the rate of fall at each point between drains. Plot the initial and final water table positions. Show a sample calculation at 0.375 m from the drain during the first time step.

Distance between drains, L	75	m
Elevation of drain above impermeable layer, d	2.5	m
Effective drain radius, re	0.0051	m
Rainfall rate	0.005	m/d
Number in series approximation, m	20	
Hydraulic conductivity	2.84	m/day

The Kirkham water table elevation vs. distance from the drain is shown in Fig. 31.18 (blue line is initial elevation). At the same steady state recharge rate as in Example 31.3, the elevation at the midpoint is 0.53 m, which is approximately

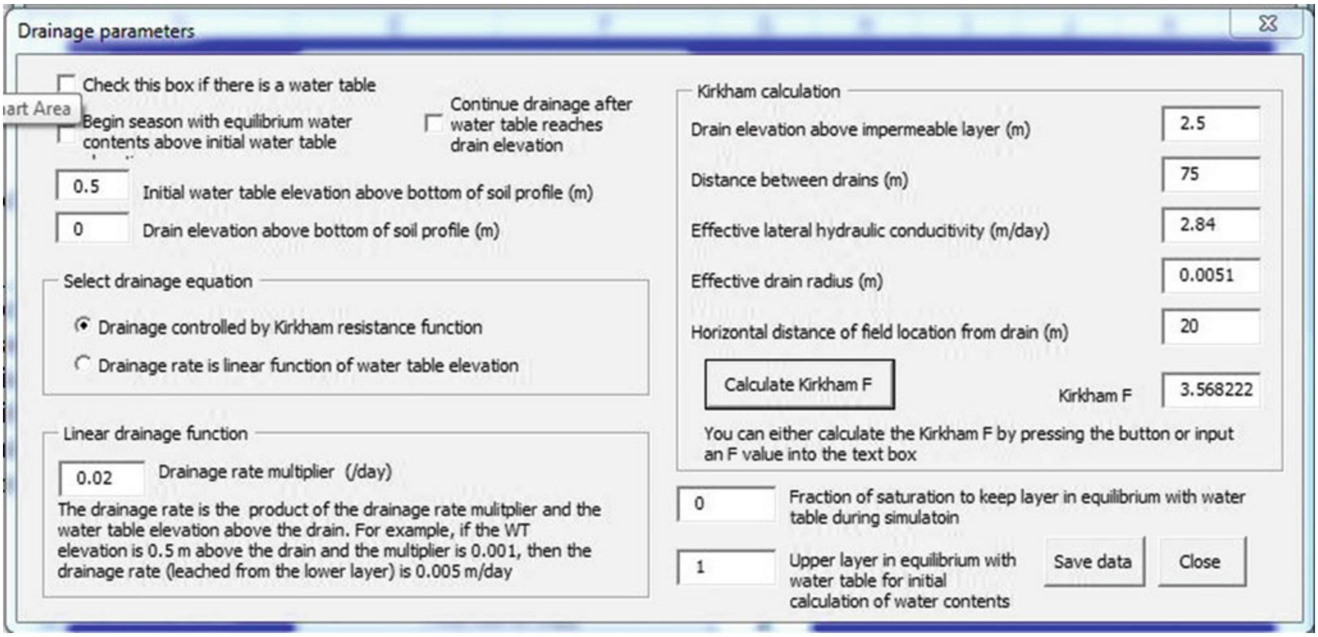


Fig. 31.19 *WINDS* model *Drainage* dialog box for drain parameters in Example 31.8

10 % lower than the elevation calculated with the Hooghoudt equation; however, there is close agreement between the Kirkham and Bower and Von Schilfegaarde (Example 31.3) solutions for the change in midpoint water table elevations over time.

The F value is calculated as 1.37 at 0.375 m from the drain. The initial water table elevation is 0.181 m at 0.375 m from the drain. The new water table elevation is calculated as follows.

$$b_{final} = b_{initial} + \frac{db}{dt} = b_{initial} + \frac{-K/f}{1 + LF/b}$$
$$= 0.181 + \frac{-2.84/0.1}{1 + 75{*}1.37/0.181} = 0.131\ m$$

The calculations for each time step and distance are made in the Streamtube worksheet. The initial and final elevations of the water table are shown in Fig. 31.18.

