Lenses, Apertures, and Resolution

6

CHAPTER PREVIEW

Electron lenses are the magnetic equivalent of the glass lenses in an optical microscope and, to a large extent, we can draw comparisons between the two. For example, the behavior of all the lenses in a TEM can be approximated to the action of a convex (converging) glass lens on monochromatic light. The lens is basically used to do two things:

- either take all the rays emanating from a point in an object and recreate a point in an image,
- or focus parallel rays to a point in the focal plane of the lens.

D. B. Williams et al., *Transmission Electron Microscopy*

The lens can't collect *all* the rays from the object and we often deliberately limit the collection angle with an aperture. We can draw ray diagrams showing how electron lenses control beams of electrons. These diagrams correspond directly to the ray diagrams used in physical optics. Of course the analogy with light fails for certain aspects, but basically it will pervade this chapter. So we'll start by reminding you of the principles of light optics insofar as they relate to electron optics. Then we'll discuss the magnetic electron lens in more detail, showing how an electron behaves as it passes through such a lens. We'll describe some actual lenses and tell you how we use different kinds of electron lenses to do different things in the microscope.

The major limit to the use of electron lenses is the fact that we aren't very good at making them. They suffer from rather severe aberrations, which we control by inserting limiting apertures. You need to understand these aberrations, since they play a major role in deciding what we can and cannot do with the microscope. In particular, the resolution of an electron lens (rather than the wavelength of the electrons) limits the resolution of the TEM. Since resolution is usually the single most important reason for buying a TEM, you need a firm understanding of this concept. Unfortunately, we electron microscopists aren't very firm in our definitions of resolution. Finally, we describe how the apertures we put in the lenses aid both the depth of field and the depth of focus of the instrument.

Lenses, Apertures, and Resolution

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6.1. WHY LEARN ABOUT LENSES?

Why should we learn about electron lenses? As in a visible-light microscope, the lenses in a TEM control all the basic operational functions of the instrument. We physically move glass lenses up and down in a light microscope to control the intensity of the illumination and the focus and magnification of the image. The focal length of a glass lens is fixed. In a TEM the positions of the lenses are fixed and we focus, etc., by changing the strength of the lenses. As you'll see, in most cases the lenses we use are magnetic, so that we change their strength by changing the magnetic field. Almost any operation we carry out on the TEM involves changing magnification or focus; we use electron lenses to magnify and focus the electron beam, the images, and the diffraction patterns.

We also use apertures in the lenses to control the beam current and the convergence of the beam hitting the specimen.

These factors are critical in imaging, diffraction, and microanalysis. An aperture is used to select different electron beams to form different images, thus manipulating the image contrast. Another aperture is used to select different regions of the specimen to contribute to the diffraction pattern.

In essence then, we control the quality of our images, diffraction patterns, and analytical signals by adjusting the lenses and their apertures. So knowing how these aperture/lens combinations work allows you to understand how we control the TEM and why we do certain operations on the microscope.

An understanding of electron lenses will help us to answer such questions as:

- Why can we see finer detail with an electron microscope than with a light microscope?
- Why can't we see as much detail as we might expect from physics?
- Why does the TEM have a better depth of field and depth of focus than the light microscope?

We'll see that the answer to these questions lies in the quality of the lenses, and how we use them. In this chapter we'll discuss the basics of how a lens/aperture combination works. Throughout the book you'll come across different uses and combinations of lenses and apertures. So this is a central chapter for the serious microscope operator.

6.2. LIGHT OPTICS AND ELECTRON OPTICS

You are already familiar with the action of a magnifying glass lens on light rays. The magnifying glass is a convex lens. It can be used in two ways to control the light rays coming through it. First, it can produce a magnified image of the object you're looking at. Second, it can focus a parallel beam of light to a point, in the focal plane of the lens. (As children, we used this latter property to set fire to a piece of paper by focusing the sun's rays.) These two actions, forming an image of an object and focusing parallel rays to a point, are all we need in order to understand how the lenses in a TEM work. The reason that we can get away with this simple approach is because the electron lenses that we use act, to a reasonable approximation, like convex glass lenses; in detail, they're often equivalent to more complex combinations of convex lenses. Remember that, at present, all magnetic lenses are convex lenses.

6.2.A. How to Draw a Ray Diagram

In traditional light optics it's customary to draw ray diagrams of the path of light rays through the lens, and we do the same for electrons and their lenses. These ray diagrams are usually drawn horizontally because the traditional optical bench on which light optics experiments are carried out is a horizontal setup. But since the electron microscope is usually a vertical instrument, we will draw all our ray diagrams vertically.

Let's start by drawing ray diagrams to illustrate the two fundamental actions of image formation and focus of parallel rays. In these and all subsequent diagrams we'll draw all the lenses in the TEM as convex lenses. We will draw all ray paths as straight lines outside the lens, and we'll start by assuming that the lenses are perfect. We'll also draw the lenses as so-called "thin" lenses, which means their thickness is small compared to their radii of curvature. Actually, we'll make the lenses *very* thin. We'll see later that all these assumptions are wrong, to a degree, but that these traditional illustrations are nonetheless useful.

The first thing we need to do is to have a base line on which to draw our diagrams; this line is called the optic axis.

> The optic axis is an imaginary line down the column of the TEM passing through the center of each lens.

Now the first action of a lens that we want to show is how it produces an image of an object. In a TEM the object will usually be the specimen itself or an image of it, but it may also be the electron source, which is an object for the illumination system. If we assume the object is a point and the radiation is emanating from that point (a so-called "self-luminous object"), then a perfect lens will gather a fraction of that radiation and form a point image. This action is shown in Figure 6.1 in which the point is on the optic axis. The fraction of the rays from the object gathered by the lens is an important variable, defined by the semiangle β in Figure 6.1. Ultimately, as you can see, β is governed by the size of the lens, but we often choose to limit β by inserting an aperture, as we'll discuss later in this chapter. You'll often see the semiangle of collection defined as α, but we will reserve α for convergence semiangles (see Section 2.7).

> So, all lenses are imperfect insofar as they cannot gather all the radiation emitted by an object and so we can never create a perfect image.

As you know from Chapters 2–4, most electrons are strongly forward scattered, so we can in practice gather a high fraction of the scattered electrons.

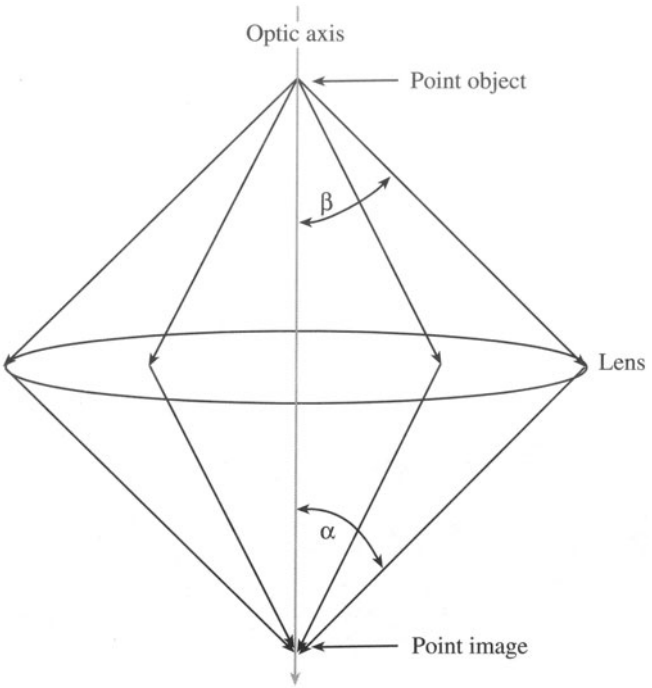

Figure 6.1. Image formation by a convex lens. A point object is imaged as a point and the collection semiangle of the lens is defined relative to the object (β) or the image (α).

> The angles in Figure 6.1 and in the other ray diagrams we'll draw are all greatly exaggerated. In practice, a typical value of β is maybe a few tens of milliradians (10 mrads = 0.57°) so if the diagrams were drawn to scale they would be many times longer than they were wide and all the ray paths would be exceedingly narrow. Since drawing to scale is impractical we always exaggerate the angles considerably in all electron ray diagrams.

If the object has a finite size, we can illustrate this by an arrow, asymmetrically positioned with respect to the optic axis, as in Figure 6.2. Then the lens creates an image of the arrow, rotated by 180°. To draw this figure, the first step is to draw line 1 from the arrowhead through the center of the lens, because rays crossing the optic axis in the lens (or "on-axis" rays which travel down the axis) are *not* affected by the lens at all and remain as a straight line. (Of course, this is a fundamental principle of how a lens works.) The second step is to draw line 2, which is a ray from the arrowhead that is parallel to the optic axis. The farther away that rays are from the optic axis, the more strongly they are bent by a convex lens, so we take line 2 and bend it toward the optic axis as it passes through the

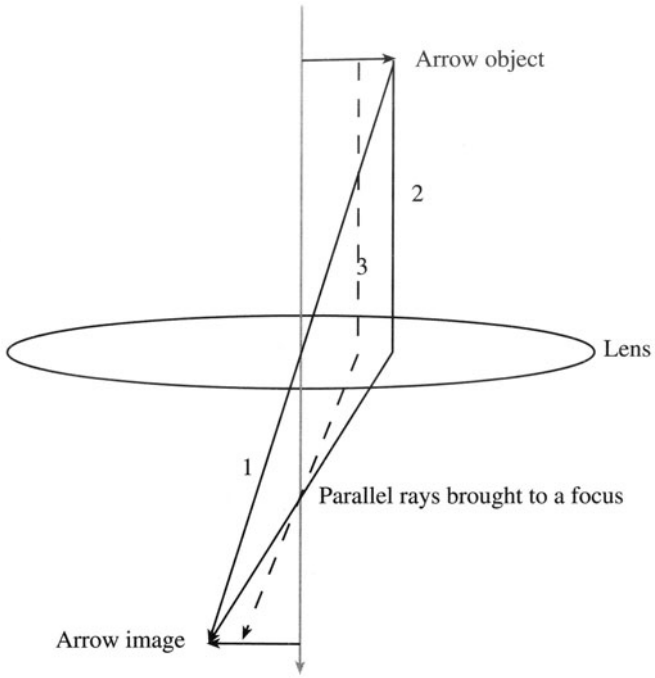

Figure 6.2. How to draw a ray diagram: first construct ray 1 through the middle of the lens, then ray 2, parallel to the optic axis, to determine the lens strength. Finally, draw line 3 parallel to 2 to define the focal plane where the parallel rays are focused. Thus an asymmetric object is imaged off axis and rotated through 180°.

lens. We can choose to make the lens as strong as we wish, and the strength determines how much the ray is bent and where lines 1 and 2 meet to recreate an image of the arrowhead. We could draw as many rays as we wished from any point on the object arrow to an equivalent point on the image arrow, such as line 3. Note that parallel rays 2 and 3 both cross the axis at the same point, illustrating the second fundamental action of a convex lens, i.e., bringing parallel rays to a focus. Again, the strength of the lens determines where the parallel electrons are focused.