The benefit of the Kirkham model over the Bower and van Schilfgaarde model is that the water table as a function of distance from the drain can be evaluated for its effect on water, nitrogen, and salinity, and crop growth. There are significant trends vs. distance from the drain in many fields, which is why farmers often decide to install drains between drains. For example, salinity is often high in the region between drains but low near the drains. The Kirkham model allows the analysis of a number of points between drains in order to see the effect of drain spacing on crop growth and other processes. This technique could be used to improve best management practices for reducing nitrate output from drains and into surface waters.

The *WINDS* model uses the Kirkham algorithm at each field position to compute water table position vs. time. For example, the parameters from Example 31.8 can be entered into the *Drainage* dialog box (Fig. 31.19). In this case, the field position is 20 m from the drain, and the drain is at the bottom of the soil specified soil profile, which is 1.4 m deep, a typical drain depth. The program calculates the Kirkham F and sets up the model to run simulations of water table elevation vs. time at the specified location.

In the following examples, the rainfed clay soil example from Chap. 29 (Example 29.5) will be modified for the presence of a water table. Because the soil is clay, and there is little change in water content with matric potential, the "fraction of saturation to keep layer in equilibrium with water table during simulation" value (Chap. 28) must be high (0.95); otherwise, all layers will remain in equilibrium with the water table during the entire simulation. This would result in evaporation (water loss) from the upper layers rapidly dropping the water table when in reality the upper layers would disconnect hydraulically from the water table.

Because clay has an extremely high field capacity, almost saturation, it is interesting to evaluate whether an underlying water table would reduce water stress during dry periods. It would seem that the water remains in the soil whether there is an underlying water table and restriction on downward water movement or not. Evaluation with the *WINDS* model indicates that with or without (Fig. 31.20) water harvesting,

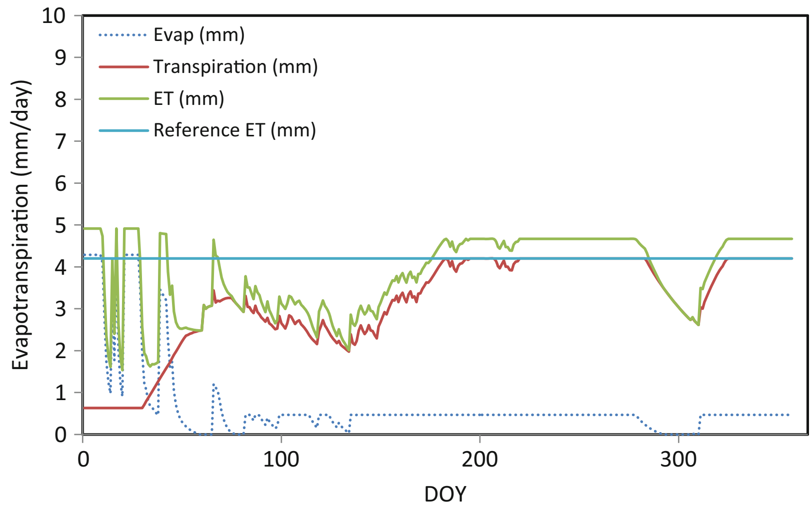


Fig. 31.20 *WINDS* model evapotranspiration for drain parameters in Example 31.8 and soil and hydrology parameters (without water harvesting) from Example 29.5