> The image formed after each lens is rotated by 180° with respect to the object.

Now a full ray diagram for an object of finite size, symmetrically positioned about the axis, combines aspects of Figures 6.1 and 6.2, as shown in Figure 6.3. In Figure 6.3, all rays from a point in the object are brought back to a point in the image and all parallel rays (whether parallel to the optic axis or not) are brought to a focus in a plane at a position depending on their angle to the axis. Note that on-axis parallel rays are focused on axis and off-axis parallel rays are focused off axis. This is a most important property, since it allows the lens to create diffraction patterns in their focal plane. We'll use this diagram to introduce you to the principal terms used in lens optics.

6.2.B. The Principal Optical Elements

From the above diagrams, we can define three important planes to which we will often refer. The first plane is the object plane, which is the plane containing the object point in Figure 6.1 or the object arrow in Figures 6.2 and 6.3. The object plane always lies above the lens in question in the diagrams in this text. The second plane is the image plane (sometimes called the Gaussian image plane), which is the plane containing the image point or arrow, and it always lies below the lens. These two planes are said to be "conjugate," which means "optically equivalent." Rays leaving a point in one plane are brought to a point (if the lens is perfect) in a conjugate plane and vice versa. In other words, the electron doesn't care which way it goes through the lens. The third plane is the focal plane of the lens, and this is the plane in which the parallel rays are brought to a focus as shown in Figures 6.2 and 6.3. In the image-forming process in a TEM, the focal plane lies after or "behind"

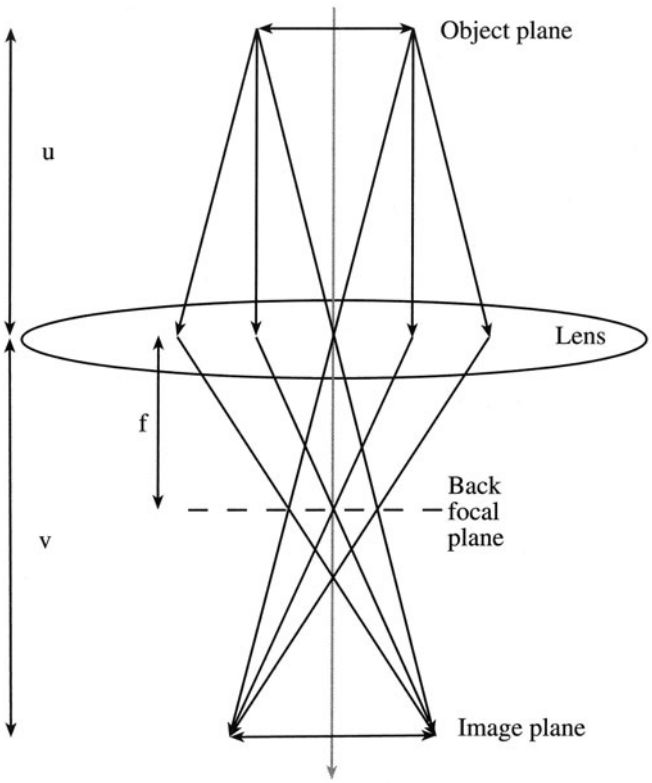

Figure 6.3. A complete ray diagram for a finite object, symmetrically positioned around the optic axis. All rays emerging from a point in the object (distance u from the lens) that are gathered by the lens converge to a point in the image (distance v from the lens) and all parallel rays are focused in the focal plane (distance f from the lens).

the lens and so the plane is sometimes called the "back focal plane" (BFP). There is also an equivalent "front focal plane," and a convex lens would take all the rays coming from a point in the front focal plane and create a parallel beam of radiation, in exactly the reverse manner to Figures 6.2 and 6.3.

6.2.C. The Lens Equation

From the above diagrams we can define three important distances, labeled in Figure 6.3: the distance from the object plane to the lens (the object distance, u), the distance from the lens to the image plane (the image distance, v), and the distance from the lens to the back focal plane (the focal length, f). Now if the lens is symmetric in strength either side of the lens plane (i.e., the front and back focal planes are the same distance from the lens), then we can write the following basic equation

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \qquad [6.1]$$

which is known as Newton's lens equation, and its proof can be found in a standard optics text such as Jenkins and White (1976). In thick lenses u and v are measured from different principal planes in the lens, but from the same plane in the middle of a thin lens, which we are assuming here. In all cases that we'll consider, the object distance (and therefore the image distance) is greater than the focal length. Thus a real image is produced on the other side of the lens beyond the back focal plane. If the object were within the (front) focal length, then a virtual image would be produced on the same side of the lens as the object, and this is often the case in light optics. Since we don't deal with virtual images in the TEM we'll ignore this aspect.

6.2.D. Magnification, Demagnification, and Focus

We can use Newton's lens equation to define the magnification of the convex lens as

$$M = \frac{v}{u} \qquad [6.2]$$

M is also approximately equal to the ratio of the collection semiangles of the lens subtended at the object (β) and at the image (α) as shown in Figure 6.1, assuming that these angles are small, as they invariably are in a TEM. In this example the magnification is unity.

Now we may sometimes want to *demagnify* an object (for example, when we want to form a small image of the electron source, to create the finest possible beam at the specimen). If that is the case, we define the demagnification as $1/M$. In an optical microscope we could change the magnification by moving the object relative to the lens or vice versa, and adjusting our eyes accordingly, but generally we rotate in another objective lens of different strength (curvature). In an electron microscope, we change magnification in this latter way by changing the strength of the lens, but you'll see that we can do this without changing the lens itself. So electron lenses differ fundamentally from glass lenses in that one lens can be adjusted to a range of strengths.

If we make the lens stronger, then the focal length is shortened as shown in Figure 6.4. If f is shortened but u is unchanged, then v must be correspondingly shorter and the image magnification is smaller, or the demagnification is larger. Under these conditions which normally occur in the TEM, strong lenses magnify less and demagnify more, which is counter to our understanding of light microscopes in which stronger lenses produce greater magnifications.

How do we get the high magnification that we need to form images of atom rows such as Figure 1.2? What we do is put the object close to the lens, making u small and M large, and repeat this for several lenses in tandem one after the other. So we end up with a multilens system like a com-

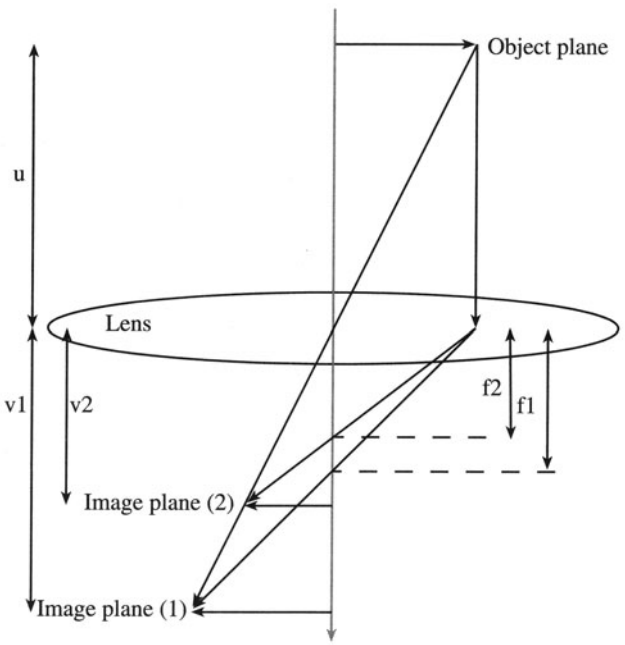

Figure 6.4. Strengthening the lens shortens the focal length f. So a weaker lens ($f1$) produces a higher magnification of the object than a stronger lens ($f2$) since the image distance v increases, but the object distance is unchanged.

pound optical microscope. (We'll discuss the details of the lens combinations in the illumination and imaging systems of the TEM in Chapter 9.) In these circumstances, the image plane of the upper lens acts as the object plane for the lower lens, assuming we want the lower lens to further magnify the image produced by the upper lens. In principle, there's nothing to stop us magnifying as much as we wish.

Don't confuse magnification with resolution.

Above a certain magnification, we will see no more information because other factors limit the image detail and therefore the resolution of the microscope. We'll discuss this point in Section 6.6. We'll also see that there are times when we want to look at an image of the focal plane (because this contains the diffraction pattern). To do this, the back focal plane of the upper lens must become the object plane for the subsequent lenses in the imaging system. When discussing focus we need another convention.

- If the lens is too weak and the image forms below the desired image plane, the image will be out of focus and the lens is said to be *underfocused.*
- If the lens is too strong and the image forms above the image plane, then we say the lens is *overfocused.*

It's very easy to confuse these two terms, unless you think in terms of the vertical frame of the microscope as shown in Figure 6.5.

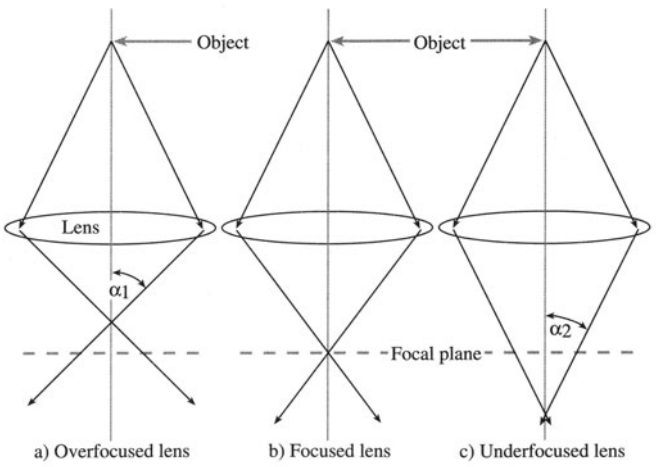

Figure 6.5. (a) Ray diagram illustrating the concepts of overfocus, in which a strong lens focuses the rays before the image plane, and (c) underfocus, where a weaker lens focuses after the image plane. It is clear from (c) that at a given underfocus the convergent rays are more parallel than the equivalent divergent rays at overfocus ($\alpha_2 < \alpha_1$).

6.3. ELECTRON LENSES

Electrons were first successfully focused by Busch in 1927; he used an electromagnet of the sort that Ruska incorporated into the first TEM. Busch also showed that it's possible to focus electrons using electrostatic fields and we've already seen how this works in thermionic electron guns in Chapter 5. In practice, magnetic lenses are superior in many respects, particularly because they are not susceptible to high voltage breakdown. So TEMs use magnetic lenses exclusively and we won't discuss electrostatic lenses further.

6.3.A. Polepieces and Coils

To make a magnetic electron lens you need two parts, and both are drawn schematically in Figure 6.6. First there is a cylindrically symmetrical core of soft magnetic material such as soft iron, with a hole drilled through it. We call this soft iron a "polepiece" and the hole is called the bore of the polepiece. In most lenses there are two polepieces (upper and lower), which can be part of the same piece of soft iron as in Figure 6.6 or may be two separate pieces. The distance between the polepiece faces is called the gap and the bore-to-gap ratio is another important characteristic of such lenses, controlling the focusing action of the lens. Some polepieces are machined to a cone shape and the cone angle is then an important variable in the lens performance.

The second part of the lens is a coil of copper wire which surrounds each polepiece. When we pass a current

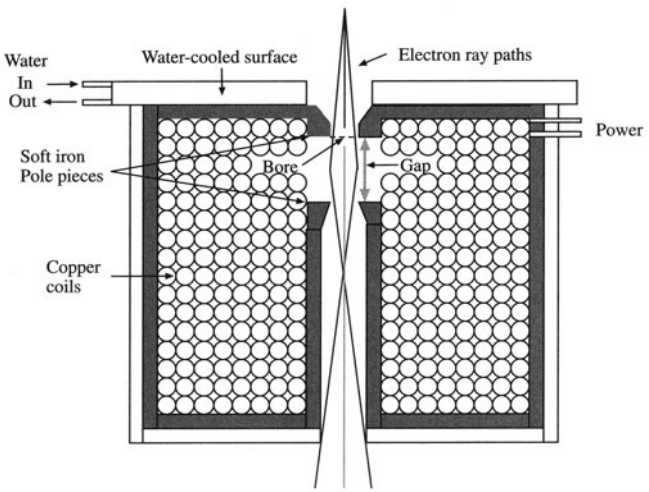

Figure 6.6. Schematic diagram of a magnetic lens. The pole pieces surround the coils and, when viewed in cross section, the bore and the gap between the polepieces are visible. The magnetic field is weakest on axis and increases in strength toward the side of the polepiece, so the electrons are more strongly deflected as they travel off axis.

through the coil, a magnetic field is created in the bore. This field is inhomogeneous along the length of the lens, but axially symmetric. The strength of the field in a magnetic lens controls the ray paths. As you can see, the electron path through the lens is a reasonable approximation to the schematic diagram back in Figure 6.1.

The resistive heating of the coil means that the lenses have to be cooled and a water recirculating system is an essential part of TEM lenses. A real lens removed from the column of a microscope is shown in Figure 6.7.

Practical hint: *You should be able to get a readout (on your TEM console) of the current through any lens coil and it is a useful thing to know the standard lens currents for your common operating modes such as imaging and diffraction and for various beam sizes.*

6.3.B. Different Kinds of Lenses

The principles that we've just described are incorporated into different kinds of lenses used in the TEM. Most lenses in the microscope are weak lenses with large gaps. Either they act to demagnify the source image onto the specimen, or they magnify the image or the diffraction pattern and project it onto the viewing screen in ways that we'll see in Chapter 9. Typically, these lenses are of the sort shown schematically in Figure 6.6 and an aperture can be introduced into the bore of the lens, as we'll discuss later.

The objective lens is the most important lens in the TEM, since it forms the images and diffraction patterns that will be magnified by all the other lenses. It is also the most difficult to construct, since the specimen must be located so close to the "plane" of this lens.

The objective lens is a strong lens. Several types exist, depending on the needs of the particular TEM. The most flexible objective lens is that in which the upper and lower polepieces are separated and have their own coil, as shown in Figure 6.8A. This geometry gives the space needed to allow us to insert the specimen and the objective aperture between the polepieces. With this type of polepiece, other instruments such as X-ray spectrometers can have relatively easy access to the specimen. For the same reason, it is straightforward to design specimen holders that do a variety of tasks such as tilting, rotating, heating, cooling, and straining, and this versatility accounts for the popularity of the split polepiece lens in TEMs.

With split polepieces it is possible to make the upper polepiece behave differently than the lower polepiece.

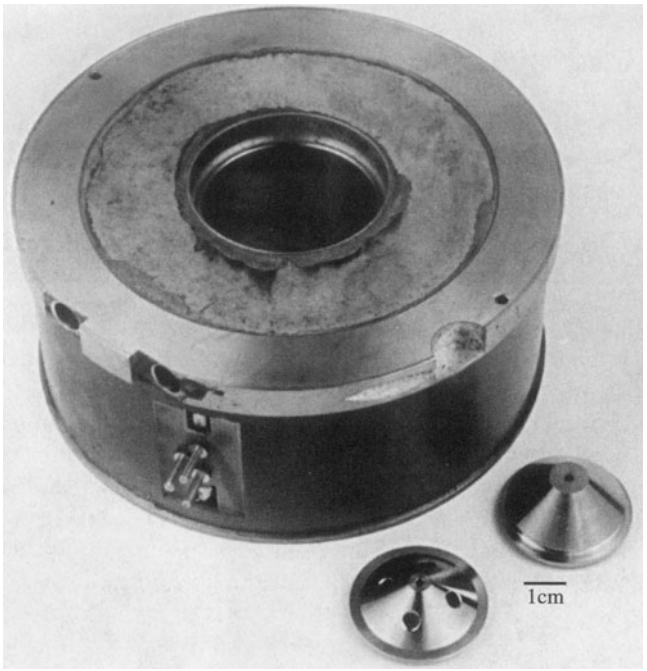

Figure 6.7. A real lens: the cylindrical shape conceals the copper wire coils. The two conical polepieces beside the lens sit inside the central hole in the lens. The three-pin electrical connections provide current to the coil to magnetize the polepieces, and cooling water is circulated in and out of the two holes in the top plate of the lens to dissipate the resistive heat generated.

The most common application of this is to excite the upper objective polepiece very strongly. This kind of lens is ideal for an AEM/STEM because it can produce both the necessary broad beam of electrons for TEM and a fine beam of electrons for AEM and STEM. We'll see how this is accomplished in more detail in Chapter 9.

If high resolution is a major requirement, then we'll see that it is essential to keep the focal length of the objective lens short and this means a very strong lens is needed. This is traditionally accomplished by an immersion lens in which the specimen is dropped into (i.e., immersed in) the center of the lens field as shown in Figure 6.8B. In such a top-entry stage the specimen is surrounded by the objective lens, and so it is a more difficult engineering feat to manipulate, heat, or cool the specimen and it is not possible to get X-ray detectors near the specimen, so analytical microscopy is very inefficient. If the focal length is kept really short to give the highest resolution, then it becomes difficult to tilt the specimen more than a few degrees. So in the highest-resolution TEMs you can't do much apart from imaging and diffraction over a restricted range of tilt (see Chapter 8 on stages). This limitation can be overcome by more recent lens designs such as the snorkel lens, as shown