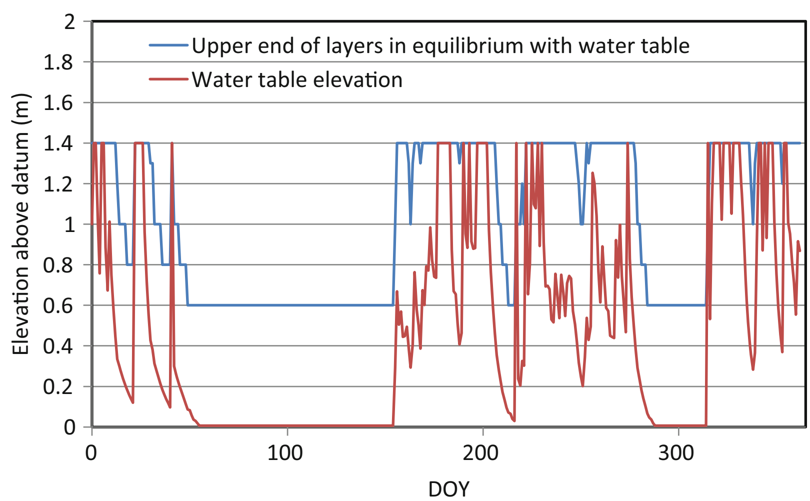


Fig. 31.21 *WINDS* model *Drainage* dialog box for drain parameters in Example 31.8

the stress is reduced with an underlying water table (compare Fig. 29.24 to Fig. 31.20).

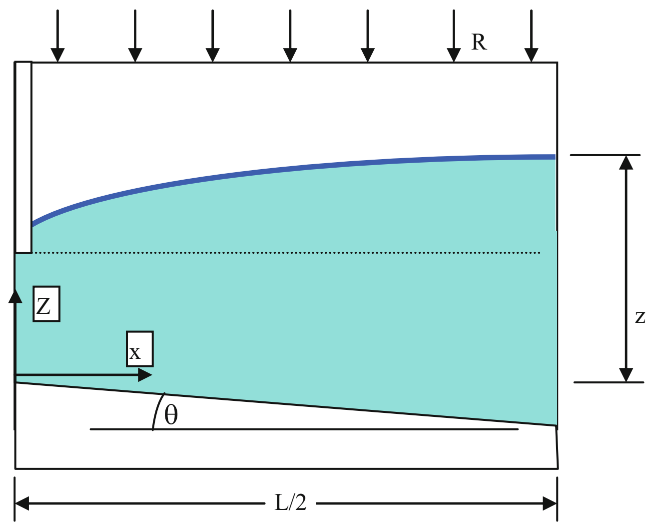
Without water harvesting, the water table quickly falls to the bottom of the soil profile and remains there. In spite of this, the underlying water table (0.4 m below the root zone) apparently adds water to the root zone during dry periods. With water harvesting, the water table frequently rises up into the root zone (Fig. 31.21).

Questions

1. Give two reasons for installing subsurface drainage in a field and discuss its importance.
2. Describe the two different types of flow to subsurface drains
3. List the water sources and sinks for drained soils.
4. Describe the Dupuit Forchheimer assumption and draw flow lines to a subsurface drain according to the Dupuit-Forchheimer assumption.
5. Discuss the following drainage systems: open parallel ditches, conventional drainage, water table control, and subirrigation.
6. Calculate the yield reduction for corn with the following water table data. Assume that CS is 0.12 from DOY 135–143 and is 0.10 from DOY 144–160. YRDMAX = 102 and DSLOPE = 0.75. When does the storm occur?

DOY	DTWT	DOY	DTWT
135	5	143	5
136	15	144	15
137	24	145	24
138	32	146	32
139	39	147	39
140	44	148	44
141	46	149	46
142	0	150	47