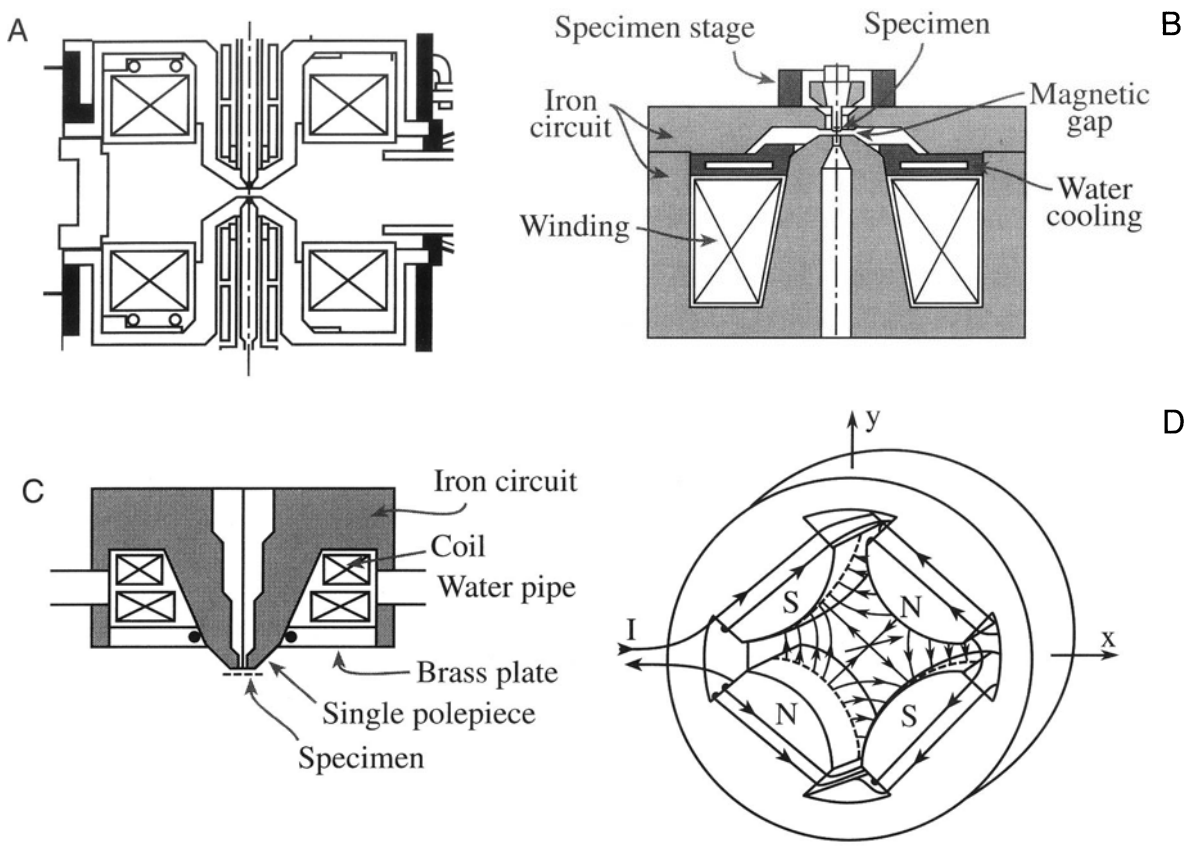

Figure 6.8. A selection of different lenses: (A) a split polepiece objective lens, (B) a top-entry immersion lens, (C) a snorkel lens, and (D) a quadrupole lens.

in Figure 6.8C, which is a single polepiece lens with a small bore to give a strong lens.

The limitations of ferromagnetic polepieces can be overcome using superconducting lenses. We cannot make soft iron polepieces stronger than their saturation magnetization and this limits the focal length and the probe-forming capability of the lens. Superconducting lenses can overcome these limitations, but since a superconductor generates a fixed field it cannot be varied in the same way as a conventional ferromagnetic lens and so it is not very flexible. Nevertheless, there is a lot of interest in these lenses because they are small, they don't need water cooling, and they cool the area around the specimen which both improves the vacuum and helps minimize contamination. They can generate intense fields (>100 T) which are very promising for forming fine probes with high-energy electrons (useful in AEM). Superconducting lenses are so strong that their aberrations (which we'll get to in Section 6.5) are reduced to the level where resolutions < 0.1 nm are feasible and they may be used in the future to construct compact TEM columns.

In addition to these variations on the theme of a single or double polepiece, it is also possible to design a quadrupole and octupole lens in which the focusing action is achieved by four or eight polepieces. Adjacent polepieces are of opposite polarity as shown in Figure 6.8D. These lenses are not used in TEMs as magnifying lenses but they are used to correct lens defects such as astigmatism, and they are used as lenses in electron spectrometers (see Chapter 37). These lenses require less power, and they don't introduce any rotation into the image which, as we'll now show, is a characteristic of standard electromagnetic lenses.

6.3.C. Electron Ray Paths through Magnetic Fields

We need a bit of mathematics to explain how the magnetic lenses actually work. When an electron with charge q ($= -e$) enters a magnetic field of strength **B** (Tesla) and an electric field of strength **E**, it experiences a force **F**, known as the Lorentz force, which depends on the velocity of the electron, **v**. All these factors are related through the equation

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad [6.3]$$

where the term in parentheses is a vector cross product. Since we are not applying an electric field within the lens, the resulting (Lorentz) force **F** is a vector normal to **v** and **B**, which are inclined to one another at an angle θ. You can easily work out the relative directions of **E**, **v**, **B**, and **F** using the right-hand rule in which your thumb represents the direction of the force acting on a *positive* charge moving in the direction of the middle finger through a field in the direction of the index finger. So the force on the electron acts in the opposite direction to the thumb.

Field: Forefinger; Velocity (Speed): Second finger; Thrust: Thumb (Right-hand rule.)

The force on an electron entering a uniform magnetic field, nearly 90° to **B** is

$$F = evB \sin\theta = evB = \frac{mv^2}{r} \qquad [6.4]$$

where r is the radial distance of the electron from the optic axis and m is the mass of the electron.

We can rearrange equation 6.4 to give an expression for

$$r = \frac{mv}{eB} \qquad [6.5]$$

Since v is a relativistic velocity, we should write this equation as

$$r = \frac{\left[2m_0E\left(1 + \frac{E}{2E_0}\right)\right]^{1/2}}{eB} \qquad [6.6A]$$

where m_0 and E_0 are the rest mass and energy of the electron, respectively. This form of the equation allows us to substitute known constants to estimate r (in meters)

$$r = \frac{3.37 \times 10^{-6}\left[V(1 + 0.9788 \times 10^{-6}\, V\right]^{1/2}}{B} \qquad [6.6B]$$

For example: if V = 100 kV and the magnetic field B is 1 Tesla, the radius is less than 1 mm.

In deriving equation 6.4, we made a rather gross oversimplification. If θ does equal 90°, the electron is traveling straight across the optic axis and is not focused! It is the deviation from θ = 90° that gives the lens effect. The next step therefore is to separate the electron velocity **v** in a magnetic field into two components, $\mathbf{v}_1$ perpendicular to, and $\mathbf{v}_2$ parallel to, the magnetic field direction **B**, as shown in Figure 6.9, where $v_1 = v \sin\theta$ and $v_2 = v \cos\theta$. The parallel component, $\mathbf{v}_2$, results in motion parallel to the optic axis in the z direction, with $z = v_2t$, while the perpendicular component produces circular motion with radius given by equation 6.5. Hence we see the very important result:

The electron spirals through the lens field with a helical trajectory.

The period of rotation (T_c) through the field gives rise to a so-called "cyclotron frequency" ω

$$\omega = \frac{2\pi}{T_c} = \frac{eB}{m} \qquad [6.7]$$

From these various relationships, we can calculate the complete ray paths through the lens. The most important equations are called the "paraxial" (near-axis) ray equations, which describe both r and the angle of rotation (θ) about the axis as the electron moves down the axis in the direction z, rotating under the influence of the cylindrically symmetrical field, B. These equations, which neglect electron trajectories far off axis, are derived in texts on electron optics. The account by Hawkes (1972) is particularly clear if you have an interest in the physics of electron lenses. As Hawkes succinctly states, "a straightforward, but quite lengthy calculation yields"

$$\frac{d^2r}{dz^2} + \frac{\eta^2B^2r}{2V^{1/2}} = 0 \qquad [6.8]$$

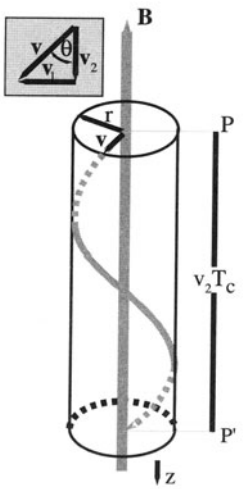

Figure 6.9. Electron trajectories in a homogeneous magnetic field, strength **B**. The electrons have velocity components parallel and perpendicular to the field, so long as they are not traveling at 90° to the direction of **B**. The Lorentz force causes electrons passing through point P on the optic axis to spiral through the field and intersect the axis again at P'. The electron's helical path defines the cyclotron radius, r.

$$\frac{d\theta}{dz} = \frac{\eta B}{2V^{1/2}} \qquad [6.9]$$

where V is the accelerating voltage of the microscope and η is $(e/2m_0c^2)^{1/2}$. You can see from equation 6.8 that the rate of change of r is smaller for more energetic electrons and larger for more intense field strengths. From equation 6.9 the rate of angular rotation increases with increasing field strength and decreases for more energetic electrons.

When the energy of the electrons increases, we must use stronger lenses (larger **B**).

When we increase **B**, the pitch of the helical path becomes steeper.

Both of these conclusions are intuitively obvious, but the implication is often missed. When we change our operating kV, we change the lenses in the microscope! (Think what this would mean in a visible-light microscope.) Thus the calibration of the microscope and lens "constants" change as we change the accelerating voltage.

While all these ray equations are approximations, they form the basis of more detailed mathematical models of electron motion through lenses. The more complete models are used in a computer program (Munro 1974), which simulates the effects of new lens shapes, bore/gap ratios, etc., and has permitted significant advances in the design of lenses to meet the more stringent demands of modern TEMs. We also use non-paraxial rays to explain the effect of spherical lens aberrations on resolution in Section 6.5.A.

6.3.D. Image Rotation and the Eucentric Plane

The electrons follow a helical path as they traverse the field along the axis of the lens. This rotation is rarely shown on standard ray diagrams. Its effects are seen in the routine operation of the TEM because the image, or diffraction pattern, rotates on the viewing screen as you try and focus or if you change magnification. This rotation may require calibration; as we'll see later, the manufacturer may have compensated for it by including an extra lens.

We've already seen in Figure 6.4 that if we change the strength of the lens, the position of the focal plane and the image plane will also change. Because of this, we have to define a standard object plane for the main imaging lens of the microscope and we call this the *eucentric* plane. Your specimen height should always be adjusted to sit in the eucentric plane because an image of an object in this plane will not move as you tilt the specimen. All other planes in the imaging system are defined with reference to the eucentric plane. If your specimen is in the eucentric plane, then the objective lens strength is always the same when the image on the screen is in focus. We'll return to this point later in the book.

6.3.E. Deflecting the Beam

There are many occasions during the operation of the TEM when we want to deflect the beam entering the lens. We may wish to deflect the beam laterally off axis or tilt it to a certain angle with respect to the optic axis. In STEM, these operations are essential to the whole process of forming a scanning image. It is also exceedingly useful in AEM to be able to blank the beam, i.e., deflect it off axis so it goes into a Faraday cup to measure the current, or to prevent the beam from hitting the specimen when no useful spectroscopic data are being gathered. The way we do this is to apply an electromagnetic field to tilt or traverse the beam, or an electrostatic field to blank it. Electromagnetic scan times are of the order of milliseconds while electrostatic blanking can occur in fractions of a microsecond. Although we are drawing the lens as having zero thickness along the optic axis, the magnetic field actually acts over a length L. The angle of deflection ε is (for small ε)

$$\varepsilon = \frac{eLB}{mv} \qquad [6.10]$$

From this equation we can show that to tilt the beam by 5° we need a coil carrying about 0.2 A and ~100 turns applied along a length of 1 cm, giving a field of 0.01 T. For electrostatic blanking we need about 20 kV/cm.

6.4. APERTURES AND DIAPHRAGMS

We mentioned earlier that an aperture is often inserted into a lens. The aperture limits the collection angle of the lens as shown schematically in the diagram in Figure 6.10, and such an aperture in the objective lens thus allows us to control the resolution of the image formed by the lens, the depth of field and the depth of focus, the image contrast, the collection angle of the electron energy-loss spectrometer, the angular resolution of the diffraction pattern, etc. Physically, the aperture may reside above, in, or below the plane of the lens as we draw it in ray diagrams. Apertures can also perform other functions which we'll come across later, such as protecting the specimen from stray radiation

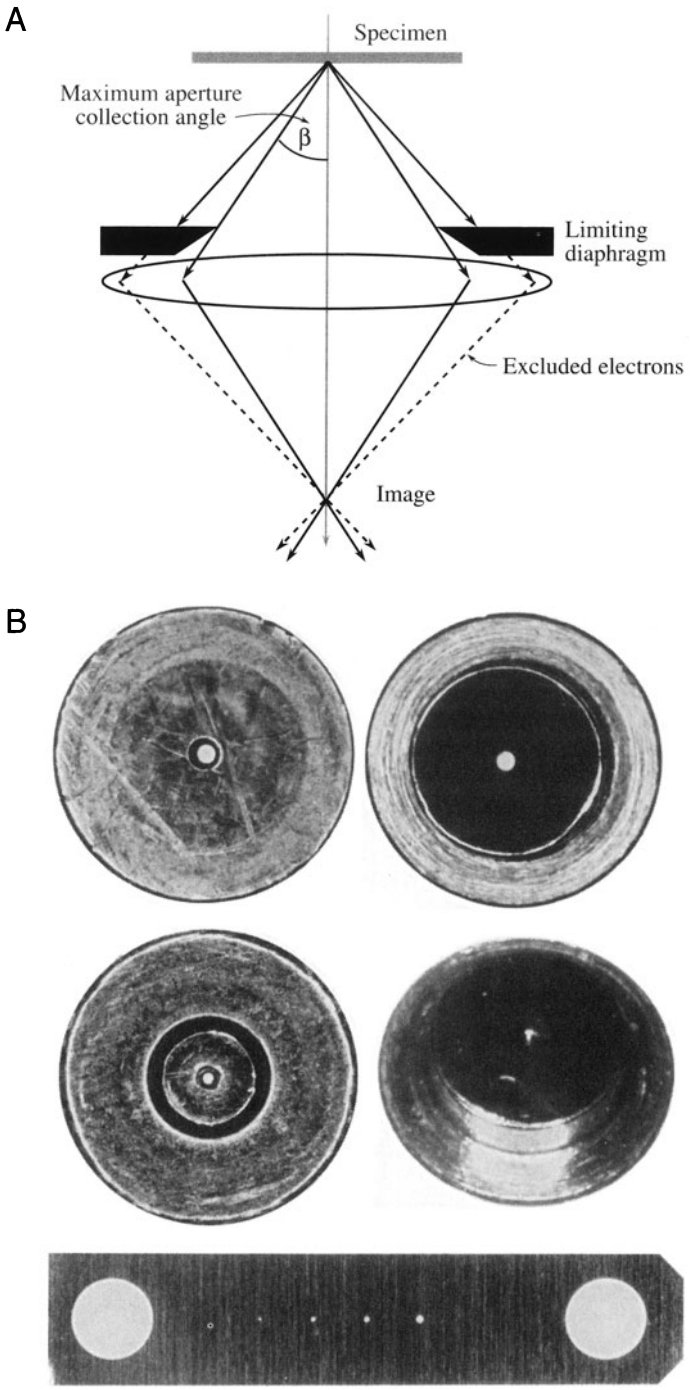

Figure 6.10. (A) Ray diagram illustrating how a diaphragm restricts the angular spread of electrons entering the lens. Only electron paths less than a semiangle β subtended by the aperture at the object are allowed through the lens (full ray paths). Electrons from the object scattered at angles >β are stopped by the diaphragm (dashed ray paths). (B) A selection of diaphragms: the top and middle left are upper and lower views, respectively, of a conventional objective diaphragm; the top/middle right are views of a "top-hat" (thick) C2 diaphragm; below is a metal strip containing several apertures. Each diaphragm is ~3 mm across.

in the illumination system. So they are really important. Usually the apertures are circular holes in metal disks and the disks are made of either Pt or Mo, which are both refractory metals.

A quick word on terminology: While the aperture is the hole in the disk, the metal surrounding the aperture is called the diaphragm (like the variable iris diaphragm in an optical microscope or your camera). We use the aperture to allow certain electrons to pass through the lens and exclude others by causing them to hit the surrounding diaphragm. This "aperture/diaphragm" wording, while strictly correct English, is a bit cumbersome, and microscopists tend to be lazy and just use the term "aperture" in both the correct sense of a hole, but also incorrectly to describe the action of the diaphragm. So we might say that "the objective aperture was used to exclude high angle scattered electrons from the image" or, as we said above, "the aperture protects the specimen from stray radiation" while, strictly speaking, the diaphragm did the excluding and protecting. We'll try to be both consistent and correct in our usage of both terms, but sometimes the precise terminology is awkward.

Diaphragms come in several forms, depending on their function and the particular microscope. They can be either individual disks with a varying diameter aperture, or they can be a series of different apertures in a single metal strip (as shown in Figure 6.10). The diameter can be as small as 10 μm, which is about the smallest circular aperture we can make consistently, up to ~ 0.3 mm (300 μm). Usually the individual diaphragms or the strips are about 25–50 μm thick, but if their job is also to prevent X-rays from hitting the specimen they may be several millimeters thick, which means they can be quite expensive if they're Pt.

Often the diaphragm collects contamination caused by the electron beam cracking residual hydrocarbons in the vacuum (as we describe in Chapter 8). The contamination tends to gather on the edges of the aperture, destroying its circular shape, and this causes astigmatism. So the diaphragms need occasional cleaning, which can be done by heating them to red heat in the central blue part of a butane flame. In some TEMs, this problem is eliminated by making the diaphragm of very thin metal foil (e.g., Au or Mo) because the foil gets hot in the electron beam and boils off any contamination. But such self-cleaning diaphragms are delicate and often crack, thus allowing electrons through other gaps, which defeats the object of the exercise of producing a well-defined aperture.

A safety note: *X-rays with energies up to the beam energy are generated within the lens wherever the electron beam hits a surface (particularly the limiting diaphragm). So substantial and carefully designed lead shielding is incorporated into the column of the TEM to prevent irradiation of the operator. Obviously, it could be very dangerous to tamper with the lenses or diaphragms of the microscope in any way and only qualified service engineers should be allowed to dismantle or in any way take apart and repair the lenses.*

6.5. REAL LENSES AND THEIR PROBLEMS

It might appear from what we've discussed so far that the analogy between electromagnetic lenses and convex glass lenses is complete, but that is not so. Over the 300 years since van Leeuwenhoek first constructed a light microscope, glass lenses have developed to a point where perfect lenses can be fabricated. In the 70 years since Busch's first magnetic lens, we haven't progressed so far and our lenses are very imperfect. We've already compared the best electromagnetic lens to the equivalent of using the bottom of a Coke bottle as a magnifying glass. Another common description is that if your own eye lens was as good as our best electromagnetic lens, then you'd be legally blind! So we have to modify all the ideal ray diagrams we've drawn to take into account the imperfections of the lenses. These imperfections all limit the resolution of the microscope but, paradoxically, help us to get better depth of focus and depth of field from the microscope.

There are ten kinds of lens defects (see Reimer 1993) and at one time or another all their effects can be seen. In practice, however, we don't need to know about all of them and we'll emphasize the ones that limit the microscope performance in substantial ways. These comprise spherical aberration, chromatic aberration, and astigmatism.

6.5.A. Spherical Aberration

The term "spherical aberration" has almost entered the popular vocabulary since its presence was discerned in the main optics of the Hubble Space Telescope (unfortunately after launch). This defect is caused by the lens field acting inhomogeneously on the off-axis rays. The further off axis the electron is, the more strongly it is bent back toward the axis. As a result, a point object is imaged as a disk of finite size, which limits our ability to magnify detail because the detail is degraded by the imaging process. Ultimately, it is this defect that limits the resolution of modern TEMs so we need to examine it carefully.