7. Why does the water table elevation in Fig. 31.5 increase rapidly after a rainstorm and then decrease more and more slowly over time? Discuss the influence of energy gradients.
8. Rainfall is 0.02 m / day, and drain spacing is 40 m. Calculate flow rate q_5, at a distance of 10 m from the drain. Find the flow rate into the drain per unit length of drain tubing.
9. A drain is placed in the field in the shape of a circle with radius 20 m. Derive equations for the flow rate to a unit length of the drain (m^2/day) at any distance from the drain. Let R represent the radius of the drain circle, r = distance from the center of the circle, D = distance from the drain, and P represent the precipitation rate. Only derive an equation for the flow on the inside of the circle.
10. Calculate the required spacing between drains. The farmer wants to maintain the water table at least 0.7 m below the soil surface. Yearly rainfall is 1 m/yr. The impermeable layer is 2.5 m below the soil surface. Hydraulic conductivity of the soil is 1 m/day. The conventional practice in the region is to install drains at 1.1 m depth below the soil surface. Drains are standard 4 in (10 cm) diameter drains. Effective drain radius for the 4 in drain is 0.51 cm
11. Set up a finite difference solution in a spreadsheet in order to solve for water table elevation vs. distance from the drain for the conditions in question 10. Solve for Δz with the finite difference solution at each position by rearranging Darcy's law and solving for Δz, and sequentially calculate the increasing drain elevation beginning at the drain and working toward the midpoint between drains. For the location close to the drain (horizontal distance from drain is less than distance from drain to impermeable layer), set up the flow to the drain as a quarter circle (or slightly more as water table increases with distance from the drain), and calculate head loss as a function of distance from the drain. Where it intersects the drain, let the water table be at the midpoint elevation of the drain. Use Dupuit-Forchheimer solution far from the drain. You should have less than 10 % difference between your solution and the 0.4 m elevation m at the midpoint between drains calculated in question 10.
12. Redo problem 11, but include a sloping geometry ($\theta = 10^0$) for the impermeable layer as shown below. Let the elevation of the drain be 1.1 m above the impermeable layer at the position of the drain. Assume Dupuit-Forchheimer flow in the region far from the drain. Compare the midpoint water table elevation to that calculated in question 11.



13. Redo question 12, but let $\theta = -4^0$ which means that the impermeable layer slopes upward from the drain rather than down. Compare to the midpoint water table elevation calculated in question 12.
14. Redo question 10 but let hydraulic conductivity of the soil within the top 1.1 m equal 0.3 m/day and below the drain equal 1 m/day.
15. With information from question 10, calculate the change in water table elevation over two weeks with the modified Hooghoudt equation. The initial elevation is 0.5 m, and the specific yield, SY_{mid}, is 15 %.
16. With parameters from question 10 and question 15 ($d_e = 1.0$), calculate the change in water table elevation over time with the USBR equation. The initial elevation is 0.5 m, and the specific yield, SY_{mid}, is 15 %. Remember to calculate D with the effective depth. Plot results and compare to the Hooghoudt transient graph from question 15. There is very little agreement because the KDt/SL^2 value is at the limit of the USBR curve from which it is derived.
17. Redo question 16 but let the equivalent depth, de = 3 for both the Hooghoudt and USBR solutions. The solutions are close because the USBR equation is in the central part of the USBR curve.
18. A field has an infiltration rate of 38 mm/hr. An irrigation with a gross application depth (volume applied divided

by field area) of 10 cm is applied, and the fraction of runoff is 0.2. Calculate the change in water table elevation for a drainage system with $d_e = 3.0$ m and K = 0.5 m/day. Plot the water table elevation (m) vs. time for four weeks of irrigation events that take place once each week. Remember to include the specific yield in the calculation of change in water table elevation and also to include drainage on the days that irrigation water is added to the water table.

19. Redo question 18, but add 20 cm gross application depth during each irrigation event. Plot the water table elevation vs. time (m).
20. Redo Example 31.9, but call drainage companies for drainage alternatives and prices in your region.
21. Redo Example 31.10, but evaluate a field soil and drainage scenario in your area, specified by the instructor.

References and Resources

Bouwer H, van Schilfgaarde J (1963) Simplified method of predicting fall of water table in drained land. Trans ASAE 6(4):288–296

Jury WA (1975) Solute travel-time estimates for tile-drained fields: I theory. Soil Sci Soc Am Proc 39:1020–1023

Kirkham D (1958) Seepage of steady rainfall through soil into drains. Am Geo U Trans 39:892–908

Kirkham D (1964) Physical artifices and formulas for approximating water table fall in tile drained land. Soil Sci Soc Am Proc 28:585–590

Kirkham D, Gaskell RE (1951) The falling water table in tile and ditch drainage. Soil Sci Soc Am Proc 15:37–42

Skaggs RW (1980) Drainmod. Reference report. Methods for design and evaluation of drainage-water management systems for soils with high water tables. North Carolina State University. Chapters 2, 3, and 5. http://www.bae.ncsu.edu/soil_water/drainmod/

van Schilfgaarde J (1963) Design of tile drainage for falling water table. J Irrig Drain Div ASCE 89(2):1–11