The effects of spherical aberration are shown in Figure 6.11. A point object P is imaged as an intense central bright region with a surrounding halo in the Gaussian image plane. This image goes by the delightful term of the "disk of least confusion" or sometimes the "disk of minimum confusion." Spherical aberration is most important in the objective lens because it degrades the image that we view in a TEM. Also, it is equally important in the condenser lenses in an AEM or STEM which we use to form the finest beam with the most current. What we can accomplish is almost always limited by spherical aberration.

From Figure 6.11 you can see why we use the term "spherical" to describe the aberration. The effect is to take

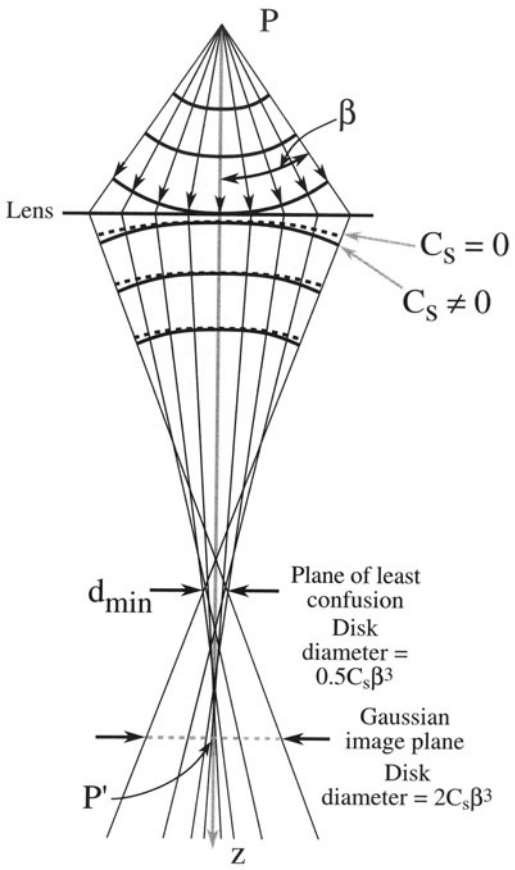

Figure 6.11. Spherical aberration in the lens causes wavefronts from a point object P to be spherically distorted. The point is thus imaged as a disk with a minimum radius in the plane of least confusion and a larger disc at P' in the Gaussian image plane.

the curved wavefronts and further curve them. Now if you go back and look at Figure 6.9, you'll see that electrons traveling through a point P on axis intersect the axis again at point P', where the distance PP' is given by (Reimer 1993)

$$\begin{aligned} PP' &= v_2 T_c = vT_c \cos\theta = 2\pi\frac{mv}{eB}\left(1 - \frac{\theta^2}{2} + \cdots\right) \\ &= L_0\left(1 - \frac{\theta^2}{2} + \cdots\right) \end{aligned} \quad [6.11]$$

In this relationship $L_0 = PP_0'$, where P_0' is the Gaussian image of the point P for very small θ (i.e., paraxial conditions). As θ increases, the distance PP' decreases because of spherical aberration and we can write

$$PP' = PP'_0 - \Delta z \quad [6.12]$$

where $\Delta z = 0.5L_0\theta^2$. Thus we get an expression describing the error, δ, in the Gaussian image position due to spherical aberration

$$\delta = \Delta z \tan\theta \sim \Delta z\theta = 0.5L_0\theta^3 \quad [6.13]$$

So the diameter of the Gaussian image of a point *formed by paraxial rays* is given by this expression, which we usually write as

$$\delta = C_s\theta^3 \quad [6.14]$$

where C_s is a constant for a particular lens called the spherical aberration coefficient.

Now this equation is for paraxial rays only, and in a real microscope the apertures are usually large enough so that paraxial conditions don't apply. As a result the spherical aberration error in the Gaussian image is broadened to a diameter of $2C_s\theta^3$. We'll see in a while that this equation is *very* important because of its effect on the resolution of the TEM, and sometimes it is written $2C_s\theta^3M$ when referred to the image plane. In a real lens the value of θ in equation 6.14 is replaced by the maximum semiangle of collection of the objective lens aperture, β. We'll use β to define the objective lens semiangle of collection in the forthcoming discussion of resolution. This contrasts with our use of α when discussing beam size in Chapter 5, since α defined a semiangle of beam convergence. Most other TEM texts use α indiscriminately for collection and convergence angles. When we discuss resolution in the TEM later in this chapter, we will use the radius rather than the diameter, and since all discussion of resolution refers back to the object, the magnification term is customarily ignored.

So we end up with an expression for the radius of the spherical aberration disk r_{sph} in the Gaussian plane under nonparaxial (i.e., realistic) conditions, given by

$$r_{sph} = C_s\beta^3 \quad [6.15]$$

From this derivation you can see that C_s has the dimensions of length. Typically, it is approximately equal to the focal length, which in most TEMs is about 3 mm, but in high-resolution instruments may be well below 1 mm. So one way to minimize this aberration is to put your money into buying a TEM with a short focal length lens.

If you look at Figure 6.11, you will see that the smallest dimension of the cone of rays formed by the lens does not occur at the Gaussian image plane. The smallest dimension is called the disk of least confusion and has a radius of $0.25C_s\beta^3$. As we'll discuss in Section 6.6.C, some texts use this smaller dimension to define the resolution limit imposed by spherical aberration.

6.5.B. Chromatic Aberration

This term is related to the "color" (i.e., wavelength and hence energy) of the electrons. We've assumed the electrons are monochromatic, but they aren't. However, we can now make very good high-tension supplies and the variation of the electron energy is usually smaller than one part in 10^6, which is 0.1 eV for a 100-keV beam. In fact, this is so good that we don't have to worry about chromatic aberration in the illumination system. In TEM images, chromatic aberration could also be safely ignored if we didn't put a specimen into the beam. Unfortunately, this rather essential action creates electrons of a whole range of energies emerging from the thin foil (for reasons we described in Chapter 4). The objective lens bends electrons of lower energy more strongly and thus electrons from a point in the object once again form a disk image (Figure 6.12). The radius r_{chr} of this disk is given by

$$r_{chr} = C_c\frac{\Delta E}{E_0}\beta \quad [6.16]$$

where C_c is the chromatic aberration coefficient of the lens (like C_s it is a length, approximately equal to the focal length), ΔE is the energy loss of the electrons, E_0 is the initial beam energy, and β is the semiangle of collection of the lens. While ΔE in the incident electron beam is < 1 eV, it is typically 15–25 eV for a good fraction of the electrons coming through a typical thin foil 50–100 nm thick. Chromatic aberration gets worse for thicker foils and better for thinner ones. So you can do something about this aberration—make good thin specimens!

6.5.C. Astigmatism

Astigmatism occurs when the electrons sense a nonuniform magnetic field as they spiral round the optic axis.

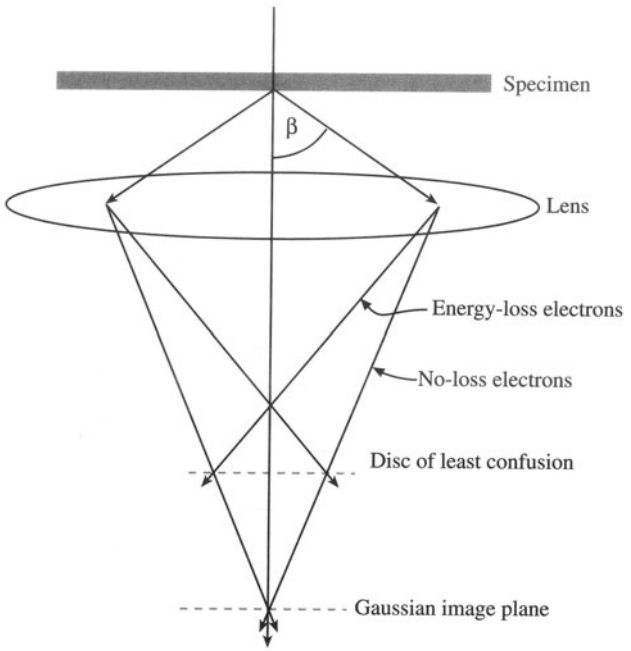

Figure 6.12. Chromatic aberration results in electrons with a range of energies being focused in different planes. Electrons emerging from the specimen with no loss of energy are less strongly focused than those that suffered energy loss in the specimen, so a point is imaged as a disk.

This defect arises because we can't machine the soft iron polepieces to be perfectly cylindrically symmetrical down the bore. The soft iron may also have microstructural inhomogeneities which cause local variations in the magnetic field strength. Even if these difficulties were overcome, the apertures we introduce into the lens may disturb the field if they are not precisely centered around the axis. Furthermore, if the apertures are not clean, the contamination charges up and deflects the beam. So there is a variety of contributions to astigmatism, which distorts the image by an amount r_{ast} where

$$r_{ast} = \beta \Delta f \qquad [6.17]$$

and Δf is the maximum difference in focus induced by the astigmatism. Fortunately, astigmatism is easily corrected using stigmators, which are small octupoles that introduce a compensating field to balance the inhomogeneities causing the astigmatism. There are stigmators in both the illumination (condenser lenses) system and the imaging system (objective lens) and we'll describe how to use them in Chapter 9.

These then are the three major defects in electromagnetic lenses. There are several minor defects, such as barrel and pincushion distortion, which are self-explanatory in terms of how they distort the image. They are occasionally seen at very low magnification where electrons traveling close to the bore of the polepiece appear in the image. Other effects, such as coma, field curvature, etc., we will ignore.

6.6. THE RESOLUTION OF THE ELECTRON LENS (AND ULTIMATELY OF THE TEM)

Another note on terminology: We electron microscopists tend to be rather imprecise in our definition and use of the words "resolution" and "resolving power" and related expressions. We've borrowed these terms from classical light optical microscopy, which is concerned with the imaging of incoherent waves through amplitude contrast. High-resolution performance in the electron microscope is a different matter and involves phase-contrast imaging of reasonably coherent electron waves, so perhaps we shouldn't be surprised if a different usage has developed. But we should at least define the terms we use. Now in light microscopy (Bradbury *et al.* 1989) the term "resolution" strictly applies to the act of displaying fine detail in an *image.* The resolving power of the microscope is the ability to make points which are closely adjacent in the object, distinguishable in the *image.* Now the minimum distance apart of these points in the *object* is the *minimum resolvable distance.* Since electron microscopists customarily talk about the resolution of the microscope in terms of distance in the *object* (usually a few Å), we should then use the term "minimum resolvable distance," but instead everyone says "resolution."

> We will use the term "resolution," but we define it to mean the "minimum resolvable distance" in the object.

Because the lens defects that we've discussed cause a point object to degrade into a Gaussian image with a finite radius (some combination of r_{sph}, r_{chr}, r_{ast}) they limit the resolution of the electron lens, and hence that of the microscope. The image resolution in the TEM is governed by the ability of the objective lens to image the object, while in the STEM the image resolution is governed by how much beam current we can put into a small image of the electron source demagnified onto the specimen. In either case, the aberrations limit the resolution.

6.6.A. Theoretical Resolution

If there are *no* aberrations at all, the resolution of *any* lens (glass, electromagnetic) is customarily defined in terms of

the Rayleigh criterion, which we introduced back in Chapter 1. Rayleigh devised a criterion for resolution which is arbitrary in the sense that it is not a fundamental physical rule but more a practical definition. This criterion gives us a figure of merit in terms of the eyes' ability to distinguish separate images of two self-luminous incoherent point sources. A single point source will not be imaged as a point, even if no aberrations or astigmatism are present. The finite size of the lens results in diffraction of the rays at the outermost collection angle of the lens, usually defined by a limiting aperture. This diffraction results in a point being imaged as a disk (called the Airy disk) which has a cross-section intensity profile as shown in Figure 6.13a. This should be familiar to anyone who has encountered basic light optics. Rayleigh stated that if the maximum from one source lies over the first minimum from the other source, as shown in Figure 6.13c, then the overall intensity profile exhibits a dip in the middle at about 80% I_{max}. The eye can discern this dip as two overlapping images, thus indicating the presence of two separate objects. Under these circumstances the distance apart of the two incoherent point sources is defined as the theoretical resolution of the lens r_{th} and is given by the *radius* of the Airy disk

$$r_{th} = 0.61\frac{\lambda}{\beta} \qquad [6.18]$$

Beware! Sometimes the diameter is used (hence the term 1.22 in equation 5.10) because it is the beam diameter which defines image resolution in SEM and STEM. Any standard text on light optics (which we've already referenced) will derive this criterion if you're interested. Now, strictly speaking, we should not use this equation for electron sources because they are not incoherent, and when dealing with true high resolution a different approach is used (see Chapter 28). But for our purposes here, we will be content with this approximation.

Obviously, then we can get higher resolution (smaller r) if we lower λ or increase β. Unfortunately, microscopists often use the expression "higher resolution" when in fact they mean "better resolution" and this can be confusing since the term "higher" is then associated with a lower number. The vacuum is also "higher" if its magnitude is smaller! The improvement in resolution with lower λ accounts for much of the drive toward intermediate and high voltage microscopes since λ decreases with keV, as we saw back in Chapter 1. But the obvious question is why don't we just increase β (i.e., use a bigger lens aperture or remove it altogether). Well, we could do this if we had perfect lenses, but that isn't the case, and the lens aberrations increase as we increase β.

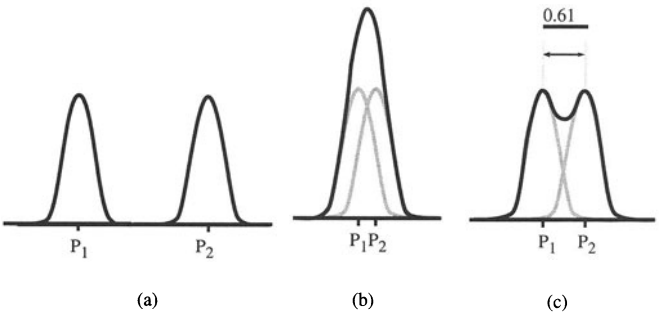

Figure 6.13. (a) The Airy disk intensity profile from two point sources P_1 and P_2 defines the resolution of a lens. In (b) the two Airy disks are so close that they cannot be distinguished, but in (c) the two are separated such that the maximum in the image of P_1 overlaps the minimum in P_2. This is the definition of resolution defined by the Rayleigh criterion.

6.6.B. Spherical Aberration-Limited Resolution (The Practical Resolution)

Let's assume first of all that we have corrected for any astigmatism and our specimen is thin enough that chromatic aberration is negligible. Under these circumstances, the spherical aberration error r_{sph} limits the resolution. Now if you go back and look at equation 6.15 you'll see that r_{sph} increases with the cube of β, a *very* strong dependence. The resolution in the object, then, is given by some combination of the Rayleigh criterion and the aberration error. Hawkes (1972) gives a particularly clear description of how this combination leads to a value for the resolution of the microscope. Since this is very often *the* figure of merit used when investing hundreds of thousands of dollars in a TEM, it is essential that you understand that the definition is not exact. We will follow Hawkes' treatment in detail.

We take the combination of the Rayleigh and spherical aberration disks in quadrature

$$r = \left(r_{th}^2 + r_{sph}^2\right)^{1/2} \qquad [6.19]$$

We can thus find how r varies with β

$$r(\beta) = \left[\left(0.61\frac{\lambda}{\beta}\right)^2 + \left(C_s\beta^3\right)^2\right]^{1/2} \qquad [6.20]$$

Since the two terms vary differently with the aperture collection semiangle β, a compromise value exists when

$$\frac{dr(\beta)}{d\beta} = 0 = -2\frac{(0.61\,\lambda)^2}{\beta^3} + 6C_s^2\beta^5 \qquad [6.21]$$

From this equation the optimum (compromise) value of β is given by

$$\beta_{opt} = 0.77 \frac{\lambda^{\frac{1}{4}}}{C_s^{\frac{1}{4}}} \qquad [6.22]$$

(Sometimes, this compromise value is determined approximately by simply equating the equations for r_{th} and r_{sph}. A quick calculation for 100-keV electrons (λ = 0.0037 nm) for an instrument with C_s = 3 mm gives a β_{opt} value of 4.5 mrads.)

If this expression for β_{opt} is substituted into equation 6.18, we get a minimum value of $r(\beta)$

$$r_{min} \approx 0.91\left(C_s\lambda^3\right)^{1/4} \qquad [6.23]$$

This is the expression we want; it gives the practical resolution of the microscope.

Typically, the value for r_{min} is ~0.25–0.3 nm, but the high-resolution instruments have r_{min} ~0.15 nm. So, as we showed back in Figure 1.2, we can resolve rows of atoms, which in most crystalline materials have a separation close to r_{min} (although low index planes in some metals are still below the resolution). It's worth noting that since your eyes can resolve a distance of ~0.2 mm, then the maximum useful magnification of the best high-resolution TEM is ~10^6. Above this magnification, no more detail will be revealed.

Hawkes (1972) reminds us that the decision to add in quadrature back in equation 6.19 was arbitrary, and simply summing r_{th} and r_{sph} is another possible way to determine r_{min} (as we'll see in Section 28.7). But any way you combine the two terms for r leads to expressions of the form of equation 6.22 for β_{opt} and 6.23 for r_{min}. So in many textbooks you will see the use of letters A and B for the constants in these two equations, and often A and B are put equal to unity.

Remember, however, that we ignored any contribution of chromatic aberration. If you have a thick specimen in which most electrons lose 15–25 eV of energy, the chromatic aberration resolution limit given by equation 6.16 will dominate. For example, at 100 keV with β_{opt}, from equation 6.16 the value of r_{chr} is ~2 nm, and you can see all the available information at a magnification of 10^5. Under these circumstances, it doesn't matter what voltage you use or how low your C_s is. If you have thick specimens, then you'll have poor resolution. So how thick is thick? Well, it depends on the voltage and the mean free path for inelastic scatter and many other factors. For good high resolution at 100 keV the specimen should be ~30 nm, while at 300 keV you can probably get away with ~50 nm. A rule of thumb given by Sawyer and Grubb (1987) is that, for biological and polymeric specimens, the resolution limit is about one-tenth the specimen thickness.

6.6.C. Confusion in the Definitions of Resolution

If you're new to the subject, you don't have to read this section because it may confuse you still further, but if you've read other TEM texts you may have noticed discrepancies in the definitions of resolution. Now we used the expression for r_{sph}, measured at the Gaussian image plane. Strictly speaking, it is only under ideal conditions (i.e., if C_s = 0) that we should use the Gaussian image as a measure of the resolution limited by the lens. Also, Reimer (1993) further points out that it is only really correct to use the Gaussian image under *paraxial* conditions, that is, with a *very* small objective aperture. As we've already noted, in the TEM β is usually large enough that paraxial conditions do not apply, and so the disk of least confusion is the relevant feature from which to define the resolution, as shown back in Figure 6.11. If this is so, why did we choose the definition of r_{sph} as the radius of the disk in the Gaussian image plane?

The answer to this question is discussed by Hawkes (1972), who notes that defocusing the image slightly, to bring the disk of least confusion to the image plane, will indeed lead to a decrease in the value of B from 0.91 to 0.43. Hawkes also comments that since this latter value is smaller, manufacturers tend to use it to define the resolution of their instrument! However, this whole treatment of resolution assumes incoherent illumination, which is *not* the case in the TEM. Also, the resolution depends on the contrast in the image and how the lens transfers information from the object to the image. As a result, Hawkes concludes that B ~ 1 (from the Gaussian image) is "a more prudent choice" (i.e., closer to reality) than B = 0.43 (from the disk of least confusion) even though the disk of least confusion refers strictly to the conditions operating in the TEM.

Beware! Electron microscopists are generally rather careless in their definition of the term (r_{sph}) that they use to describe the effects of spherical aberration on resolution. We use the Gaussian image radius referred back at the object plane, i.e., $r_{sph} = C_s\beta^3$.

Part of this confusion arises out of an uncertainty whether to use the Gaussian image or the disk of least confusion for the definition of r_{sph} and it seems to be a matter of opinion which is more correct. Fortunately, it doesn't really matter too much since, in the end, the choice only alters the value of the constants A and B, which are often approximated to unity. For example, the value of A will depend on exactly which of the several quoted expressions was used for r_{sph}, i.e., if there was 0.25, 0.5, or 1 in front of $C_s\beta^3$. After these various terms are fed into equations and the value of β_{opt} is extracted, A only varies by about ±15%. A small variation in B will occur also, for the same reason.

We have consistently used the radius of the Airy disk and the radius of the aberration error. Obviously, it doesn't really matter whether you use the radius or the diameter, so long as you are consistent. Occasionally, however, you will find the Airy disk *radius* is used in combination with the *diameter* of the disk of least confusion or the Gaussian image, so this also contributes much to the discrepancy between various TEM texts.

6.7. DEPTH OF FOCUS AND DEPTH OF FIELD

Because of the poor lens quality we have to use small apertures to minimize their aberrations. This generally means that we cut out many of the electrons that would otherwise be gathered by the lens. Fortunately, our electron sources are so intense that we can live with substantially reduced beam currents hitting our specimen. In fact there are advantages to using small apertures, despite the price we pay in image intensity and resolution. These advantages come in the form of better depth of focus and better depth of field. These terms can be confusing and, once again, the TEM literature is variable. So we need to go back to light optics to find the correct definition of these terms.

Basically, we are trying to find out how much of the object (the specimen) is in focus at the same time and over what range the image is in focus. (This latter question is irrevelant in SEM and STEMs where we don't use lenses to form the image, so both terms are equivalent.) In TEM both terms are important.

The depth of field, D_{ob}, is the depth of sharpness in *object space.* It's the distance along the axis on both sides of the object plane within which the object can be moved without detectable loss of sharpness in the image. The depth of focus, D_{im}, is the depth of sharpness in the *image plane,* i.e., the distance along the axis on both sides of the image plane within which the image appears sharp (assuming the object plane and objective lens are fixed).

We can derive expressions for these definitions using Figure 6.14. Imagine that ray 1 originates at the highest point up the column where the object can appear to be in focus within the resolution, and that this ray arrives at the furthest point down the column where the image can appear to be in focus. Ray 2 represents the other extreme but travels at the same inclination to the optic axis. If these two rays appear to come from the same point (to within the resolution of the lens) the distances d_{ob} and d_{im} correspond to the smallest distances which we can resolve in the object or image, respectively. Now we can show that angles α_{im} and β_{ob}, which are both small, are given by

$$\alpha_{im} \sim \tan \alpha_{im} = \frac{\frac{d_{im}}{2}}{\frac{D_{im}}{2}} \qquad [6.24]$$

and

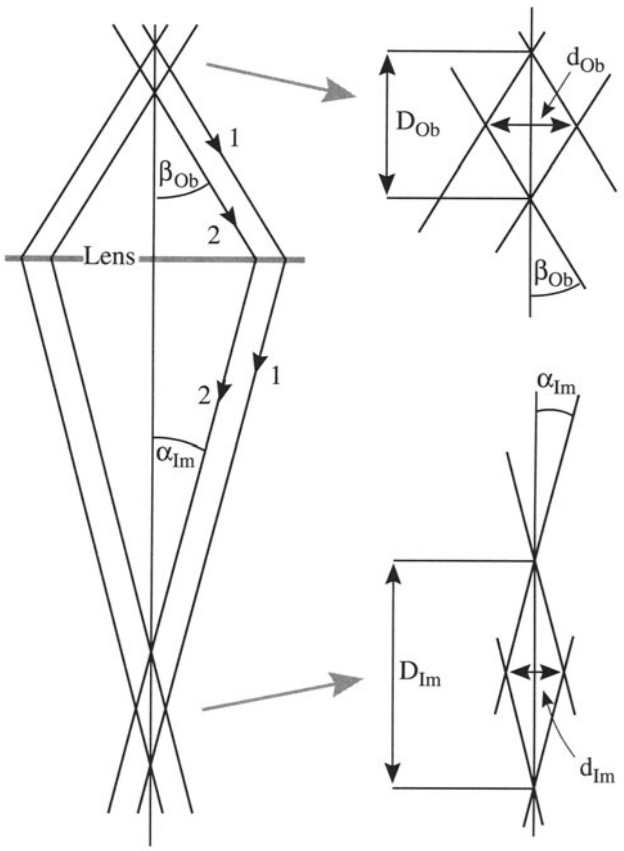

Figure 6.14. The definition of the depth of field and the depth of focus. Rays 1 and 2 represent the extremes of the ray paths that remain in focus when emerging ± $D_{Ob}/2$ either side of the specimen. The same rays define the depth of field over which the image is in focus ± $D_{Im}/2$ either side of the image plane. The resolution in the object is d_{Ob} and that in the image is d_{Im}.

$$\beta_{ob} \sim \tan\beta_{ob} = \frac{\frac{d_{ob}}{2}}{\frac{D_{ob}}{2}} \quad [6.25]$$

The angular magnification is thus

$$M_A = \frac{\alpha_{im}}{\beta_{ob}} \quad [6.26]$$

and the transverse magnification (which we simply call the magnification) is

$$M_T = \frac{d_{im}}{d_{ob}} \quad [6.27]$$

If these two magnifications are related in the usual way by

$$M_T = \frac{1}{M_A} \quad [6.28]$$

then we can say that the depth of focus is given by

$$D_{im} = \frac{d_{ob}}{\beta_{ob}} M_T^2 \quad [6.29]$$

and the depth of field is

$$D_{ob} = \frac{d_{ob}}{\beta_{ob}} \quad [6.30]$$

So we get a much increased depth of focus and field by using small apertures which reduce β. Notice that for a correct calculation of either D_{ob} or D_{im} you must be careful to select the right value of β. Under different circumstances the limiting semiangle is defined by the illumination aperture α (in the C2 lens) or the objective aperture β_o (in the objective lens). In thin specimens, which scatter weakly, most electrons emerge from the specimen in a cone closer to that defined by α_{im}, which is often small ($\sim 10^{-4}$ rad). In a thicker, more strongly scattering specimen, the objective aperture defines the angle and β_o is usually 10^{-2} rad.

For a collection semiangle, β_{ob}, of 10 mrad and a d_{ob} of 2 Å, equation 6.30 tells us that the depth of field will be 20 nm, i.e., a specimen of this thickness can all be in focus at the same time. If you only need 2-nm detail in your image, then you can use a specimen which is 200 nm thick and it will all be in focus.

If we want to see detail at the 2Å level, we need to use a magnification of about 500,000×. Equation 6.29 tells us that, under these conditions, the depth of focus will then be 5 km! If we only need to see 2 nm, we can use a magnification of 50,000× and the depth of focus is 5 m. In either case, we have tremendous latitude in where we put the photographic plate (or other recording media) because it would still record a focused image many meters either side of the screen. This explains why we can use a 35-mm camera which often sits below the final projector lens, and still get a focused image with a TV camera well below the plate camera. In fact, the TEM image would be in focus on the floor under the microscope if you projected it there but M_T would be different!

CHAPTER SUMMARY

We've introduced you to the principles of how an electromagnetic lens works, and how we describe its functions via simple ray diagrams. There are two principal operations: either we use the lens to create an image of an object, or we use it to bring parallel rays to a focus. We'll see in later chapters that the former operation is used to create magnified images of the specimen on the screen of the TEM and also used to create fine electron probes (demagnified images of the electron source) at the plane of the specimen in a STEM or SEM. The latter operation is used to create diffraction patterns in the back focal plane of the objective lens.

Our lenses are rather abysmal in their performance, resulting in the need for small limiting apertures. The lens aberrations limit the resolution of the microscope, and we need an optimum aperture to get the minimum resolution. The small apertures cut down the electron beam intensity, but also give us remarkable depth of focus and depth of field in our images and specimen, respectively.

Points to be wary of when reading about definitions of resolution:
Grundy and Jones (1976), Watt (1985), and Sawyer and Grubb (1987) use the Gaussian image radius referred back at the object plane, as we just did, i.e., $r_{sph} = C_s\beta^3$.
Beware: *Edington (1976) implies, and Thomas (1962), Murr (1970) and Hirsch* et al. *(1977) state, that* $C_s\beta^3$ *is the radius of the disk of least confusion, which it is not, since by definition it must be less than the Gaussian image radius (see Figure 6.11).*
Beware: *von Heimendahl (1980) defines the diameter of the disk of least confusion as* $C_s\beta^3$, *which is also incorrect.*

Points to be wary of when reading about depth of field and depth of focus
Bradbury et al. *(1989) give a particularly clear discussion of the topic.*
Grundy and Jones (1976), Thomas and Goringe (1979), and Sawyer and Grubb (1987) use the conventional definition given here.
Reimer (1993) uses the term "depth of focus" for the "depth of field," a rare inconsistency!
The terms are used interchangeably in SEM because there is no lens between the object and the image.

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